Seismology of Pulsating Ap Stars:
Results From The Past Decade, Prospects For The Next

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Abstract

Since the discovery of the first rapidly oscillating Ap (roAp) star in 1978 by Kurtz, this class of magnetic chemically-peculiar pulsators has grown to over two dozen. The eigenfrequency spectra of roAp stars (with periods of $\sim 6-15$ min) are consistent with nonradial p-modes of low degree ℓ and high overtone n, not unlike the Sun's five-minute oscillations seen in integrated light. However, unlike the Sun, the strong global dipole fields of roAp stars significantly affect the pulsations.

Although much of the effort in the last decade has been towards detecting new roAp candidates and refining the frequencies of known variables, initial "seismic" analyses have already yielded important results. Measurements of fundamental frequency spacings (ν_0) constrain the luminosities and radii of some roAp stars. In addition, mode splitting provides: (1) an independent determination of rotation period, even in the absence of longer-term light variations; (2) limits on the rotational inclination i and magnetic obliquity β; and (3) an indication of the relative *internal* field strengths of certain roAp stars. Very recently, the temperature – optical depth structure of the atmosphere of HR 3831 was inferred from optical and IR photometry of its oscillations.

Judging from current developments, the next decade promises exciting results on both observational and theoretical fronts. Several roAp stars have now been monitored for over a decade, allowing us to investigate long-term period changes due to evolution, binarity, etc. Eigenfrequency models for stars in the mass and radius range appropriate for Ap stars are becoming available, as well as explicit treatments of the perturbations due to magnetic fields. Armed with these, we may be able to place some roAp stars on a theoretical $\nu_0 - \delta\nu_{21}$ (or "asteroseismological H–R") diagram to derive independently their masses and main-sequence ages.

1. Introduction

Imagine a star with a magnetic field thousands of times stronger than that of the Sun; a star whose spectrum is dominated by lines of holmium; a star which – although it shouldn't be unstable to pulsation at all – is in fact vibrating persistently in something like the 25th overtone of its fundamental resonance period. Now imagine that such a bizarre object may actually reveal something useful about 'normal' main-sequence stars.

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What you've pictured is Pryzbylski's Star (HD 101065). Even though this star is possibly the most peculiar example of a class already considered "peculiar" relative to other A–F stars, its pulsations are not unique. HD 101065 was the first star to join the ranks of the *rapidly oscillating Ap* or *roAp* stars. These variables were all identified by their photometric oscillations with short periods ($\sim 4 \leq P \leq \sim 20$ min) – compared to expected fundamental periods of several hours – and low amplitudes ($\Delta B \leq 0.015$ mag).

The observed characteristics of roAp stars have been well summarised in various reviews, including Kurtz (1990), Matthews (1991), and Weiss (1986). Shibahashi (1987) has reviewed some of the theoretical aspects of these stars. Therefore, I'd like to concentrate here on the applications of roAp stars as probes of stellar astrophysics, through the techniques of asteroseismology.

2. The *p*-mode pulsation spectrum

Many of the roAp stars are multiperiodic; two of them – HR 1217 (HD 24712) and HD 60435 (see Figure 1) – have very rich eigenspectra in which it is clear that the frequencies are nearly equally spaced from each other. The regular frequency spacing seen in both the roAp and solar oscillations is the signature of nonradial *p*-mode pulsations where the modes are of high overtone; i.e. $n >> \ell$. According to the asymptotic pulsation theory of Tassoul (1980), the frequencies of such modes can be expanded in a fit of the form (Christensen-Dalsgaard 1986):

$$\nu_{n,\ell} \simeq \nu_0(n + \frac{\ell}{2}) - D_0[\ell(\ell+1)] + \text{higher order terms} \qquad (1)$$

where

$$\nu_0 = [2\int_0^R \frac{dr}{c(r)}]^{-1} = (\text{sound travel time across the star})^{-1} \qquad (2)$$

and

$$D_0 \propto [\frac{c(R)}{R} - \int_0^R \frac{dc}{dr}\frac{1}{r}dr]^{-1}. \qquad (3)$$

$c(r)$ is the local sound speed at radius r. The higher-order terms in equation (1) and the proportionality constant in equation (3) depend on the detailed structure of the star. Due to the properties of the expansion, the term D_0 can be expressed as $\frac{1}{6}(\nu_{n,\ell} - \nu_{n-1,\ell+2})$. The term in brackets is often designated $\delta\nu_{02}$: the fine-splitting between the same overtone of an $\ell = 0$ and and $\ell = 2$ mode.

