BULL. AUSTRAL. MATH. SOC. 34K15, 34K30, 47H99, 47N20
VOL. 58 (1998) [525–526]

Semi-hyperbolic mappings in Banach spaces

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The definition of semi-hyperbolic dynamical systems generated by Lipschitz continuous and not necessarily invertible mappings in Banach spaces is presented. Like hyperbolic mappings, they involve a splitting into stable and unstable spaces, but a slight leakage from the strict invariance of the spaces is possible and the unstable subspaces are assumed to be finite dimensional.

Bi-shadowing is a combination of the concepts of shadowing and inverse shadowing and is usually used to compare pseudo-trajectories calculated by a computer with the true trajectories. The concept of bi-shadowing in a Banach space is defined and proved for semi-hyperbolic dynamical systems generated by Lipschitz mappings. As an application to the concept of bi-shadowing, linear delay differential equations are shown to be bi-shadowing with respect to pseudo-trajectories generated by nonlinear small perturbations of the linear delay equation. This shows robustness of solutions of the linear delay equation with respect to small nonlinear perturbations.

Complicated dynamical behaviour is often a consequence of the expansivity of a dynamical system. Semi-hyperbolic dynamical systems generated by Lipschitz mappings on a Banach space are shown to be exponentially expansive, and explicit rates of expansion are determined. The result is applied to a nonsmooth noninvertible system generated by delay differential equation.

It is shown that semi-hyperbolic mappings are locally ψ-contracting, where ψ is the Hausdorff measure of noncompactness, and that a linear operator is semi-hyperbolic if and only if it is ψ-contracting and has no spectral values on the unit circle. The definition of ψ-bi-shadowing is given and it is shown that semi-hyperbolic mappings in Banach spaces are ψ-bi-shadowing with respect to locally condensing continuous comparison mappings. The result is applied to linear delay differential equations of neutral type with nonsmooth perturbations.

Received 2nd March, 1998
Thesis submitted to Deakin University October, 1997. Degree approved March, 1998. Supervisors: Professor Peter E. Kloeden and Professor Alexei V. Pokrovskii.

Finally, it is shown that a small delay perturbation of an ordinary differential equation with a homoclinic trajectory is 'chaotic' and exhibits a complicated behaviour.

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https://doi.org/10.1017/S0004972700032512 Published online by Cambridge University Press