Heterogeneity of Beliefs and Trade in Experimental Asset Markets

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Abstract

We investigate the relationship between traders’ expectations and market outcomes with experimental asset market data. The data show that those who have high price expectations buy more frequently and submit higher bids, and those who hold low price expectations sell more frequently and submit lower bids. Traders who have more accurate expectations achieve greater earnings. Simulations using only belief data reproduce the pricing patterns observed in the market well, indicating that the heterogeneity of expectations is a key to explaining market activity.

I. Introduction

It has long been recognized that expectations are an important input in economic choices. One obvious example of this is asset trading, in which purchase and sale decisions are presumed to be governed, at least in part, by expectations of future prices. For example, if expectations are homogeneous, as is frequently assumed in theoretical work, and individuals do not differ in terms of risk attitudes, a no-trade theorem typically applies (Milgrom and Stokey (1982), Tirole (1982)). On the other hand, a substantial literature suggests that the heterogeneity of expectations is the key to explaining most trades in asset markets (see the survey of Hong and Stein (2007)). Varian ((1992), p. 668) contends that “just as it
takes differences of opinion to make horse races, it is likely that a substantial portion of trade in actual financial markets is due to different ... beliefs.”¹ Different beliefs can result from heterogeneity in information, but they can also result from differing interpretations of the same information, differences in tastes, or different views about the future. Keynes (1936) highlights the distinction between tastes and beliefs in his famous “beauty contest” metaphor, where he emphasizes the differences between one’s own tastes and one’s beliefs about the tastes of others.² In their model of the beauty contest, Biais and Bossaerts (1998, p. 309) conjecture that “disagreement may lead to trades in which agents with low private valuations buy the asset from agents with higher valuations, because of optimism about the resale potential of the asset.”³ Testable implications of the literature on heterogeneous beliefs are that shares are purchased and held by the more optimistic traders (Hirshleifer 1975, Harrison and Kreps 1978), that belief dispersion implies an increase in transaction volume (Varian 1989), and that price levels are greater when short sales are prohibited (Miller 1977).

In this paper, we conduct an analysis of experimental asset market data, in which we consider the relationship between individual beliefs and decisions. Theories of heterogeneous beliefs, heterogeneous agent models, and learning-to-forecast experimental designs that elicit beliefs and incorporate them as part of trading strategies (e.g., Marimon, Spear, and Sunder 1993, Brock and Hommes 1997, 1998, Hommes, Sonnemans, Tuinstra, and van de Velden 2005, Hommes 2006, LeBaron 2006, and Hommes and Lux 2013) build on the assumption that individual expectations and actions are aligned.⁴ So far, the empirical evidence supporting these conjectures is scarce and a direct test seems overdue. In nonmarket experiments, empirical evidence has been reported that supports the conjecture that actions are rational given their beliefs, at least to some degree. Nyarko and Schotter (2002) conclude that in 2-person, constant-sum games, subjects usually play a best response to their stated beliefs.⁵

¹Throughout the paper, we employ the terms “expectation” and “belief” synonymously.
²In her experimental study of the beauty contest game, Nagel (1995) suggests that differences in subjects’ responses under identical information can result from heterogeneous beliefs about the other subjects’ cognitive abilities.
³Biais and Bossaerts (1998) call these trades controversial since each trader thinks that the other party makes a bad deal, given their speculative valuations. The presence of similar, speculative trades are also suggested in Harrison and Kreps (1978) and Scheinkman and Xiong (2003), where the latter, theoretical study links heterogeneous beliefs with overconfidence. The model of Barberis, Greenwood, Jin, and Shleifer (2015) suggests that most changes in prices may be driven by changes in beliefs.
⁴Key features of the heterogeneous agent model of Brock and Hommes (1997), (1998) are that agents have the same information, but nevertheless have different opinions and heterogeneous expectations about the price of a risky asset, and switch between different forecasting rules based upon their relative performance. Hommes (2006) and LeBaron (2006) provide literature reviews of heterogeneous agent models. The learning-to-forecast experimental design pioneered by the work of Marimon et al. (1993), who study a macroeconomic Overlapping Generations model, has also been applied to asset pricing experiments (with a long-lived asset and a constant fundamental) in Hommes et al. (2005). Another, more recent, relevant contribution is Bao, Hommes, and Makarewicz (2017), who compare trading behavior and individual expectations in learning-to-forecast and learning-to-optimize asset pricing experiments. They find a positive correlation between individuals’ beliefs about future returns and trading decisions, though fewer than 25% of the subjects trade optimally (i.e., consistent with mean-variance utility maximization) conditional on expectations.
⁵See also the literature review in Schotter and Trevino (2014).
Neugebauer, Perote, Schmidt, and Loos (2009) report evidence from a linear public goods game, and they find that subjects’ beliefs about the actions of others are highly correlated with their own actions, although their own actions are closer to equilibrium play than these beliefs (Fischbacher and Gächter (2010) report confirming evidence). Blanco, Engelmann, Koch, and Normann (2010) suggest that first movers best respond given their elicited beliefs about second movers in a sequential prisoners’ dilemma game. In ultimatum games, Trautmann and van de Kuilen (2014) obtain significant positive correlations between beliefs and actions, using different approaches of belief elicitation. On the other hand, Costa-Gomes and Weizsäcker (2008) and Rey-Biel (2009) find that subjects in $3 \times 3$ matrix games play a best response to their beliefs only infrequently. Lahav (2015) also finds some inconsistency between beliefs and actions in beauty contest experiments with belief elicitation. The correlation between the dispersion of beliefs and activity in experimental asset markets has also been investigated in recent work that shows that the manipulation of subjects’ beliefs in the direction of more homogeneity leads to smaller mispricing in experimental asset markets (Kirchler, Huber, and Stöckl (2012), Cheung, Hedegaard, and Palan (2014)).

One advantage of experimental methods is that expectations can be measured directly using protocols in which individuals have an incentive to truthfully report their beliefs. In this paper, we reanalyze the data collected by Haruvy, Lahav, and Noussair (HLN) (2007). HLN elicit predictions of future prices from participants in repeated experimental asset markets with a 15-period horizon. They find that price expectations in the laboratory are formed in an adaptive, backward-looking manner. With repetition, the average expectation converges to the expected dividend value, but does so only after the price has converged close to the expected dividend value. HLN (2007) do not discuss the dispersion and heterogeneity of beliefs in detail. The work reported here extends the work of HLN in view of several objectives. The first objective is to study heterogeneous beliefs in a market environment, where subjects have identical information regarding the risks and returns of the asset, and to consider the fundamental assumption of finance theory that subjects make trades in accordance with their expectations. In this context, we report the elicited beliefs about both short- and long-term future prices and explore the interaction between the two. The second is to explore a possible connection between accurate beliefs and trading profits. The third is to study how

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6 A large number of experimental studies in finance manipulate beliefs by offering traders different pieces of information about the uncertain state of the world (see the literature survey by Sunder (1995)). More recently, Hey and Morone (2004) and Palfrey and Wang (2012) report that mispricing can occur in markets with noisy private or public information, respectively. In contrast to this literature, however, our analysis uses elicited expectations rather than manipulated expectations.

7 Confirming empirical evidence has been reported in a recent study of naturally occurring markets by Greenwood and Shleifer (2014). They use time series of six surveys of expectations as proxies for beliefs and investigate the correlation of market expectations with market returns. Their data reject the rational expectation hypothesis, according to which expected future returns should equal expected realized returns. Their survey data show, like HLN (2007), that expectations of future returns positively correlate with past returns and past price levels and, therefore, correlate negatively with model-based expected returns.

8 In this paper, we use the terminology expected dividend value to denote the expected value of the future stream of dividends that a unit of the asset yields to its holder. In much of the experimental literature, the expected dividend value is termed the “fundamental value.”
accurately the market activity can be simulated by using belief and order quantity data only. The fourth is to test the market implications of the heterogeneity of beliefs on market prices and transaction volume.9

Our assets have the feature that the expected value of the future stream of dividends declines over time. This decreasing trend in expected dividend value characterizes some assets in the field, such as options, bonds that have a coupon payment, assets that depreciate in value, and nonrenewable extractable resources. Our view is that a decreasing trajectory, which is conducive to mispricing in the lab, is suitable for the study of belief dispersion. A constant fundamental value exhibits less of a tendency to misprice (Noussair, Robin, and Ruffieux (2001), Kirchler et al. (2012)) and, thus, might be likely to exhibit less heterogeneity of expectations. Some authors (Lei et al. (2001), Lei and Vesely (2009), Kirchler et al. (2012), and Charness and Neugebauer (2018)) have noted that declining expected dividend values are relatively challenging for experimental subjects to comprehend the first time they are exposed to them. This suggests that any connection between expectations and trading behavior would presumably be at least as strong in environments that are more easily understood to the decision maker. Thus, a link between expectations and trading behavior would be particularly noteworthy in our environment.