The higher-order terms on the right side of equation (1) are small, so to first order, *one expects a comb of equally spaced frequencies. If a set of consecutive overtones* n is present for modes of *only* odd or even degree ℓ, then the spacing will be ν_0. If modes of *both* odd and even degree are present, the predicted spacing is $\nu_0/2$. The second-order term introduce slight deviations from this equal spacing. In Figure 1, ν_0 could be either 26 or 52 μHz (2.246 or 4.492 d^{-1}). Tentative mode identifications and other arguments suggest $\nu_0 \simeq 52$ μHz for HD 60435.

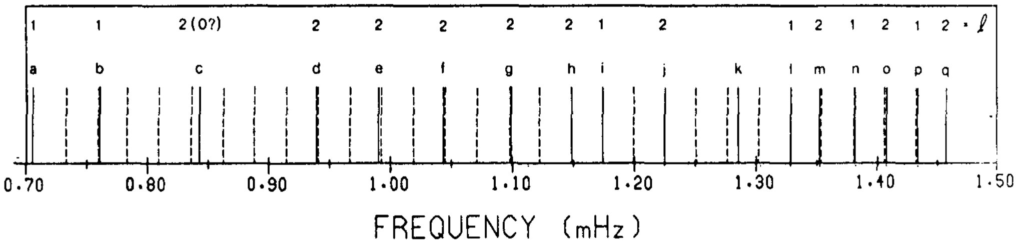

Figure 1. Oscillation frequencies identified in HD 60435, compared to the expected values for exactly equal spacing (from Matthews *et al.* 1987).

From equation (2), it is clear that the observed frequency spacing ν_0 depends on the average sound speed across the diameter of the star, which is proportional to its mean density. On the other hand, the fine structure of the frequency spectrum, due to the $\frac{1}{r}$ dependence in equation (3), is sensitive to conditions near the core of the star.

3. Measuring luminosities

A basic parameter like luminosity is extremely difficult to determine for an Ap star. The normal photometric indicators of log g do not apply to the heavily line-blanketed flux distributions of such stars.

However, we can exploit the dependence of ν_0 in equations (1) and (2) to place certain roAp stars on a theoretical H–R diagram. Gabriel *et al.* (1985) have shown that, for A–G V stars, $\nu_0 \sim 0.20\,(\frac{GM}{R^3})^{\frac{1}{2}}$. Hence, lines of constant ν_0 are straight lines in a plot of log L vs. log $T_{\rm eff}$; in fact, they are lines of nearly constant radius. By measuring ν_0 in a multiperiodic roAp star, and finding an independent determination of its $T_{\rm eff}$, one can in principle derive its luminosity and radius.

Of course, there are problems with this approach. Many of the multiperiodic roAp stars have only two or three observed frequencies, so it is unclear if the observed frequency spacings are in fact directly related to ν_0. Even in the cases of HD 60435 and HR 1217, whose p-mode eigenspectra are rich enough to make the pattern more obvious, there is an ambiguity. Is the observed spacing ν_0 (indicating modes of only even *or* odd ℓ) or $\frac{\nu_0}{2}$ (modes of even *and* odd ℓ)? Estimates of the evolutionary state of the star through long-term period changes may resolve the ambiguity (see the discussion in §6) but such measurements are themselves confused by the multiperiodicity of the star.

There may be another way to decide upon the actual value of ν_0. I had noticed (Matthews 1988, 1991) that the ratio $\frac{<\nu>}{\nu_0}$ observed in roAp stars tends to fall near two preferential values. Even the solar value of this ratio fits into this pattern. This tendency has endured as new stars have been added to the original sample. The ratios are shown in Table 1, in which values of mean frequency and possible spacings have been drawn from a variety of published sources. Be warned that, while some of these values are well established, others are still uncertain.

