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PRESSURE TRANSIENTS IN AN IDEALISED HORIZONTAL TWO FLUID RESERVOIR*

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Two compressible immissible fluids, possibly water and gas, are confined within a horizontal permeable reservoir whose vertical height is small relative to both the horizontal reservoir dimensions and reservoir depth below sea level, say. Mass conservation, a simplified Darcy's Law, Bousinesq averaging over height and the Dupuit approximation on fluid velocities result in two coupled non-linear parabolic equations for interface pressure and interface height. Linearisation yields two diffusivities; one associated with the initial pressure response to reservoir exploitation, while the other is much smaller in magnitude and of Buckley-Leverett type, being associated with the initial interface response. Some numerical results are presented of upconing and pressure drawdown in a bounded one-dimensional reservoir.

1. Introduction

The aim of this paper is to describe pressure drawdown and water rise in an idealised horizontal porous reservoir, in which compressed gas overlies compressed water.

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In Section 2 we shall present the relevant equations and simplifying assumptions; in Section 3 we linearise the equations and derive two diffusivities which describe the initial pressure and interface response to reservoir exploitation, while Section 4 contains some numerical solutions of the non-linear equations. We summarise our results in Section 5. SI units are used throughout.

2. The simplified two fluid equations

In a Cartesian coordinate system, (x, y, z), with z aligned vertically upwards, the equations of mass conservation away from fluid sources or sinks are

$$(1) \qquad \varepsilon \frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{u}_i) + \frac{\partial}{\partial z}(\rho_i w_i) = 0 \,, \quad i = 1, 2 \,,$$

where ρ_i, $\mathbf{u}_i$, w_i, ε, $\nabla \cdot$ and t are the respective fluid density, horizontal volumetric fluid flux per unit area, vertical fluid flux per unit area, porosity, horizontal divergence operator and time. Fluids 1 and 2 occupy the respective regions $z_1 < z < \zeta$ and $\zeta < z < z_2$, where $z = \zeta$ is the fluid-fluid interface (see Figure 1). The two boundaries $z = z_i$ $(i = 1, 2)$ are assumed impermeable, and have as normals

$$(2) \qquad n_i = (1 + \nabla z_i \cdot \nabla z_i)^{-\frac{1}{2}}\left(-\frac{\partial z_i}{\partial x}, -\frac{\partial z_i}{\partial y}, 1\right) \,,$$

where ∇ is the horizontal gradient operator. Consequently, requiring that fluid velocities are tangential at the impermeable boundaries yield

$$(3) \qquad \mathbf{u}_i \cdot \nabla z_i = w_i \quad \text{at} \quad z = z_i \,,$$

while at the fluid-fluid interface

$$(4) \qquad \varepsilon \frac{d\zeta}{dt} = w_i = \varepsilon \frac{\partial \zeta}{\partial t} + \mathbf{u}_i \cdot \nabla \zeta \quad \text{at} \quad z = \zeta \,.$$

Then, integrating equation (1) with respect to z between z_i and ζ, and using equations (3) and (4), yields

$$(5) \qquad \frac{(\zeta - z_i)}{\bar{\rho}_i}\left[\mathbf{v}_i \cdot \nabla \bar{\rho}_i + \varepsilon \frac{\partial \bar{\rho}_i}{\partial t}\right] + \varepsilon \frac{\partial \zeta}{\partial t} + \nabla \cdot \left((\zeta - z_i)\mathbf{v}_i\right) = 0 \,,$$

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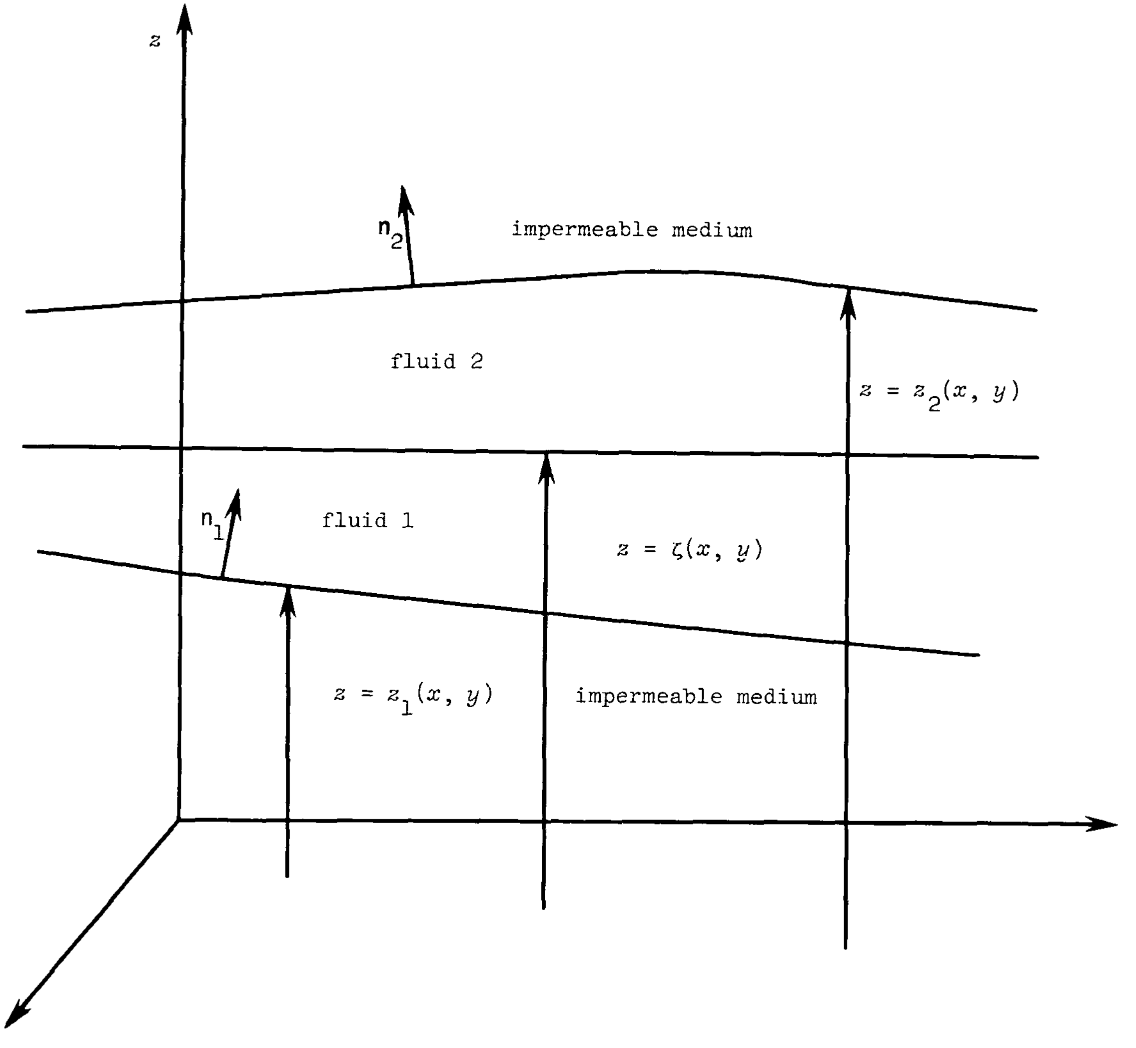