The rest of the paper is organized as follows: In Section II, we briefly survey the data and our statistical approach. In Section III, we report our findings. In Section IV, we describe the results of simulation market behavior using belief data. Section V concludes.

II. Data and Procedures

A. The Data

In our test of heterogeneous beliefs, we analyze the experimental data of HLN (2007), who elicited individual beliefs during six sessions of experimental asset markets in the standard single asset market design of Smith, Suchanek, and Williams (1988).10 In each session, n = 9 subjects participated in

9Gillette, Stevens, Watts, and Williams (1999) also record the heterogeneity of beliefs in an asset market experiment. They report regression results, according to which trading volume decreases in the contemporaneous dispersion in forecasts. Their experimental setting is quite different from ours, as their focus is on the updating of beliefs about the final expected dividend value following public announcements (that affect cumulative expected dividend value). The cumulative expected dividend value is announced every third period, but only the final expected dividend value is paid out at the end of the experiment (i.e., after 15 periods). Gillette et al. elicit (long-term) beliefs about the final expected dividend value in each period. Since all information is public, they argue, beliefs should be homogeneous so that in theory no trade should occur in their markets. They conjecture that trading follows from the speculation motive. However, Lei, Noussair, and Plott (2001) show that some trading occurs even in the absence of speculation opportunities. In this study, we provide a rationale for trading and accompanying evidence by linking observed transaction volume and price to individual beliefs (short term and long term) and the overall heterogeneity of beliefs.

10HLN (2007) use call market trading rules, while Smith et al. (1988) employ continuous double auction rules. However, according to Van Boening, Williams, and LaMaster (1993), pricing behavior is not different in asset markets conducted under the two rules. Call markets, in which there is only one uniform price for all transactions in a period, are more conducive to studying price predictions.
Each market consisted of $T = 15$ periods of a call auction. Thus, the data set includes 345 market periods.

The structure follows design 4 of Smith et al. (1988). Subjects were endowed with cash and shares. At the end of each period, a dividend was independently drawn from the set \{0, 4, 14, 30\} francs, each number being equally likely to be chosen. Given 15 dividend draws per market, the initial expected value of the future dividend stream of each share was 180 francs. The expected value of the initial endowment in each market was 652 francs for each subject, including shares and cash. Three subjects were endowed with one share of asset, three other subjects were endowed with two shares, and the last three subjects were endowed with three shares. Other than shares, the remainder of the initial endowment value of 652 francs was cash. Dividend payments and revenue from sales (over the course of a market) increased cash holdings, while expenditures of purchases decreased them. Throughout the experiment, no borrowing was possible; asset purchases on margin and short sales were disabled.

In each period, subjects submitted one “bid” (an offer to buy) and one “ask” (an offer to sell) order. The ask consisted of a sale price and a quantity of shares offered for sale, and a bid consisted of a proposed purchase price and a quantity of shares demanded for purchase. Both quantities were required to be nonnegative, but could equal 0, and the proposed sale price had to exceed the proposed purchase price. At the end of each period, the market cleared, the market price was determined by the intersection of submitted market demand and market supply, and shares were exchanged between winning sellers and buyers at the market price. The data contain 2,009 submitted bid orders and 1,554 ask orders of single or multiple units. The majority of positive order submissions were for single units, including 58% of bids and 61% of asks.

Cash and asset holdings were reinitialized at the same starting levels at the beginning of each of the four markets. Within a market, participants carried over cash and shares from one period to the next.

Beliefs were elicited by providing monetary incentives to subjects to reveal their expectations about the trajectory of future prices. At the beginning of each period $t = \{1, 2, \ldots, 15\}$, before submission of bids and asks for period $t$, each subject predicted the clearing prices of both current and future periods (all periods $s \geq t$) within the current market $m = \{1, \ldots, 4\}$. Thus, the task involved 120 predicted prices per subject and market, 15 predictions of the current period (below, we refer to short-term beliefs) and 105 predictions of future periods (below, we refer to the aggregate as long-term beliefs). Short-term beliefs were about equally often above (51%) and below (49%) the actual realized price in markets 2–4, but

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11One session had only eight subjects, and another session had only three repeated markets.
12To make sure that all subjects recognized the role of the expected per-period dividend in the determination of the overall expected dividend value of the asset, each subject received a table that specified the expected dividend value of the asset at the end of each period.
13Submissions for two and three units occurred with relative frequencies 20% and 8% for bids and 20% and 11% for asks when positive quantities were submitted, respectively. Finally, 31% of bids and 39% of asks involved zero quantities when positive submissions were possible (accounting for the individual price expectations and liquidity constraints).
more often below (61%) the realized price in market 1. Monetary incentives offered salient rewards to subjects for the accuracy of each prediction.

At the end of the last market, subjects received their total earnings of all markets in cash. Total earnings included payment received for the prediction task and cash balance in their possession at the end of the repetition. The cash balance depended on the collected dividends and the capital gains from trade.

Market prices in the data exhibit the bubble and crash pattern typically observed in this experimental design (see the survey by Palan (2013)), starting below the expected dividend value at the beginning of the market, increasing gradually and crossing the value after a few periods. After another few periods during which the asset price exceeds the expected dividend value, it reaches a peak. The price subsequently plummets to roughly track its fundamental value in the end phase of the market. This pattern is observed in all sessions when traders are inexperienced (market 1). With increased experience in repeated markets, the magnitude of the bubble shrinks, the asset price peaks earlier, and it more closely tracks its expected dividend value (Figure 4).

B. Measures Used in the Analysis

We construct a number of variables from the data. In particular, we organize the elicited individual expectations about future prices from each period. Individual beliefs are characterized by the price level they indicate, both in the short term, defined as the upcoming market period, and the long term, which is the average over the remaining periods in the life of the asset.

1. Ranked Short-Term Beliefs

For each period, we rank subjects by their expectations on future market prices, from highest to lowest. The lowest price expectation is assigned rank $[.,] = 1$, the second lowest rank $[.,] = 2$, etc. In case of a draw, we assign the mid-rank. For the analysis of short-term belief in period $t$, we use the price-expectation rank of subject $i$ within group $g$ (denoting one of the experimental sessions 1–6) for period $t$ in market $m$. We write $i$’s short-term belief (STB) as

$$\text{STB}_{mit} = B_{mit}^g,$$

where $B_{mit}^g$ denotes the belief of subject $i$ from group $g$ during market $m$ (denoting the current market) and period $t$ (which denotes both the submission and forecast period). If subject $i$’s short-term price expectation submitted in period $t$ is largest within group $g$, we write rank[$\text{STB}_{mit}$] = 9. Consequently, the high ranks indicate a high level of optimism, while the low ranks indicate less optimism about the price in the current period. The ranking procedure serves two main purposes. First, it provides a measure of the optimism of beliefs that is not sensitive to the declining time trend of expected dividend values, as the range of ranks is always constant. Second, it is unbiased by outliers (e.g., extremely high or low beliefs).

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14 Short- and long-term beliefs were below the fundamental values 42% and 23% of the time, respectively. Long-term beliefs were above the actual observed price three times as often as they were below. In other words, the average long-term belief was too optimistic.
2. Ranked Long-Term Beliefs

Recall that each trader submits beliefs for the current period as well as for all of the remaining future periods of the current 15-period market. As we are interested in the ranked long-term beliefs, we define the long-term belief (LTB) of trader $i = \{1, 2, \ldots, 9\}$ as the average deviation of the trader’s beliefs from the asset value in the remaining periods as follows:

$$\text{LTB}_{mgit} = \frac{1}{T-t} \sum_{k=1}^{T-t} \frac{B_{mg,t+k} - f_{t+k}}{f_{t+k}}. \quad (2)$$

$B_{mgit}$ denotes the subject’s price forecast for the period $t+k$ ($1 \leq k \leq T-t$), submitted in the current period $t$. $T = 15$ is the total number of periods in a market, and $f_{t+k}$ is the expected dividend value in the forecasted period. As with short-term beliefs, we rank the long-term beliefs and, therefore, write $\text{rank}[\text{LTB}_{mgit}] = 1$ if the LTB of subject $i$ is the lowest number within group $g$.

3. Belief Dispersion

We use the coefficient of variation as a measure for current belief dispersion within a trader cohort. This measure accounts for the changing expected dividend value over time better than the simple standard deviation. Hence, we define the short-term belief dispersion as the ratio of the standard deviation of short-term beliefs in group $g$ and its average in the same period.

$$\text{STBD}_{mgt} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left( \frac{B_{m,i}^{mgt} - \overline{B}_{mgt}^{m,i}}{\overline{B}_{mgt}^{m,i}} \right)^2}, \quad (3)$$

where $\overline{B}_{mgt}^{m,i} = \sum_{i=1}^{n} B_{mgit}^{m,i}/n$ and $n$ is the number of participants in group $g$. The long-term belief dispersion, $\text{LTBD}_{mgt}$, is defined in the same way using long-term beliefs.