To first order, from equation (1), the ratio $\frac{<\nu>}{\nu_0}$ is roughly equal to the average overtone $< n >$ of the modes, as long as $n >> \ell$. In other words, it appears at first glance that both roAp stars and the Sun choose to pulsate in two restricted ranges of overtone, near $n \sim 25$ and 40. If confirmed by increasing numbers of roAp variables, or if a simple theoretical mechanism can explain it, then we may be able to use this rule to decide between two possible values of ν_0.

To demonstrate the diagnostic potential of ν_0, Kurtz (1992) has taken published estimates of ν_0 for roAp stars – wary of their ambiguities and varying quality – to try and locate these stars on the H–R diagram. He has adopted effective temperatures based on the H_β calibration by Moon & Dworetsky (1985), which should be relatively insensitive to metallicity. Kurtz's results are shown in Figure 2.

Table 1.

Star	$< \nu >$ (μHz)	ν_0 (μHz)	$\frac{< \nu >}{\nu_0}$
Sun	3300	135	24
33 Lib	2105	80	25
HD 101065	1373	58	24
γ Equ	1370	58	24
HD 60435	1380	52	26
10 Aql	1385	51	27
HD 203932	2800	72	39
HR 1217	2710	68	40
HD 218495	2240	61	37
HD 119027	1930	52	37
HD 166473	1870	50	37

Note that all the roAp stars appear to lie within the boundaries of the lower instability strip defined by the δ Scuti pulsators, arguing for a common κ driving mechanism. Also, the stars seem to be relatively evolved, lying well above the ZAMS. However, some of the values Kurtz takes as ν_0 would appear to be $\frac{\nu_0}{2}$ according to the pattern of Table 1. This would bring at least two of the "most highly evolved" stars much closer to the ZAMS, and one would fall outside the classical instability strip.

Fortunately, the question may soon be settled when accurate parallaxes become available for some of these stars, courtesy of the Hipparcos satellite. Then we can

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invert the process: specifying the luminosities and using ν_0 to determine $T_{\rm eff}$ to high accuracy. This will have the added benefit of testing the validity of the H_β temperature calibration for these peculiar stars.

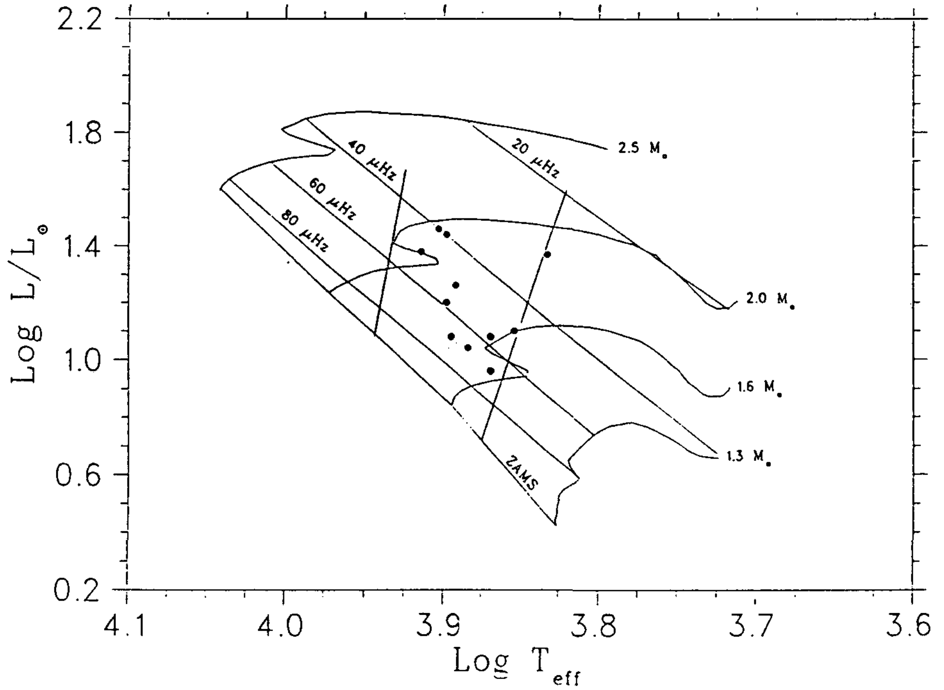

Figure 2. A theoretical H–R diagram (taken from Kurtz 1992) which shows tentative locations for roAp stars for which ν_0 has been estimated. The parallel diagonal lines are contours of constant ν_0; the borders of the δ Scuti instability strip are also shown.

