

# AN INTERCOMPARISON OF CONNECTED-ELEMENT INTERFEROMETER AND LUNAR LASER EARTH ROTATION PARAMETERS

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## ABSTRACT

The determination of Earth orientation parameters using techniques which promise an improvement in the precision of their measurements requires a careful evaluation of the systematic errors present in each series of observations. It is now possible to begin to intercompare two of these sets of observations of the rotation of the Earth (UT1-UTC) obtained routinely and independently. These are the data obtained from laser ranging to the Moon and those derived from the connected-element interferometer information of the U. S. Naval Observatory. Although these data are available for a relatively short time span, it is possible to begin to intercompare the observations in an effort to derive possible systematic errors existing in the data. This has been done using data from July 1979 to March 1980 in comparison with Bureau International de l'Heure (BIH) data. Using this information, the hypothesis that there is no correlation between the differences of the interferometer and laser ranging data with respect to the BIH must be rejected. This indicates that both sets of observations show that systematic errors do exist in the BIH data. The nature of the correlation of the two series is examined to establish possible models of the systematic errors. A much longer series of data from both techniques is necessary to improve the estimates, and the results will depend critically on adopted models used in the reductions and the degree of smoothing to which the raw data are subjected.

## INTRODUCTION

The use of new, high-precision techniques to observe the orientation of the Earth within a quasi-inertial reference frame promises the possibility of improved accuracy in the determination of polar motion and Universal Time. To achieve this improvement in accuracy, however, it is absolutely necessary to evaluate possible systematic errors which may exist in the parameters derived from the observations of each technique. As the reductions of the observations from

each new method become more sophisticated there is an increasing reliance on the models adopted in the reductions. These include attempts to model the effects of the Earth's variable atmosphere on the delay of electromagnetic signals, refraction, gravitational and non-gravitational forces affecting the motion of observed solar system bodies and many more. Errors inherent in each of these models contribute in different ways to the errors in the Earth orientation parameters derived from different techniques. To evaluate the effect of possible systematic errors then, it is necessary to intercompare observations made with different techniques in order to establish if differences do occur and where the cause of the differences may lie. Perhaps more important is the establishment of the existence of time variations in the systematic differences. This, of course, requires a sufficient period of overlapping observations for a good comparison. This type of intercomparison must be the concern of all those dealing with the representation of the orientation of the Earth in a fixed reference frame.

Recently, it has become possible to intercompare the results from two techniques whose observations may be available on a routine basis for the determination of the Earth's orientation. These are observations derived from laser ranging to the Moon and from the connected-element interferometer of the U. S. Naval Observatory. In this paper the UT1-UTC information derived from each of these techniques is compared in an effort to establish if systematic differences exist between these two sets of data and between them and the UT1-UTC determined by the Bureau International de l'Heure (BIH).

#### OBSERVATIONAL DATA - LUNAR LASER RANGING

The lunar laser ranging data chosen for this comparison is that from Fliegel, *et al.* (1981). The results listed in their work are smoothed with a Gaussian filter (using the function  $e^{-t^2/2\theta^2}$ ,  $\theta = 15$  days). BIH polar coordinates were used to correct the observed Universal Time (UT0) to UT1. According to the authors, these data are characterized by a precision of approximately one millisecond, and values are available at ten-day intervals.

#### OBSERVATIONAL DATA - CONNECTED-ELEMENT INTERFEROMETER

The UT0-UTC data derived from the observations of the U. S. Naval Observatory connected-element interferometer (McCarthy, *et al.*, 1979, 1980) which overlap the dates given for the lunar laser ranging data were chosen for the comparison. Systematic corrections derived from past observations made with the connected-element interferometer were applied to the data in the study. This correction consists only of an empirical annual periodic term of approximately five milliseconds in half-amplitude. This may be due to annual motion of one antenna with respect to the other or to possible systematic errors in the

modelling of the differential signal delay through the atmosphere. The latter possibility seems to be able to account for this effect, and it is expected that all of the inteferometer data will soon be re-reduced with an improved atmospheric model.

Smoothing identical to that used with the laser ranging data was applied to the interferometer data. This was done in an effort to minimize any artifacts due to the smoothing techniques involved. Values were determined for UT1-UTC for each tenth day for comparison with the laser data. Again, BIH polar coordinates were used to derive values of UT1-UTC from the observed UT0-UTC. The intercomparison then makes use of the residuals of UT1-UTC from the BIH values at ten-day intervals from 6 July 1979 through 16 March 1980, from both the laser-ranging data (LURE) and the connected-element radio interferometer (CERI). Figure 1 shows the data used in the study.

