

---

*Fifth Meeting, March 13th, 1896.*

---

Dr PEDDIE, President, in the Chair.

On Curve Tracings.

By G. DUTHIE, M.A.

Note on Four-Dimensional Figures.

By J. D. HÖPPNER.

By assuming that multiplication by a line is the true operation corresponding to the passing from space of  $n$  dimensions to space of  $n + 1$  dimensions we may arrive very simply at certain well-known results in geometry of higher dimensions.

A finite straight line may be symbolised by

$$1.a^1 + 2a^0$$

which indicates that the line consists of one line quantity and (+) is fixed by two non-dimensional quantities. For simplicity, we may write the above symbol in the form

$$a + 2.$$

The algebraic square of this quantity is

$$a^2 + 4a + 4$$

and this is also the symbol of a geometrical square, having 1 area bounded by 4 sides and 4 points. Raising  $(a + 2)$  to the 3rd power we obtain

$$a^3 + 6a^2 + 12a + 8$$

which represents 1 volume, bounded by 6 faces, 12 lines, and 8 points.

Passing on to a higher dimension, we obtain as the symbol of the four-dimensional figure

$$(a + 2)^4 = a^4 + 8a^3 + 24a^2 + 32a + 16$$

consisting of 1 four-dimensional region bounded by 8 cubes, 24 squares, 32 lines, and 16 points.

The method admits of other applications and of obvious extension to higher dimensions.