4. Measuring internal magnetic fields?

The oscillations of several roAp stars exhibit periodic amplitude modulation and $\pi^{\rm rad}$ phase shifts synchronised with their magnetic variations. This behaviour can be explained by the *Oblique Pulsator Model* (Kurtz 1982, 1990): the nonradial pulsation pattern is tilted towards the magnetic dipole axis, rather than being aligned with the rotation axis. (The observations are consistent with zonal pulsation patterns; in particular, the $\ell = 1$, $m = 0$ mode). In the Fourier domain, a single pulsation frequency $\nu_{n,\ell}$ would be split into a multiplet of $2\ell + 1$ frequencies, centred about the rest-frame frequency and spaced by the rotation frequency $\Omega_{\rm rot}$. If the magnetic field were to completely dominate over rotation, then the magnetic and pulsation poles would be identical. In this case, the Fourier multiplet would be symmetric in amplitude.

However, the pulsations will also feel Coriolis forces due to the star's rotation which would tend to disrupt this alignment and bring the pattern back towards the

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rotational pole. When neither the Lorentz forces ($\propto B^2$) nor the Coriolis forces ($\propto \Omega_{\rm rot}$) totally dominate, the pulsation and magnetic poles will be at the same stellar longitude, but not exactly aligned. (In other words, the pulsation pole will be at a point directly between the magnetic and rotational poles.) In this instance, the Fourier multiplet is asymmetric in amplitude. The amplitude differences of the sidelobes contain information about the orientation of the magnetic field (i.e., inclination i and obliquity β) and its average strength through the pulsating *interior* of the star.

Analytical treatments of the magnetic perturbations on the spherical harmonics of the pulsation pattern have been developed by Dziembowski & Goode (1985), Kurtz, Shibahashi & Goode (1990), and Shibahashi & Takata (these proceedings), among others. Several roAp oscillations show triplet structure in the Fourier domain, consistent with $\ell = 1$. Estimates of the *relative* internal field strengths based on the triplet amplitudes are available for four stars (see, e.g., Matthews 1991): HD 6532, HR 1217, HD 60435 and HR 3831. The results suggest that HD 60435 has the weakest global field of the four[1] and HR 3831, the strongest.

5. Measuring atmospheric structure

(a) Critical frequencies

By considering the pulsations of an roAp star to be standing waves in a potential well (defined by the stellar structure), one can treat them as a Sturm-Liouville-type problem:

$$\frac{d^2v}{d^2r} + \frac{1}{c^2}\,[\sigma^2 - \phi(r)]\; v = 0 \tag{4}$$

valid for

$$\sigma^2 >> \ell(\ell+1)\,\frac{c^2}{r^2} \tag{5}$$

where σ = angular frequency, $\phi(r)$ = potential, $v = \rho^{\frac{1}{2}} c r \xi_r$, and ξ_r = radial displacement.

In this picture, there must be an acoustic cutoff frequency for p-modes above which the waves will not be reflected by the density falloff in the upper atmosphere, and the mode becomes evanescent. This *critical* frequency

$$\nu_{\rm crit} \sim \frac{c}{2H_p} \tag{6}$$

(where $\nu = \sigma/2\pi$ and H_p is the pressure scale height) is a sensitive function of the atmospheric structure.