4. Bubble Measure: Relative Deviation

To measure bubble magnitudes, we use the relative deviation (Stöckl, Huber, and Kirchler (2010)) of the price trajectory. The relative deviation (RD) is calculated as follows:

$$\text{RD}_{mg} = \frac{1}{T} \sum_{t=1}^{T} \frac{P_{mgt} - f_{t}}{\overline{f}}, \quad (4)$$

where $P_{mgt}$ denotes the clearing price in group $g$ and market $m$, $f_{t}$ is the fundamental value in period $t$, and $\overline{f}$ is the average dividend value across the 15 trading periods. Positive values of the measure indicate overpricing and negative values reveal underpricing, relative to fundamentals. A low absolute value means that prices are adhering closely to fundamentals.
III. Results

This section is organized into three parts. The first considers the relationship between reported expectations, on the one hand, and individual decisions and market behavior, on the other. The second covers the connection between the accuracy of beliefs and earnings. The third concerns the relationship between belief dispersion and market outcomes.

A. Individual Beliefs and Behavior

In this section, we report a number of findings concerning the connection between individual expectations and subsequent actions. We study both short- and long-term beliefs and show how both affect traders’ buying and selling behavior. Our first observation summarizes our results on the relationship between individual expectations and purchase/sale decisions.

Observation 1. Subjects who believe that prices will be higher tend to be buyers and subjects who believe that prices will be lower tend to be sellers, in the market. Consequently, share holdings are positively correlated with beliefs.

Support for Observation 1. Figure 1 shows subjects’ net purchases according to their ranked short-term beliefs. Net purchases of those individuals with low ranked short-term beliefs (less optimistic subjects) are lower than those of highly ranked ones (more optimistic subjects). Figure 1 suggests that more optimistic subjects purchase shares from the less optimistic ones.

Figure 1 shows the relationship between the net purchases of individual traders and the ranking of their beliefs in a given period. The horizontal axis classifies individuals based on their submitted short-term belief compared to others in the same period. 1 indicates the lowest predicted price and 9 the highest. The vertical axis indicates the individual’s net purchases in a period. A positive value means that the individual purchased more units than he or she sold in the period. The figure includes the data for each period and each participant.

Figure 2 shows subjects’ average share holdings per period according to their ranked short-term beliefs. As Figure 2 shows, more optimistic investors hold more shares than less optimistic ones. Interestingly, both graphs are not linear. Figure 1 suggests that more optimistic traders make purchases while relatively pessimistic ones make sales. Figure 2 suggests that there is a limit to the extent of the purchases that optimists can make, perhaps due to cash constraints.
Figure 2 illustrates the relationship between short-term beliefs and the number of shares held. The horizontal axis classifies individuals based on their submitted short-term belief compared to others in the same period. 1 indicates the lowest predicted price and 9 the highest. The vertical axis shows the number of shares held by the average individual with each ranking.

To check for statistical significance of these observations, we report generalized least squares (GLS) regression results, where net purchases ($\Delta S_{mgit}$) and share holdings ($\Delta S_{mgit}$) are the dependent variables and the ranked short-term belief is the independent variable. The results are summarized in Table 1 for short-term beliefs and in Table 2 for long-term beliefs.

The results reported in Table 1 clearly show that ranked short-term belief is a significant predictor of net purchases and total share holdings, with more positive beliefs associated with greater net purchases. The $\beta$ coefficient is significant at the 5% level in every market and across markets for either dependent variable. Long-term beliefs have a similar relationship and are significant in markets 1, 2, and 4, for both net purchases and total holdings, and in market 3 for share holdings.

Observation 1 supports theoretical work that assumes that individual beliefs can be heterogeneous, even with all traders having identical information, and that more optimistic agents hold more shares in the market (e.g., Miller (1977),

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15We conduct regressions on these ranked beliefs using panel data methods. We pretest the data using the Hausman specification test (Wooldridge (2010)) and choose the specification of our generalized least squares (GLS) regression model depending on the results of the test. If the null hypothesis of the Hausman test that a random-effect specification is the appropriate model is rejected at a 5% significance level, we report the results of the fixed-effects model and indicate this specification with the subscript “f”. Otherwise, we report the results of the random-effects model. The GLS model accounts for group effects and is denoted as follows:

$$y_{mgit} = \alpha + x_{mgit} \beta + \eta_{mg} + \epsilon_{mgit},$$

where $\alpha$ and $\beta$ are the intercept and slope of the regression, respectively, and the indices $m$, $g$, $i$, and $t$ denote the market repetition, the session (or group), the trader, and the period, respectively. $x_{mgit}$ is the explanatory variable (e.g., within-session rank of the subject’s short-term price expectation), $y_{mgit}$ is the dependent variable (e.g., the subject’s net purchases in period $t$ of market $m$), and $\epsilon_{mgit}$ is an independent and identically distributed (i.i.d.) error term. With the fixed-effects model, the term $\eta_{mg}$ denotes the group-specific, time-invariant error term. In contrast to the fixed-effects model, the random-effects model supposes that the time-invariant error term $\eta_{mg}$ in the regression equation is also i.i.d. and, therefore, that the two error terms are mutually independent. We estimate the GLS model using Stata.
Harrison and Kreps (1978)). The observed positive relationship between share holdings and short-term beliefs, however, may have an endogenous element. While it can be presumed that optimistic traders increase their holdings and, therefore, tend to hold more units, it is possible that large shareholders become more optimistic because of wishful thinking (Forsythe, Rietz, and Ross (1999)), but we find no support of such a relationship in the data.16

16To check the “wishful thinking” hypothesis, we test whether initial individual beliefs depend on the initial (random) endowment of shares. The GLS regression of ranked short-term beliefs on the initial share holdings for the first period shows no significant positive effect; the \( t \)-statistic is \(-1.17\).
An analogous result to Observation 1 applies to the submission of bids and asks. Observation 2 reports that there is a positive relationship between beliefs and offer levels. In our analyses, we consider bids and asks of subjects who may expect a trade, that is, those whose bids exceed their beliefs and whose beliefs exceed their asks.\footnote{The bids and asks considered in the analysis represent intended transactions. If, instead of the intended transactions, we consider all bids and asks in the analysis, we arrive at similar conclusions to those reported here. The signs of the coefficients remain the same, but the levels of significance drop.}

Observation 2. Subjects who believe that prices will be higher submit higher bids and asks, and subjects who believe that prices will be lower submit lower bids and asks.

Support for Observation 2. The analysis involves i) an individual consistency test for each subject and ii) a consistency test for each group and market.

i) We conduct a Spearman rank-order correlation coefficient test on the individual data, including up to 60 short-term beliefs and up to 60 bids and 60 offers. This test on individual bids and asks is restricted to bids below and asks above the submitted short-term price expectation. Under the null hypothesis, the nominal short-term beliefs and the individual bids and asks are random. Our result supports the alternative hypotheses that bids and asks are positively correlated with beliefs; only for 2 of the 53 individual order submissions (i.e., 3.8\% of our sample) are the correlations between bids and short-term beliefs and between asks and short-term beliefs not significant at the 5\% level. The average correlation coefficients between bids and short-term beliefs and asks and short-term beliefs are 0.730 and 0.721, respectively. Thus, the data show that the bids and asks and the short-term beliefs of the same subject are significantly positively correlated over all periods.

ii) Tables 3 and 4 show regression results where the dependent variable is the rank of the submitted bid or ask. The independent variables are ranked short-term (Table 3) and long-term (Table 4) beliefs. The estimates indicate a significant relationship between the submitted offer and beliefs, both short term and long term.

To check the robustness of short-term and long-term beliefs as determinants of individual behavior, we include control variables that have been shown to impact beliefs. HLN suggest that beliefs are formed in an adaptive manner, influenced by past price changes. In Tables A1 and A2 in the Appendix, we show that both long-term and short-term beliefs influence trading behavior even when we control for previous price changes, as well as share and cash holdings.

A natural question to ask is whether long-term beliefs have additional predictive power for actions if one controls for short-term beliefs. Our main finding in this regard is reported as follows.

Observation 3. Short-term beliefs are better determinants of trading behavior than long-term beliefs.