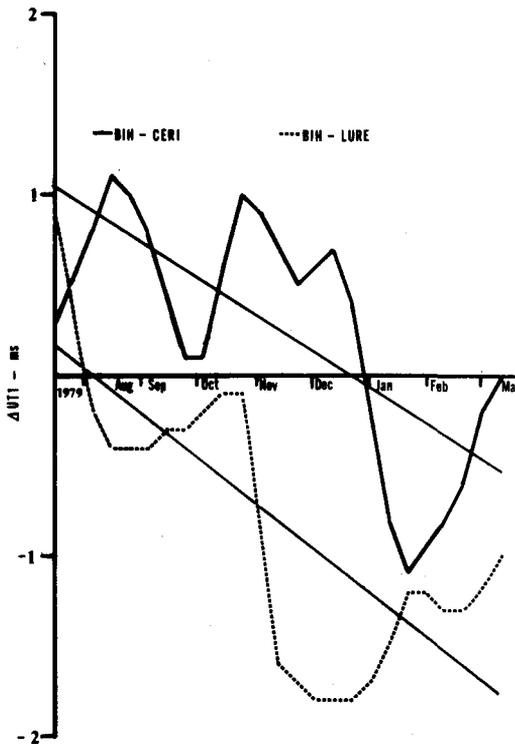


Figure 1. UT1-UTC residuals from connected-element interferometer (CERI) and lunar ranging (LURE).

DATA ANALYSIS MODEL

For the connected-element interferometer the data consist of points every ten days which were computed using

$$R = UT0_o - [UT1_B - (UT1-UT0)_B], \tag{1}$$

where  $UT0_o$  is the observed Universal Time (McCarthy, *et al.*, 1980),  $UT1_B$  is the smoothed BIH Circular D values and  $(UT1-UT0)_B$  is the correction for polar motion computed from the BIH Circular D smoothed polar coordinates. The expression (1) can be rewritten in the form

$$R = UT0_o + (UT1-UT0)_B - UT1_B, \tag{2}$$

representing the residuals in  $UT1$ . We can then compare the residuals,  $R$ , with similar values,  $R'$ , obtained from lunar laser ranging.

In treating the data we assume that

$$e_R = e_{Tr} + e_{Ts} + (e_x \sin \lambda - e_y \cos \lambda) \tan \phi/15$$

where  $e_{Ts}$  is the systematic error in the observed time residual,  $e_{Tr}$  is the random error, and  $e_x$  and  $e_y$  are the errors in the assumed polar coordinates,  $x$  and  $y$ . Similarly, for lunar laser ranging data,

$$e'_R = e'_{Tr} + e'_{Ts} + (e_x \sin \lambda' - e_y \cos \lambda') \tan \phi'/15.$$

If  $\lambda = 124^\circ.8333$ ,  $\lambda' = 104^\circ.0217$ ,  $\phi = -44^\circ.2100$ , and  $\phi' = 30^\circ.6717$ , and if we assume that the errors  $e_x$ ,  $e_y$ ,  $e_{Tr}$ , and  $e_{Ts}$  are not correlated with each other we find that

$$\sigma_{RR}^2 = \sum_1^N e_R^2/N = \sigma_{Tr}^2 + \sigma_{Ts}^2 + 0.0532^2 \sigma_x^2 + 0.0370^2 \sigma_y^2.$$

Now if we let

$$\begin{aligned} \sigma_x^2 &= \sigma_y^2 = \sigma_p^2, \text{ then} \\ \sigma_{RR}^2 &= \sigma_{Tr}^2 + \sigma_{Ts}^2 + 0.0649^2 \sigma_p^2. \end{aligned}$$

Similarly,

$$\sigma_{R'R'}^2 = \sigma_{Tr'}^2 + \sigma_{Ts'}^2 + 0.0395^2 \sigma_p^2.$$

and 
$$\sigma_{RR'}^2 = \sum_1^N e_R e_{R'}/N = \sigma_{TsTs'}^2 - 0.0439^2 \sigma_p^2.$$

The theoretical correlation coefficient is then found

$$\rho = \sigma_{RR'}^2 / (\sigma_{RR}^2 \sigma_{R'R'}^2)^{1/2}. \tag{3}$$

## LINEAR TRENDS

Least squares straight lines were fitted to each of the two sets of smoothed data. The results show an expected systematic constant difference between the two sets. This difference in the sense of LURE-CERI is 0.87 ms ( $\pm 0.29$  ms). This is not unexpected and could represent a systematic error in the adopted baselines of the interferometer or the adopted longitude of the laser ranging observatory at McDonald Observatory.

The least squares analysis also shows that the data can be represented by a linear secular variation during this period of time. For BIH-LURE this amounts to  $-0.0077$  ms/day ( $\pm 0.0014$ ) and for BIH-CERI is  $-0.0062$  ms/day ( $\pm 0.0014$ ). Both values for the linear terms are significant at the 99% significance level suggesting that we must reject the hypothesis that there is no linear secular difference between the BIH and the LURE or CERI results during this time. There is also no significant statistical difference between the CERI and the LURE estimates for this linear term. It appears that both the LURE and CERI data are in agreement with the hypothesis that the BIH UT1 data were subject to a secular systematic error during this period of time. This possibility reflects a systematic error in the length of the day derived by the BIH during this time.