Shibahashi & Saio (1985) first recognised that this was a potentially useful diagnostic of the atmosphere of an roAp star. They found that standard Kurucz model atmospheres for $2M_\odot$ stars produced values of $\nu_{\rm crit}$ of about 50–75% of the highest

[1] Attempts to measure the longitudinal field of HD 60435 by Landstreet and coworkers (*cf.* Matthews 1987; Matthews *et al.* 1987) have produced only low upper limits.

frequencies observed in HR 1217 and HD 60435. Since those frequencies were long-lived (recurring in observations spaced by years), they could not be evanescent modes. Hence, the true critical frequencies of these atmospheres must be higher than inferred from standard models (or those modes must experience incredibly strong driving!). Shibahashi & Saio noted that $\nu_{\rm crit}$ could be raised by *steepening* the $T-\tau$ gradient of the atmosphere; i.e, making the surface temperature $T(\tau=0)$ cooler than one would infer from $T_{\rm eff}$.

(b) Limb darkening

Support for this interpretation came from an unlikely source: rapid multicolour photometry of the roAp star HR 3831 (HD 83368). Observers had already recognised that the oscillation amplitude of an roAp star dropped rapidly with increasing wavelength – much more so than for other known pulsators. Based on their ESO K-band photometry of HR 3831, Matthews *et al.* (1990) argued that this was a result of limb darkening.

The pulsations of HR 3831 appear to be dominated by an $\ell=1$, $m=0$ mode. Simulations show that limb darkening acts as a filter to enhance the integrated amplitude of that mode, no matter how much the pulsation pole is inclined to the observer (except for 90°, where the amplitude is zero). Stronger limb darkening at shorter wavelengths leads to higher apparent amplitudes at those wavelengths. By measuring the amplitude at various wavelengths, one can estimate the limb darkening coefficients and infer the general features of the $T-\tau$ structure of the star's atmosphere.

Matthews *et al* (1992a, 1992b) have applied this approach to simultaneous rapid photometry of HR 3831 in *vbyRI* and *JHK* bandpasses. They find that the stellar atmosphere must have a steeper $T-\tau$ gradient than the solar atmosphere in order to account for the data, just as Shibahashi & Saio predicted. They also find evidence for a temperature inversion near $\log\tau_{5000}\sim-0.7$, suggesting an additional source of opacity at that optical depth.

6. The immediate future

(a) Period changes

As the baseline for observations of known roAp stars increases, we can begin to investigate their long-term behaviour. Heller & Kawaler (1988) have predicted the rates of period change for models of stars in the roAp mass range, based on evolution in and beyond the main-sequence band. Values of $|dP/dt|\sim10^{-12}-10^{-13}$ are expected.

However, an $O-C$ diagram spanning roughly 400,000 cycles of the oscillation of HD 101065 (Martinez & Kurtz 1990) implies a rate of period change about $10-100\times$ higher than expected through evolution (see Figure 3). If due to light-time-delay effects in a binary system, the unseen companion would have a mass $M\leq0.1M_{\rm Earth}$! Perhaps the $O-C$ diagram in Figure 3 doesn't represent a continuous smooth change in period, but abrupt shifts due to mode switching, which may be occurring in other roAp stars (e.g., HD 217522; Kreidl *et al.* 1991). Clearly, our unusual friend HD 101065 holds more surprises in store for us.

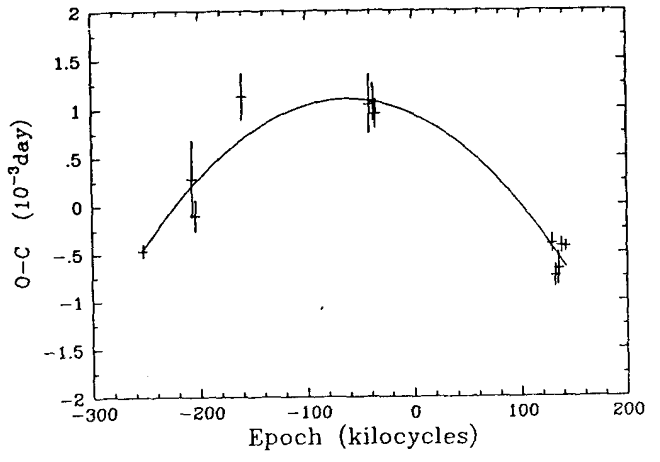

Figure 3. $O - C$ **diagram for the oscillations of HD 101065 (from Martinez & Kurtz 1990).**

HR 3831 also exhibits anomalous phase shifts over long intervals of time (Kurtz 1992), which are as yet unexplained. One must be cautious, though; cycle-count ambiguities are always a hazard when dealing with periods of a few minutes and gaps in the data sets of months or years. An apparent phase shift of f cycles could easily be $N + f$ cycles, where $N = \pm 1, 2, 3...$, depending on the accuracy of the period. Thus, the shape of the $O - C$ curve may be difficult to specify.