Support for Observation 3. When including the short-term belief as an independent regression variable, the long-term belief is no longer a significant determinant.
of bids and asks (Tables 5 and 6). These results suggest that subjects make their decisions based on their short-term, rather than their long-term, expectations. Both short-term and long-term beliefs remain significant in regressions of net purchases and share holdings using the pooled data from all markets.18

18The average individual Spearman rank-order correlation coefficient between the short-term and long-term belief over all periods is 0.51, 0.49, 0.32, and 0.39 for markets 1, 2, 3, and 4, respectively. Apparently, subjects who are optimistic about short-term market behavior are also optimistic for the long term.
Table 5 shows the GLS regression results of net purchases ($\Delta S_{mgit}$) and share holdings ($S_{mgit}$) on ranked short-term (rank[STB]) and ranked long-term beliefs (rank[LTB]). STB$_{mgit}$ denotes the short-term price expectation, and LTB$_{mgit}$ denotes the long-term price expectation of subject $i$ within group $g$ for period $t$ in market $m$, as defined in equation (2). Based on Hausman tests, we report results of random-effects regressions. * , **, and *** indicate significant coefficients, according to 2-tailed $t$-tests at the 10%, 5%, and 1% levels, respectively ($t$-statistics reported in parentheses).

Table 6 shows the GLS regression results of ranked bids (rank[bid]) and ranked long-term beliefs (rank[LTB]). STB$_{mgit}$ denotes the short-term price expectation, and LTB$_{mgit}$ denotes the long-term price expectation of subject $i$ within group $g$ for period $t$ in market $m$, as defined in equation (2). The regression includes observations in which bids exceed short- and long-term beliefs and short- and long-term beliefs exceed asks; other bids and asks are treated as missing observations. Based on Hausman tests, we report results of fixed-effects or random-effects regressions. The subscript “f” indicates fixed effects; otherwise, they are random effects. * and ** indicate significance different from 0 at the 5% and 1% levels, respectively ($t$-statistics reported in parentheses).

Table 6 suggests that beliefs are a more important influence on purchase than on sale decisions. Because purchases typically involve anticipation of a future action (resale), beliefs may be given greater consideration in decisions to buy. The profitability of a sale, once it is concluded, is not affected by future prices. On the other hand, even after a purchase is made, beliefs affect the perceived value of the unit that was purchased. Our results show that individual choice depends on short-term beliefs only.

To conclude this section on individual beliefs and behavior, we report on the interactions between short- and long-term beliefs. The following observation concerns the dynamics of the updating of expectations, that is, how subjects adjust their long-term beliefs in light of new experiences. The sign of the deviation of their short-term belief from the observed market price seems to play a critical role.
If the market turns out to set the price above (below) the subject’s reported short-term expectation, the subject reacts by upward (downward) adjustment of the forecasted future price levels. The adjustment behavior of reported expectations upon arrival of new price information is quite similar for all subjects. It indicates that higher than expected returns generally lead to more optimistic forecasts. The finding is summarized in Observation 4.

**Observation 4.** Individuals increase (decrease) the price estimates in their long-term belief profile when their short-term belief has turned out to be below (above) the realized market price. The short-term price estimates behave in the same manner.

**Support for Observation 4.** We compare the short-term belief with the realized price for every individual in each period. When their short-term belief is below the realized price, subjects tend to increase their reported long-term beliefs. When their short-term belief is above the realized price, subjects tend to decrease their reported long-term beliefs. More formally,

\[
STB_{mgt-1} - P_{mgt-1} \leq 0 \Rightarrow LTB_{mgt} - LTB_{mgt-1} \succ 0,
\]

where the subscript indexes the time of belief submission and the superscript indicates the first forecast period.\(^{19}\) We say that subjects receive an upward price impulse if their short-term belief falls short of the realized price and a downward price impulse if their short-term belief exceeds the realized price. Figure 3 shows the relative frequency of long-term belief adjustments in the direction of the price impulse, on average, over all markets and cohorts. The reported numbers are representative of the whole sample. In no market does the relative frequency of belief adjustments in the direction against the price impulse reach or exceed 30%. On the individual subject level, we find that 51 of 53 subjects (96%) adjust their long-term beliefs in the direction of their current price impulse in a majority of periods.

**FIGURE 3**

Relative Frequency of Individual Adjustments of Long-Term Beliefs and of Short-Term Beliefs in the Direction of and against the Received Price Impulse

Figure 3 shows the relative frequency of individual adjustments of revealed beliefs from one period to the next, both long term and short term, following the received price impulse. If the prior short-term belief exceeds (falls short of) the realized price, the subject receives a downward (upward) price impulse. The chart shows how frequently subjects change their belief profile from that of the prior period in the direction of the impulse, versus in the other direction.

\(^{19}\)This adjustment behavior is in the spirit of impulse response theory (Selten (2004)) that proposes adaptive behavior in the direction of the ex post best response.
The result is significant, as the probability of 51 or more subjects of 53 behaving in this way due to pure chance is close to 0.

The data also support the claim that subjects adjust their short-term beliefs in the direction of the impulse; that is,

\[ \text{STB}_{mgt-1} - P_{mgt-1} \leq 0 \Rightarrow \text{STB}_{mgt} - B^*_{mgt-1} \leq 0. \]

Figure 3 shows the evidence for this claim; only 13% of the short-term belief adjustments between periods are in the other direction. The short-term belief adjustments are usually in the direction of the price impulse; 52 of 53 subjects (98%) adjust their short-term beliefs more frequently in that direction than in the opposing one. Thus, this result is significant at any commonly accepted level.

**B. Beliefs and Earnings**

In this section, we examine the relationship between earnings and predictions. We test the hypotheses that higher profits are associated with greater belief accuracy and with the level of optimism that a trader exhibits.

With respect to the relationship between profits and the accuracy of beliefs, there are two plausible conjectures. The first is that subjects with forecasts closer to actual prices will earn higher profits. The second is that traders with predictions closer to expected dividend values will earn higher profits. We observe both relationships holding in the data.

*Observation 5.* Subjects who accurately forecast asset prices, and subjects who expect prices close to fundamentals, earn higher profits.

*Support for Observation 5.* We employ a metric called the *relative belief-price deviation* (RBPD), which we define as a measure of the absolute difference between short-term beliefs and prices.

\[ \text{RBPD}_{mgi} = \frac{1}{T} \sum_{i=1}^{T} \left| \text{STB}_{mgi} - P_{mgi} \right| P_{mgi}. \]

Lower values of the relative belief price deviation indicate that short-term beliefs are, on average, closer to the realized prices. Analogously to equation (7), we define the measure *relative belief-value deviation*, which indicates how far the short-term belief differs from the expected dividend value.\(^{20}\) A GLS regression analysis with ranked total profits across the 15 periods as the dependent variable and ranked belief deviation measures as the independent variable is summarized in Table 7 for each measure of short-term beliefs, separately. Larger deviations of short-term beliefs from both fundamentals and prices are associated with lower profits when measured across all markets, as indicated in the table by the negative coefficients. The deviation of short-term beliefs from prices has a significant negative effect on profits in markets 1 and 4. The deviation of short-term beliefs from fundamentals has a significant negative effect on profits in market 1.

\(^{20}\) We also tested the effect of short-term beliefs at the beginning of the market (belief regarding the price in period 1) on profits. We found that, when inexperienced (during market 1 only), traders who were more accurate in predicting the price of period 1 earned higher profits. We did not find a significant effect for long-term beliefs.
TABLE 7
GLS Regression of Ranked Profits on Ranked Relative Belief-Price Deviation and Ranked Relative Belief-Value Deviation

Table 7 shows the GLS regression results of ranked profits (\( \text{rank}[\text{profit}_mg] \)) on ranked relative short-term belief-price deviation (\( \text{rank}[\text{RBPD}_mgit] \)), as defined in equation (7), and on ranked relative short-term belief-value deviation, (\( \text{rank}[\text{RBVD}_mgit] \)), where \( \text{RBVD}_mgit = \frac{1}{T} \sum_{i=1}^{T} |\text{STB}_mgit - f_t|/f_t \)). Based on a Hausman test, we report results of random-effects regressions. * and ** indicate significance different from 0 at the 5% and 1% levels, respectively (\( t \)-statistics reported in parentheses).