## FURTHER STATISTICAL TESTS

The residuals from the least squares straight line fit to both sets of data are plotted in Figure 2. The correlation coefficient for these data was computed to be  $\rho = -0.37$ , and was found to be significant at the 95% level of significance. From this it appears that the residuals are significantly anti-correlated. Referring to the expression (3) we find that the correlation coefficient,  $\rho$ , may be approximated by

$$\rho = -0.0439^2 \sigma_p^2 / (\sigma^4 + 5.772 \times 10^{-3} \sigma_p^2 \sigma^2 + 6.572 \times 10^{-6} \sigma_p^4)^{1/2}$$

where we have assumed that  $\sigma^2 = \sigma_{Tr}^2 + \sigma_{Ts}^2 = \sigma_{Tr}'^2 + \sigma_{Ts}'^2$ , and that  $\sigma_{TsTs}'^2 = 0$ . Then if  $\rho = -0.37$  it can be shown that the apparent anti-correlation can be explained if  $\sigma = 0.09089 \sigma_p$ . Quantitatively, if  $\sigma_p = 0.0061$  as suggested by the BIH Annual Report for 1979 then  $\sigma = 0.0006$ . Thus it seems that the apparent anti-correlation can be readily explained by the known errors in the assumed polar coordinates of the BIH. The correlation coefficient is consistent with an error model in which the accuracy of the polar coordinates is  $\pm 0.0061$  and the accuracy of the LURE and the CERI residuals is  $0.0006$ .

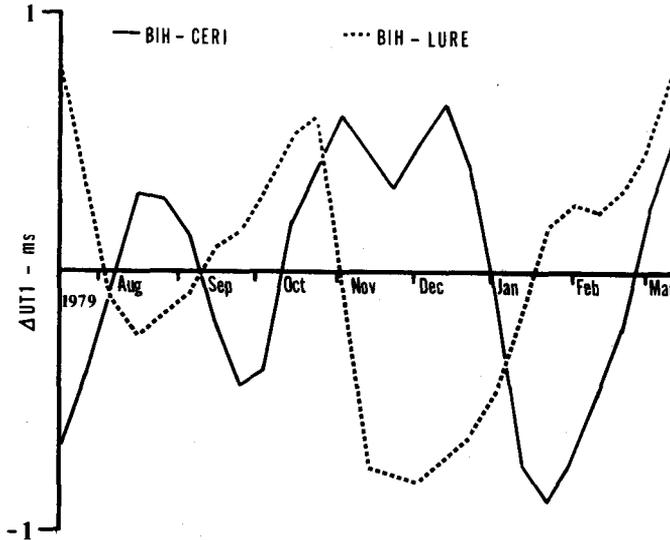


Figure 2. UT1-UTC residuals after adjustment for linear trend.

#### CONCLUSION

From the rather limited sample of overlapping observational Earth orientation data from lunar laser ranging and connected-element interferometry it is possible to come to the following conclusions for the time period from 6 July 1979 through 16 March 1980.

1. Significant constant bias exists in the smoothed residuals of the LURE and CERI data with respect to the BIH. A significant bias exists between the LURE and the CERI data.
2. Both the LURE and CERI data show the same linear drift in the smoothed BIH values for UT1-UTC. This indicates a systematic difference between the length of the day as computed by the BIH and that computed from CERI or LURE information.
3. After correcting the residuals for the systematic difference in the length of the day the residuals appear to be anti-correlated. Because of the geometry of the interferometer baseline a component of polar motion which is opposite in sign to that used with the lunar laser data must be applied to the interferometer results. This fact together with estimated accuracies of approximately  $\pm 0.006$  in the BIH polar coordinates and  $0.0006$  in the LURE and CERI values of UT0-UTC explain this effect.

It must be emphasized that these conclusions are drawn from a relatively small sample of smoothed data. While it is believed that these do represent indications of the nature of possible systematic effects in the observations of Earth orientation, it should be realized that a much larger sample of overlapping data must be used to evaluate these effects completely.

#### REFERENCES

- Fliegel, H. F., Williams, J. G., Dickey, J., 1981, "Earth Rotation from Lageos Lunar Laser Ranging Data Analysis", Paper presented at Third Annual NASA Geodynamics Program Review.
- McCarthy, D. D., Klepczynski, W. J., Kaplan, G. H., Josties, F. J., Branham, R. L., Westerhout, G., Johnston, K. J., Spencer, J. H., 1979, 1980, "Variation of Earth Orientation Parameters from Changes in the Orientation of the 35-km Baseline of the Green Bank Interferometer", Bureau International de l'Heure Annual Reports for 1979, 1980.

#### DISCUSSION

*Eckhardt : The Connected Element Radio-Interferometry (CERI) may provide useful information to geophysicists studying Earth tilts over long (for tilt measurements) baselines. This information is included in the differences between CERI and other Earth rotation measurement techniques. Another type of geophysical information of interest from CERI is the nature of the correlations between ionospheric corrections at either end of the baseline. Are these correlations being studied ?*

Klepczynski : Yes, the Naval Observatory is studying such ionospheric correlations.