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THE EQUIVALENCE OF LINEAR SYSTEMS

DAVID JOHN CULLEN

The theory of polynomial realisations of a rational matrix developed by Rosenbrock [5] can be used to study systems of ordinary, linear, constant coefficient differential equations which relate the behaviour of a set of internal variables to that of the control inputs and the observation outputs of the system. Strict system equivalence is an equivalence relation on the set of polynomial realisations of a rational matrix which preserves the structure of the finite frequency modes of the dynamical systems associated with polynomial realisations and which also preserves their coupling to the system inputs and outputs.

Coppel [2, 3] has clarified the nature of the results of this theory by extending it to matrices with elements from the field of fractions of an arbitrary principal ideal domain. The results of the thesis are significant applications of this generalisation within systems theory itself.

Verghese [6], amongst others, has pointed out that if a dynamical system is formed as a result of switching caused by component failure in some other system then it becomes important to consider not only the finite frequency behaviour but also the impulsive solutions which may arise in response to the unconstrained nature of the inputs. Verghese used dynamical considerations to define 'operations of strong equivalence' on a special class of polynomial realisations, *generalised state space*

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realisations. These operations preserve both the finite and the infinite frequency modes of the dynamical systems and their coupling to the inputs and outputs. It was left as an open problem to define 'operations of strong equivalence' for arbitrary polynomial realisations. This problem is solved in the thesis.

We introduce and study, motivated by valuation theory, an equivalence relation on the set of polynomial realisations of a rational matrix, *strong system equivalence*, which preserves the structure of not only the finite, but also the infinite, frequency modes of the dynamical systems and their coupling to the system inputs and outputs.

We show that every polynomial realisation is strongly system equivalent to a generalised state space realisation and that two generalised state space realisations are strongly system equivalent if and only if they are *constant system equivalent*. That is, we show that generalised state space realisations and constant system equivalence play roles with respect to strong system equivalence analogous to those played by state space realisations and similarity with respect to strict system equivalence.

We also use the related algebraic concept of the localisation of a ring at a prime ideal to study the local properties of realisations. We show how the global results of Rosenbrock's theory can be constructed from this local information. This allows us to define a potentially useful equivalence relation, *stable system equivalence*, on the set of polynomial realisations of a rational matrix which preserves the structure of the modes of the dynamical systems and their coupling to the system inputs and outputs at a specified set of frequencies, for example, the unstable frequencies, but not necessarily elsewhere.

Many of the results of the theory of strong system equivalence were first obtained in collaboration with Professor B.D.O. Anderson and W.A. Coppel and have appeared in Anderson, Coppel and Cullen [*1*] and Coppel and Cullen [*4*]. However, the results are obtained in the thesis via a different approach.

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Centre for Mathematical Analysis,
Australian National University,
GPO Box 4,
Canberra,
ACT 2601,
Australia.