(b) Estimating main-sequence ages

Recall equations (2) – (3), which describe the frequencies of the high-overtone pulsations of roAp stars and the Sun. The fundamental frequency spacing ν_0 is a function of the star's mean density; hence, mass and radius, which change very slowly during the mean-sequence lifetime of a star. The deviations from that spacing, represented by D_0 (equation 3), are most sensitive to the sound-speed gradient $\frac{dc}{dr}$ near the core of the star. The convective core of an Ap star should be isothermal; the only changes in sound speed are due to the changing composition of the gas during core H-burning. Thus, the second-order spacing in the eigenfrequency spectrum should be sensitive to the main-sequence age of the star. Ulrych (1986) and Christensen-Dalsgaard (1986) suggested that the $\nu_0 - D_0$ plane would be a mass – age diagram.

Such "seismological H-R diagrams" have been generated for models in a narrow mass range around $1M_\odot$, but to date, only one solar-type p-mode oscillator has been detected with certainty: the Sun. On the other hand, observations of several roAp pulsators supply the data for such a diagram, but the theoretical calculations for masses near $2M_\odot$ were until recently unavailable.

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That situation is changing. Audard & Provost (1992) have produced eigenspectra for their models of 1.5 and 2.0 $M_{\odot}$ stars. They find values of ν_0 and D_0 which are quite consistent with the available roAp observations. Pedersen & Vandenberg (work in progress) are pursuing similar lines. The prospects are bright that we may be able to place roAp stars on seismological H-R diagrams to test our developing notions of their ages and other global properties.

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Discussion

T. J. KREIDL: HD 134214 also has $\nu > \nu_{crit}$ for its pulsation mode. It appears to be both unstable as far as its pulsation frequency *and* its phase coherence are concerned. Might the fact that $\nu > \nu_{crit}$ have an influence on the pulsational stability of roAp stars?

J. M. MATTHEWS: It's possible that some observed modes, particularly in roAp stars with rich eigenspectra, might be evanescent. Certainly a star like HD 60435 seems to show mode growth and decay on timescales of a day or less. If such a mode dies out and is re-excited, there is no reason for it to maintain its original phase. However, it should return with the same frequency. A pronounced shift in frequency would suggest that a *different* mode has appeared to replace it.

B. CARROLL: You mentioned that $\ell = 1$, $m = 0$ is the dominant mode for these stars. Is it that the roAp stars actually prefer this mode, or that you could not detect them in they were pulsating in another mode?

J. M. MATTHEWS: The evidence for this mode lies in the amplitude modulation and phase shifts which are observed. For maximum oscillation amplitudes to coincide with magnetic extrema and π^{rad} phase jumps to take place at zero cross-overs of the B_{eff} field, one must be seeing something very much like a zonal pulsation mode. A different pulsation pattern would produce quite different modulation and phase behaviour during the star's rotation cycle, as well as different fine-splitting in the Fourier domain.

C. AERTS: (1) The Coriolis force induces more than 1 spherical harmonic for one mode. Do you take this into account? (2) What are the rotation periods of these stars?

J. M. MATTHEWS: (1) We do allow for the additional spherical harmonics. In fact, recent observations of HR 3831 by Kurtz *et al.* (1992) require a decomposition of its oscillation amplitude spectrum into a series of 7 spherical harmonics. A treatment of the effects of rotation and magnetic field by Shibahashi & Takata (these proceedings) predicts this level of complexity. (2) Typical rotation periods of Ap stars range between a few days to a few weeks, although periods as long as years have been inferred for some stars. Generally, Ap stars are slower rotators than their "normal" counterparts.