<table>
<thead>
<tr>
<th>Deviation from Prices</th>
<th>Deviation from Expected Dividend Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{rank}[\text{profit}_mg] = \alpha + \text{rank}[\text{RBPD}_mgit] \beta )</td>
<td>( \text{rank}[\text{profit}_mg] = \alpha + \text{rank}[\text{RBVD}_mgit] \beta )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>( \beta )</td>
</tr>
<tr>
<td>Market 1</td>
<td>(N = 53)</td>
</tr>
<tr>
<td>7.21**</td>
<td>–0.44**</td>
</tr>
<tr>
<td>(10.18)</td>
<td>(–3.52)</td>
</tr>
<tr>
<td>Market 2</td>
<td>(N = 53)</td>
</tr>
<tr>
<td>5.72**</td>
<td>–0.14</td>
</tr>
<tr>
<td>(7.32)</td>
<td>(–1.04)</td>
</tr>
<tr>
<td>Market 3</td>
<td>(N = 53)</td>
</tr>
<tr>
<td>5.65**</td>
<td>–0.13</td>
</tr>
<tr>
<td>(7.22)</td>
<td>(–0.94)</td>
</tr>
<tr>
<td>Market 4</td>
<td>(N = 44)</td>
</tr>
<tr>
<td>6.45**</td>
<td>–0.29*</td>
</tr>
<tr>
<td>(7.74)</td>
<td>(–1.96)</td>
</tr>
<tr>
<td>Overall</td>
<td>(N = 203)</td>
</tr>
<tr>
<td>6.25**</td>
<td>–0.25**</td>
</tr>
<tr>
<td>(16.23)</td>
<td>(–3.66)</td>
</tr>
</tbody>
</table>

Observation 5 suggests that profits are made when traders either i) make decisions that reflect accurate forecasts of the price of the next period or ii) trade on fundamentals. The first strategy is profitable when speculating on price changes between one period and the next. The second strategy is more profitable than average earnings in the long run. In market 1, both of these are profitable, as there is a positive correlation between earnings and the adherence of beliefs to each benchmark.

C. Belief Dispersion and Market Behavior

Theory suggests that heterogeneous beliefs can push up prices if short sale constraints are binding (Miller (1977)) and that belief dispersion can lead to higher transaction volume (Varian (1989)), as disagreement encourages trade. Our data support the former statement that the price increases with belief dispersion, but they are inconsistent with the second conjecture that belief dispersion and transaction volume are positively correlated.

We measure belief dispersion in each period by means of the coefficient of variation (equation (3)). Figure A1 in the Appendix displays the average dispersion of short- and long-term beliefs over all 15 periods. Earlier research in asset market experiments has shown that the number of transacted shares tends to decrease with repetition (Palan (2013)), but belief dispersion has not been measured in this context before. However, if transaction volume is associated with belief dispersion, as suggested by Varian, then, as a consequence, dispersion would decrease over time too. Indeed, we find that both belief dispersion and transaction volume do decrease as traders accumulate experience.


Support for Observation 6. In experimental asset markets, participants gain experience by participating in several consecutive markets. In the HLN data, there are four levels of experience. During the first market, participants are
inexperienced. During the fourth market, participants are highly experienced. The average short-term belief dispersion decreases from 0.297 to 0.244, 0.212, and 0.174, and the average share turnover, the percentage of the total stock of units that trades, decreases from 0.145 to 0.113, 0.095, and 0.106 per period, from markets 1–4.\textsuperscript{21} On average, across 60 periods, the short-term belief dispersion declines by 1.96\% and the transaction volume by 1.90\% per period. Regressing belief dispersion and transaction volume on period number shows a significant decline across periods and markets.\textsuperscript{22} Thus, it seems as if short-term expectations become less heterogeneous with experience.\textsuperscript{23} Taking a second look at the averages of belief dispersion and transaction volume in the fourth market, nonetheless, it appears to us unlikely that the markets would converge to a no-trade, zero belief-dispersion market in a reasonable amount of time.

Next, we turn the focus to the theoretical implications of the magnitude of belief dispersion on market price and transaction volume. The effect of belief dispersion on transaction volume has the expected positive sign, but its magnitude is not significant at conventional levels. When controlling for the time trend, the GLS regression of transaction volume on belief dispersion shows that belief dispersion is not a significant determinant of volume, as suggested by theory (e.g., Varian (1989)). The \( t \)-statistic of belief dispersion is 0.46 \((N = 345)\) in a random-effects regression of transaction volume on short-term belief dispersion and period. This result is included in the following observation.

Observation 7. Belief dispersion has no significant effect on transaction volume. Belief dispersion is associated with higher prices. The initial belief dispersion can be indicative of the later market price level.

Support for Observation 7. The regression results of transaction volume on both belief dispersion and lagged transaction volume are presented in Table 8. As the table shows, the regression coefficients are not significant for any market. Therefore, the data suggest no significant relationship between belief dispersion (both short and long term) and transaction volume. Thus, we fail to support the theoretical prediction.

In line with theory (e.g., Miller (1977)), however, higher prices are associated with greater belief dispersion in the data. One possible reason for this is the fact that an increase in short-term belief dispersion is associated with a greater number of bids,\textsuperscript{24} though not of asks. To test this, we compute the average short-term belief dispersion and the average long-term belief dispersion for each market in accordance with equation (3). For the GLS regression of RD (equation (4)) on

\textsuperscript{21}There is no significant reduction in the dispersion of long-term beliefs across markets. The average long-term belief dispersions for markets 1, 2, 3, and 4 are 0.465, 0.493, 0.456, and 0.346, respectively.

\textsuperscript{22}The \( t \)-statistics obtained when regressing transaction volume, short-term belief dispersion, and long-term belief dispersion on market number equal \(-3.24 \ (p < 0.01), -2.79 \ (p < 0.01), \) and \(-1.66 \ (p < 0.1), \) respectively. When regressing on period number, the \( t \)-statistics equal \(-3.95 \ (p < 0.01), -2.81 \ (p < 0.01), \) and \(-4.37 \ (p < 0.01), \) respectively. \((N=345, 345, \text{ and } 322).\)

\textsuperscript{23}We also measured homogeneity of beliefs by counting the frequency of identical short-term (long-term) belief submissions within a period of a market by different subjects. According to this measure, the data would suggest no drop in belief heterogeneity.

\textsuperscript{24}\( t \)-statistic \( =4.27 \ (p < 0.01).\)
beliefs, we have one observation per market and four per session. The $t$-statistic for the short-term belief dispersion in this regression is 2.72 ($p < 0.01$) and that of long-term belief dispersion is 2.80 ($p < 0.01$, $N = 23$). Thus, the data support the theory that belief dispersion appears to be a good indicator of price levels in the asset market experiment. The price level can be anticipated already in the first period because the RD significantly depends on the short-term belief dispersion in the first period.\(^{25}\)

This result indicates that belief dispersion is correlated with higher prices and may reflect the parameters used in our markets, which are the ones typically employed in the literature. In the experiment, because short sales are disabled, the largest short position pessimistic traders can take is to sell all of their holdings. On the other hand, because of the relatively high ratio of cash to current prices in the experiment, an optimistic trader typically has the capacity to take larger long positions. This means that an increase in belief dispersion would typically increase price level, as suggested in the theoretical literature (Miller (1977)).

**Observation 8.** Belief dispersion affects the relative size of price changes. The relative size of past price changes affects the dispersion of beliefs. The latter effect is stronger than the former.

**Support for Observation 8.** To show the relationship between belief dispersion and price changes, we conduct a GLS regression with both belief dispersion and prior price changes as explanatory variables and the current price change from the prior period as the dependent variable. The results are presented in Table 9, which shows that short-term belief dispersion is a significant determinant of the

\(^{25}\)The GLS regression of the RD on the short-term beliefs of period 1 leads to a $t$-statistic of 2.53 ($p < 0.01$). The same regression analysis using the first period long-term belief as the dependent variable leads to a $t$-statistic of $-0.39$.  

---

**Table 8**

GLS Regressions of Transaction Volume on Belief Dispersion and Lagged Transaction Volume

<table>
<thead>
<tr>
<th></th>
<th>Short-Term Belief Dispersion</th>
<th>Long-Term Belief Dispersion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$ $\beta_1$ $\beta_2$</td>
<td>$\alpha$ $\beta_1$ $\beta_2$</td>
</tr>
<tr>
<td>Market 1</td>
<td>2.69** 0.55 $-0.13$</td>
<td>3.50** $-0.80$ $-0.20^*$</td>
</tr>
<tr>
<td>($N = 84, 78$)</td>
<td>(6.85) (0.86) ($-1.14$)</td>
<td>(6.00) (0.92) ($-1.80$)</td>
</tr>
<tr>
<td>Market 2</td>
<td>2.55** $-0.99^*$ $-0.15$</td>
<td>2.24** 0.41 $-0.15$</td>
</tr>
<tr>
<td>($N = 84, 78$)</td>
<td>(7.94) ($-1.72$) ($-1.28$)</td>
<td>(5.94) (0.82) ($-1.33$)</td>
</tr>
<tr>
<td>Market 3</td>
<td>1.39** 0.32 0.14</td>
<td>1.53** 0.58 $-0.02$</td>
</tr>
<tr>
<td>($N = 84, 78$)</td>
<td>(4.78) (0.39) (1.28)</td>
<td>(4.96) (1.14) ($-0.18$)</td>
</tr>
<tr>
<td>Market 4</td>
<td>1.64** 0.11 0.09</td>
<td>1.76** $-0.34$ 0.11</td>
</tr>
<tr>
<td>($N = 70, 65$)</td>
<td>(4.61) (0.08) (0.74)</td>
<td>(5.49) ($-0.60$) (0.81)</td>
</tr>
<tr>
<td>Overall</td>
<td>1.95** $-0.09$ 0.05</td>
<td>2.06** $-0.09$ 0.03</td>
</tr>
<tr>
<td>($N = 322, 299$)</td>
<td>(11.79) ($-0.26$) (0.82)</td>
<td>(10.74) ($-0.33$) (0.46)</td>
</tr>
</tbody>
</table>

The GLS regression of the RD on short-term beliefs of period 1 leads to a $t$-statistic of 2.53 ($p < 0.01$). The same regression analysis using the first period long-term belief as the dependent variable leads to a $t$-statistic of $-0.39$.  

---
A. Fitting Transaction Data with Full Belief-Based Information

Table 9 shows the GLS regression of magnitude of price change on belief dispersion (STBD$_{mgt}$ and LTBD$_{mgt}$), as defined in equation (3), controlling for the lagged magnitude of price change (|P$_{mgt-2}$ - P$_{mgt-1}$|/P$_{mgt-2}$). Fixed-effects or random-effects regressions are reported based on Hausman tests. The subscript ‘f’ indicates fixed effects; otherwise, they are random effects. * and ** indicate significance different from 0 at the 5% and 1% levels, respectively (t-statistics reported in parentheses).

### TABLE 9
GLS Regression of Magnitude of Price Change on Belief Dispersion and Magnitude of Prior Period's Price Change

<table>
<thead>
<tr>
<th></th>
<th>Short-Term Beliefs</th>
<th>Long-Term Beliefs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$ $\beta_1$ $\beta_2$</td>
<td>$\alpha$ $\beta_1$ $\beta_2$</td>
</tr>
<tr>
<td>Market 1</td>
<td>0.13** 0.15** 0.01</td>
<td>0.11** 0.02 0.37*</td>
</tr>
<tr>
<td>(N = 78, 72)</td>
<td>(3.50) (2.82) (0.14)</td>
<td>(2.85) (0.26) (2.39)</td>
</tr>
<tr>
<td>Market 2</td>
<td>0.12* 1.08** -0.14</td>
<td>0.19* -0.38* 0.67**</td>
</tr>
<tr>
<td>(N = 78, 72)</td>
<td>(2.04) (5.49) (-1.23)</td>
<td>(2.15) (-2.04) (4.32)</td>
</tr>
<tr>
<td>Market 3</td>
<td>0.22* 0.61 -0.01</td>
<td>0.41** -0.29 0.05</td>
</tr>
<tr>
<td>(N = 78, 72)</td>
<td>(2.12) (1.46) (-0.02)</td>
<td>(3.21) (-1.11) (0.32)</td>
</tr>
<tr>
<td>Market 4</td>
<td>0.17** 0.44 0.02</td>
<td>0.17** 0.01 -0.03</td>
</tr>
<tr>
<td>(N = 65.60)</td>
<td>(4.18) (0.30) (0.18)</td>
<td>(4.08) (0.22) (-0.22)</td>
</tr>
<tr>
<td>Overall</td>
<td>0.16** 0.48** 0.04</td>
<td>0.25** -0.16* 0.27**</td>
</tr>
<tr>
<td>(N = 299, 276)</td>
<td>(4.87) (4.42) (0.68)</td>
<td>(5.64) (-1.98) (3.22)</td>
</tr>
</tbody>
</table>

The results are preserved when the regression does not control for lagged belief dispersion.

HLN (2007) show that recent past price changes affect beliefs and establish the result for average beliefs. Here, we also find that the magnitude of prior price changes affects belief dispersion. To show this, we conduct a GLS regression of the current belief dispersion on the lagged absolute price change (controlling for lagged belief dispersion as an additional explanatory variable). Table 10 shows the results for the GLS regression. The magnitude of the price change is a significant determinant of the short-term belief dispersion in each market and of the long-term belief in market 1, market 3, and overall. The lower significance levels in Table 9 compared to Table 10 suggest that the belief dispersion is adaptive vis-à-vis the size of the price change rather than vice versa.

IV. Market Behavior, Belief Data, and Forecasting Market Behavior

A. Fitting Transaction Data with Full Belief-Based Information

In this section, we use a simulation approach to test whether beliefs can be employed to predict future prices and transaction volumes. With this approach, we indirectly test the validity of experimental designs (e.g., Marimon et al. (1993)) that use belief data for market clearance rather than bids and asks. Our preceding observations indicate that subjects’ actions are aligned with their expectations. Subjects who are optimistic about the asset’s risk and return submit bids to buy in the market, and less optimistic subjects submit asks to sell. However, we have also reported that order prices and expected prices are generally not identical. So, we address the question whether, despite these existing discrepancies, the knowledge magnitude of price changes between consecutive periods in markets 1 and 2 and in the data overall, while long-term belief dispersion is not significant in any market.

The results are preserved when the regression does not control for lagged belief dispersion.
of the market participants’ expectations is valuable information for successfully predicting observed market behavior, including for a bubble and crash pattern.

The simulations implement the following thought experiment. Suppose that an observer has the full profile of the short-term belief data and the bid and ask quantities submitted (of those who intend to trade as well as those who do not). Can the observer predict prices and quantities transacted accurately in advance? Of special interest is whether the observer can predict a market crash in advance.

The simulation model uses the same quantities of the bids and asks of the subject, while the reservation prices of these bids and asks (i.e., the proposed purchase price of the bids and the sale price of the asks) are replaced by the short-term expected prices of individual traders, STBmgit, and STBmgit + 0.01. The model simulations are then compared to the realized experimental data.

In the simulation, the submitted short-term beliefs are used to generate bids and asks. For each trader i in period t of market m in session g, a bid at price STBmgit and an ask at STBmgit + 0.01 francs are submitted. The quantities specified in i’s bid and ask are equal to the quantities the subject submitted in the corresponding period of the experiment. Demand and supply curves are constructed from these orders and the market clears according to HLN’s call market rules. The implied transactions are concluded and cash balances and asset inventories are updated accordingly. Dividends are realized and paid out at the end of each period in the same way as in the experiment.27

Observation 9. Simulated prices (and quantities) based on the short-term belief profile resemble the actual ones observed in the experiment.

---

27If investors have identical beliefs and only some of the shares can be traded at the market clearing price, the traders whose shares are exchanged are randomly determined, in line with the experimental design. In the case of a period without trade, the highest bid price plus one unit of experimental currency is treated as the market price.
Support for Observation 9. Figure 4 shows the resulting prices, averaged over the six simulated sessions. Table 11 records the RDs, the average RD measures, of the simulated and observed price trajectories and also the simulated and observed average transaction volumes per market. The figure and table suggest that the simulated prices and quantities resemble the actual values observed in the experiment. The correlation between the simulated and observed price trajectories over all markets, averaged over all cohorts, is 0.91. The correlation between the simulated price path and the observed price trajectory is considerably greater than that between the expected dividend value and the price trajectory, 0.389. The simulated price trajectory is significantly closer to the data than the theoretical benchmark is.

**FIGURE 4**
Simulated and Observed Average Market Prices

Figure 4 shows the observed market prices (solid black line) in the experiment and simulated market prices (solid gray line) averaged over the six sessions. Periods 1–15, 16–30, 31–45, and 46–60 correspond to markets 1, 2, 3, and 4, respectively. The simulated market prices account for order submission and short-term belief of each trader in each period (with the short-term belief determining subjects’ bids and asks) and are computed according to the HLN market clearing rules.

**TABLE 11**
Simulated and Observed Average Transaction Volume and RDs

<table>
<thead>
<tr>
<th>Market</th>
<th>RD</th>
<th>Transaction Volume</th>
<th>RD</th>
<th>Transaction Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market 1</td>
<td>0.46</td>
<td>14.50</td>
<td>0.52</td>
<td>10.37</td>
</tr>
<tr>
<td>Market 2</td>
<td>0.45</td>
<td>11.29</td>
<td>0.47</td>
<td>10.06</td>
</tr>
<tr>
<td>Market 3</td>
<td>0.09</td>
<td>9.51</td>
<td>0.12</td>
<td>9.38</td>
</tr>
<tr>
<td>Market 4</td>
<td>−0.08</td>
<td>10.59</td>
<td>−0.05</td>
<td>10.44</td>
</tr>
<tr>
<td>Overall</td>
<td>0.24</td>
<td>11.47</td>
<td>0.28</td>
<td>10.07</td>
</tr>
</tbody>
</table>

The timing of the crash is also reproduced quite well with our simulation approach, as is shown in Figure 4. The exception is market 3, in which the average simulated market crashed one period too late. The quantities transacted in the
simulated markets are close to (though, on average, somewhat below) the actual observed quantities.\footnote{The average transaction volume over all markets is not significantly different between the actual and the simulated markets at the 10\% significance level ($p = 0.109$).}

B. Fitting Prices with Partial Belief Information

The simulation reported in Section IV.A predicts the time path of prices well, but the approach is arguably demanding in terms of the belief information that it assumes the market to possess. The immediate questions that arise are whether a good fit of the price trajectories can be achieved with fewer requirements on the belief data and if a good fit can be obtained using only lagged data. For this purpose, we propose four simple alternative models with lower data requirements than the model of Section IV.A. These four models are called MedBel, MedBel3Trad, AdapPer, and AdapIni. The first two models use a median short-term belief rather than the full vector of beliefs, while the last two derive the current belief endogenously from belief data from prior periods only.

MedBel: This model employs the median short-term belief within the cohort in a given period as the market price. The median is a simple summary statistic of market expectations. We propose the median belief rather than the average belief because of its robustness to outliers. The average Spearman correlation coefficient of the median belief and the price is 0.87.

MedBel3Trad: In this model, we rank subjects within a cohort by their total profits from trading over the four markets, to determine the best three traders of the cohort. We then take the median short-term belief of these three traders as the market price. These traders can be thought of as the relatively skilled investors and their median belief can be interpreted as an informed consensus. The average Spearman correlation coefficient of this median and the price is 0.88.

AdapPer: This model employs only lagged belief data and requires no information about current beliefs. As in equation (6), current short-term beliefs are updated based on observed prices. Writing the qualitative relationship in linear form, we conjecture that the stated belief changes proportionally to the forecasting error of the last period.

\begin{equation}
\frac{B_{mgit}^{mgt}}{B_{mgit-1}^{mgt}} = \alpha \left( \frac{P_{mgit-1}^{mgt}}{B_{mgit-1}^{mgt}} - 1 \right).
\end{equation}

The ratio on the left-hand side represents the adjustment of current short-term beliefs from prior beliefs about the current price. In the parentheses on the right side is the observed prediction error. This is the observed price relative to its short-term prediction. We take natural logarithms and estimate the following linear model, where $b$ and $p$ denote the logarithm of $B$ and $P$.\footnote{This formulation is very similar to that of standard adaptive learning dynamics, as, for instance, formulated in Hommes et al. (2005). However, we use logarithms here because of the nonlinearity of equation (8).}

\begin{equation}
\ln(STB_{mgit}) \equiv b_{mgit}^{mgt} = -0.202^{**} + 0.597 b_{mgit-1}^{mgt}^{***} \\
+ 1.033 p_{mgit-1}^{mgt}^{***} - 0.591 b_{mgit-1}^{mgt}^{***}.
\end{equation}
With this random-effects regression (** indicates significance at 1%), we fit the price trajectory with 1-period-ahead estimation. The model has weaker data requirements than the two prior models because it requires no contemporaneous beliefs, but rather only lagged beliefs from the past. The predicted belief trajectory of this estimated model has an average Spearman rank correlation coefficient of 0.85 with the actual price trajectory.

AdapIni: This approach further lowers the informational requirements by making use of median initial beliefs before period 1 only. As in equation (9), we estimate a linear function based on the median belief within the cohort and market, but we only use the beliefs that were submitted before period 1, \( B_{mgi1} \). Thus, the following equation is estimated.

\[
\ln(STB_{mgit}) \equiv b_{mgit}^{mgt} = -0.168^{**} + 0.213b_{mgit}^{mgt}^{**} \\
+ 1.064p_{mgit-1}^{***} - 0.245b_{mgit-1}^{mgi1}^{***}. \tag{10}
\]

With this random-effects regression, we estimate the price trajectory for each period with lagged price information and the initial median belief. The predicted belief trajectory has an average Spearman correlation coefficient of 0.84 with the actual price path.

A comparison of the four models, plotted in Figure A3 in the Appendix, reveals that the fit of the price trajectory is nearly as good if we allow for less information. The median models, which use current beliefs, fit the data modestly better than the lagged information models that use prior beliefs only. Yet all models have a significantly better fit to observed prices than the expected dividend value does (at the 5% level). The significance results are the same if we compare the correlation coefficients as reported here or the mean squared error between the fitted models and the price trajectory.

C. Selling Strategies from Belief-Based Data

In this section, we consider the potential value to an investor of the belief data in terms of trading profits. To judge the value of trading based on the simulated trajectory, we derive a selling strategy involving the simulated data from the model in Section IV.A. We compare this to reasonable alternative trading strategies that do not employ the belief information. We are particularly interested in whether the belief data can be used to help traders in selling their asset holdings at a correct time before a crash occurs.

Figure 4 reveals that a crash occurs shortly after the simulated trajectory surpasses the observed price trajectory. In our strategy, this surpassing of the simulated price trajectory above the expected dividend value generates an intention to sell. As soon as this intention to sell is enabled, our strategy specifies a sale during the next period in which the price is above the expected dividend value \( f_t \). Our measure of profitability of a trading strategy is calculated as the following ratio. The numerator is equal to the sale price minus the expected dividend value at the time of sale. The denominator is calculated as the peak difference, over the course of the market, between a period price and the expected dividend value. Trade profitability of 1 means that the trader sells in the period \( t \) in which
\( P_{\text{me}} - f \) reaches a maximum value within market \( m \) that group \( g \) participates in. Trade profitability of 0 is attained if the trader sells at a price exactly equal to the expected dividend value. Our simulation-based selling strategy in Section IV.A has a trader profitability of 0.63.

To evaluate the excess return generated by our strategy, we require good alternative models for the purpose of comparison. As one alternative model for the selling strategy, we take the “rational benchmark” model that implies a sale at the first price above expected dividend value. This selling strategy achieves a trader profitability of 0.23. Another alternative model for the selling strategy is constructed on the basis of the observed excess demand, which is defined as the total number of bids minus the total number of asks submitted by all traders in each period. This measure was proposed by Smith et al. (1988) as a predictor of subsequent price movements. Smith et al. find that negative excess bids (a greater quantity of offers to sell than to buy) herald a market crash. The alternative selling strategy generates an intention to sell as soon as the excess bid is negative for the first time and is executed in the first subsequent period in which price is above expected dividend value. The excess-bids–based selling strategy achieves 0.59 of the maximum possible profit. Our simulation-based (belief-based) selling strategy, thus, secures larger, though not significantly so, excess gains than the alternative excess-bids model.

We also investigate the profitability of the less-information-demanding trading strategies discussed in Section IV.B. As assumed previously, our strategy generates a sell signal as soon as the model prediction for the next period exceeds the expected dividend value. This sale occurs in the next period at the prevailing market price. Trading based on the MedBel model has a trade profitability of 0.77; MedBel3Trad, 0.79; AdapPer, 0.74; and AdapIni, 0.63. Thus, trading based on these partial information models generates profits in excess of those of the alternative model and is also better than the full information model of Section IV.A. All our selling strategies (including the excess-bids strategy) achieve a significantly larger profitability than the rational benchmark.³⁰

V. Conclusion

Theories in economics and finance build on assumptions about decision makers’ beliefs. To study individual beliefs, experimental designs elicit predictions of future prices by offering salient rewards to participants for accurate predictions. Revisiting the data of HLN (2007), our study shows that individual beliefs are heterogeneous and consistent with individual trading behavior. Our study provides empirical evidence that heterogeneous beliefs result in heterogeneous actions and, thus, correlate with trade. The fundamental empirical result that traders in the asset market act in line with their beliefs is observed strongly in our study. Net purchases, share holdings, and submitted orders depend on subjects’

³⁰The \( p \)-values resulting from the pairwise 2-tailed \( t \)-tests comparing the excess gains from the rational benchmark strategy and our lagged response models are significant at the 5% level: 0.014 (simulation based), 0.030 (excess bids), 0.000 (MedBel), 0.000 (MedBel3Trad), 0.000 (AdapPer), and 0.005 (AdapIni).
short- and long-term beliefs. Hence, we find that trade occurs because more optimistic traders purchase from those who are relatively pessimistic.

Despite the fact that theories have long suggested this relationship (e.g., Varian (1985), Biais and Bossaerts (1998)), the experimental approach can be used to show empirically that the fundamental relationship between expectations (i.e., short- and long-term beliefs) and actions holds in financial asset markets and that expectations are indeed heterogeneous. With homogeneous beliefs, theory predicts no trade. A no-trade theorem presumably applies to experimental markets where there are no liquidity needs or surpluses and, thus, no motive to trade. Since beliefs are heterogeneous and subjects act in accordance with their beliefs, and given our statistical data analysis, we conclude that, in the HLN (2007) asset market experiment, many transactions occur because subjects have heterogeneous beliefs, even though no liquidity needs exist.

Theory also suggests the existence of a measurable effect of belief dispersion on transaction volume (Varian (1989)) and that price bubbles increase with belief dispersion in the market for given short-sale restrictions (Miller (1977)). We find that both belief dispersion and transaction volume decline over periods and markets. However, despite the aligned behavioral dynamics, the data show no significant correlation between transaction volume and belief dispersion.\(^3\) Nonetheless, we do find evidence that the mispricing increases with the measured belief dispersion.

Our simulation analysis in Section IV makes several points. The first is that data on traders’ expectations can provide good predictions of subsequent price movements. The second is that simulation models experience only a modest decrease in prediction accuracy as the amount of belief information available is reduced. The third is that belief information can be very profitable for a trader in the market.

In the field, heterogeneous beliefs can result from different types of available information, or from different interpretation of the same information. Therefore, a controlled test of homogeneous beliefs in the laboratory is meaningful. In the experiment, all individuals face the same instructions and receive the same information. Under these conditions, including identical information and perfect knowledge of fundamentals, one might assume that the beliefs of subjects must be homogeneous, but they are not. Individual beliefs are heterogeneous in each period and market. Even with repetition, beliefs seem not to converge, although belief dispersion does decrease over time. Nevertheless, heterogeneity of beliefs persists, and we find that markets are not able to homogenize expectations and do not reach a no-trade equilibrium.

\(^3\)Biais and Bossaerts (1998) investigate the correlation between belief dispersion and transaction volume in statistical simulation models. Their results do not indicate any significant correlation between dispersion and volume.
FIGURE A1
Average Short- and Long-Term Belief Dispersion

Figure A1 shows the average short- and long-term belief dispersion (STBD\textsubscript{mgt} and LTBD\textsubscript{mgt}), as defined in equation (3), averaged over all sessions, each market separately. The figure shows, for all markets and all periods, the average belief dispersions within trader cohorts. Periods 1–15, 16–30, 31–45, and 46–60 correspond to markets 1, 2, 3, and 4, respectively. Graph A shows short-term belief dispersion, and Graph B shows long-term belief dispersion.
Figure A2 shows (corresponding to Figure 4), for each session, the observed market prices (black solid line) of the experiment and simulated market prices (gray solid line). Periods 1–15, 16–30, 31–45, and 46–60 correspond to markets 1, 2, 3, and 4, respectively. The simulated market prices account for order submission and short-term belief of each trader in each period (with the short-term belief determining subjects’ bids and asks) and are computed according to the HLN market clearing rules.
FIGURE A3
Observed and Simulated Average Prices for the Partial Belief Models MedBel, MedBel3Trad, AdapPer, and AdapIni

Figure A3 shows the observed prices (solid black line) and simulated average prices (solid gray line) for the partial belief models. The partial belief models are simulated market prices based on the following rules. MedBel: Median short-term belief as the market price. MedBel3Trad: Median short-term belief of the best three traders as the market price. AdapPer: Uses only past belief data, according to equation (9). AdapIni: Uses only belief data from the first period, according to equation (10). Periods 1–15, 16–30, 31–45, and 46–60 correspond to markets 1, 2, 3, and 4, respectively.

TABLE A1
GLS Regression of Net Purchase, Ranked Bids, and Ranked Asks on Ranked Short-Term Beliefs with Controlling for Price Changes, Share Holdings, and Cash Holdings

Table A1 shows the GLS regression results of net purchase ($\Delta S_{mgit}$), ranked bids ($\text{rank}[\text{bid}]$), and ranked asks ($\text{rank}[\text{ask}]$) on ranked short-term beliefs ($\text{rank}[\text{STB}]$), past period price change ($P_{mgit-1} - P_{mgit-2}$), share ($S_{mgit}$), and cash holdings ($C_{mgit}$), for all individuals and markets. Our GLS regression approach is in line with equation (1), but instead of having the ranked beliefs as a unique explanatory variable, we add the observed price change, cash, and share holdings as additional variables. Table A1 reveals that, although the amount of cash and share holdings is also a significant determinant of trading behavior, short-term beliefs are significant determinants of net purchases, bids, and offers. In line with the Hausman test, we report results of the fixed-effects regression or random-effects regression. The subscript ''f'' indicates fixed effects; otherwise, they are random effects. ** and *** indicate significance different from 0 at the 5% and 1% levels, respectively ($t$-statistics reported in parentheses).

$$y_{mgit} = \alpha + \text{rank}[\text{STB}] \beta_1 + (P_{mgit-1} - P_{mgit-2}) \beta_2 + S_{mgit} \beta_3 + C_{mgit} \beta_4$$

<table>
<thead>
<tr>
<th></th>
<th>Intercept $\alpha$</th>
<th>$\text{rank}[\text{STB}] \beta_1$</th>
<th>$(P_{mgit-1} - P_{mgit-2}) \beta_2$</th>
<th>Shares $S_{mgit} \beta_3$</th>
<th>Cash $C_{mgit} \beta_4$</th>
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</thead>
<tbody>
<tr>
<td><strong>Net purchase</strong> ($\Delta S = y$)</td>
<td>($N = 2,633$)</td>
<td>$-0.88^{***}$</td>
<td>0.02**</td>
<td>0.01</td>
<td>0.21***</td>
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<tr>
<td>Bids ($\text{rank}[\text{bid}] = y$)</td>
<td>($N = 596$)</td>
<td>$5.72^{***}$</td>
<td>0.21***</td>
<td>0.01***</td>
<td>0.21***</td>
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<tr>
<td>Asks ($\text{rank}[\text{offer}] = y$)</td>
<td>($N = 445$)</td>
<td>1.24***</td>
<td>0.14***</td>
<td>$-0.01^{***}$</td>
<td>0.03</td>
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GLS Regression of Net Purchase, Ranked Bids, and Ranked Asks on Ranked Long-Term Beliefs with Controlling for Price Changes, Share Holdings, and Cash Holdings

Table A2 shows the GLS regression of net purchase ($\Delta S_{\text{mgit}}$), ranked bids (rank[bid]), and ranked asks (rank[ask]) on ranked long-term beliefs (rank[LTB$_{\text{mgit}}$]), past period price change ($P_{\text{mgit}-1} - P_{\text{mgit}-2}$), share ($S_{\text{mgit}}$), and cash holdings ($C_{\text{mgit}}$) over all markets. Our GLS regression approach is in line with equation (1), but instead of having the ranked beliefs as a unique explanatory variable, we add the observed price change, cash, and share holdings as additional variables. Table A2 reveals that, although the amount of cash and share holdings is also a significant determinant of trading behavior, long-term beliefs are significant determinants of net purchases, bids, and offers. In line with the Hausman test, we report results of the fixed-effects regression or random-effects regression. The subscript “f” indicates fixed effects; otherwise, they are random effects. ** and *** indicate significance different from 0 at the 5% and 1% levels, respectively (t-statistics reported in parentheses).

$$y_{\text{mgit}} = \alpha + \text{rank}[\text{LTB}_{\text{mgit}}] \beta_1 + (P_{\text{mgit}-1} - P_{\text{mgit}-2}) \beta_2 + S_{\text{mgit}} \beta_3 + C_{\text{mgit}} \beta_4$$

<table>
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<tr>
<th>Variables</th>
<th>Intercept ($\alpha$)</th>
<th>rank[LTB] ($\beta_1$)</th>
<th>$P_{\text{mgit}-1} - P_{\text{mgit}-2}$ ($\beta_2$)</th>
<th>Shares ($\beta_3$)</th>
<th>Cash ($\beta_4$)</th>
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<tbody>
<tr>
<td>Net purchase ($\Delta S = y$)</td>
<td>$-1.02^{**}$</td>
<td>$0.02^{***}$</td>
<td>$0.01$</td>
<td>$0.22^{***}$</td>
<td>$0.01^{***}$</td>
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<tr>
<td>(N = 2,430)</td>
<td>($-17.11$)</td>
<td>($3.21$)</td>
<td>($0.89$)</td>
<td>($20.74$)</td>
<td>($13.41$)</td>
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<tr>
<td>Bids (rank[bid] = y)</td>
<td>$5.81^{***}$</td>
<td>$0.13^{***}$</td>
<td>$-0.01^{***}$</td>
<td>$0.25^{***}$</td>
<td>$0.01^{**}$</td>
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<tr>
<td>(N = 568)</td>
<td>($24.36$)</td>
<td>($4.60$)</td>
<td>($-3.58$)</td>
<td>($7.42$)</td>
<td>($2.51$)</td>
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<tr>
<td>Asks (rank[offer] = y)</td>
<td>$1.34^{***}$</td>
<td>$0.08^{***}$</td>
<td>$-0.01^{***}$</td>
<td>$0.11^{**}$</td>
<td>$0.01^{**}$</td>
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<tr>
<td>(N = 403)</td>
<td>($6.17$)</td>
<td>($3.05$)</td>
<td>($-3.27$)</td>
<td>($2.08$)</td>
<td>($2.42$)</td>
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References