FIGURE 1. Definition sketch

where

$$(6) \qquad (\zeta - z_i)\bar{\rho}_i = \int_{z_i}^{\zeta} \rho_i dz \ ,$$

and

$$(7) \qquad (\zeta - z_i)\bar{\rho}_i \mathrm{v}_i = \int_{z_i}^{\zeta} \rho_i \mathrm{u}_i dz \ .$$

We shall end this section by deriving the flow equations when the reservoir boundaries are almost horizontal, since this requires little extra effort, although we shall discuss only horizontal boundaries for the remainder of the paper.

Let (x', y', z') be a Cartesian coordinate system with z' along a reservoir boundary normal, and write $(x^1, x^2, x^3) = (x, y, z)$, $\alpha, \beta \in \{1, 2, 3\}$, $a \in \{1, 2\}$; and similarly for primed indices. Then, if permeability is isotropic in surfaces locally tangential to the impermeable boundaries but may vary normal to them, Darcy's Law is

$$(8) \qquad u_i^{a'} = -\lambda_i \left(\frac{\partial P}{\partial x^{a'}} + \bar{\rho}_i g \frac{\partial P}{\partial x^{a'}} \right) ,$$

$$(9) \qquad u_i^{3'} = -\Lambda_i \left(\frac{\partial P}{\partial z'} + \bar{\rho}_i g \frac{\partial z}{\partial z'} \right) ,$$

where λ_i, Λ_i are the respective fluid mobilities, P pressure, g gravitational constant, and we have replaced ρ_i by $\bar{\rho}_i$ in equations (8) and (9).

In the vertical coordinate system

$$(10) \qquad u_i^{\alpha} = \frac{\partial x^{\alpha}}{\partial x^{a'}} u_i^{a'} + \frac{\partial x^{\alpha}}{\partial z'} u_i^{z'}$$

$$= -\lambda_i \frac{\partial x^{\alpha}}{\partial x^{a'}} \frac{\partial x^{\beta}}{\partial x^{a'}} \left(\frac{\partial P}{\partial x^{\beta}} + \bar{\rho}_i g \delta_{\beta,3} \right) + \frac{\partial x^{\alpha}}{\partial z'} u_i^{z'}$$

$$= -\lambda_i \left(\frac{\partial P}{\partial x^{\alpha}} + \bar{\rho}_i g \delta_{\alpha,3} \right) + \left(1 - \frac{\lambda_i}{\Lambda_i} \right) \frac{\partial x^{\alpha}}{\partial z'} u_i^{z'} \ ,$$

where we have used standard transformation laws, the orthogonality of transformations between Cartesian coordinate systems, and equations (8) and (9). Here $\delta_{\alpha,3}$ is the Kronecker delta function.

The Dupuit approximation that fluid velocities are everywhere tangential to impermeable boundaries $\left(u_i^{z'} = 0\right)$ implies from equation (10) that

(11) $$\mathrm{u}_i = -\lambda_i \nabla P \ ,$$

and from equation (2) that

(12) $$\mathrm{u}_i \cdot \nabla z_i = u_i^3 \ .$$

The solution of these equations is (to order terms quadratic in horizontal derivatives of z_i)

(13) $$P(z) = P_\zeta + \left(\bar{\rho}_i g - \nabla z_i \cdot \left(\nabla P_\zeta + \bar{\rho}_i g \nabla \zeta\right)\right)(\zeta - z) \ ,$$

provided we take

(14) $$\begin{aligned} \mathrm{u}_i &= -\lambda_i\left(\nabla P_\zeta + \left(\bar{\rho}_i g - \nabla z_i \cdot \left(\nabla P_\zeta + \bar{\rho}_i g \nabla \zeta\right)\right)\right)\nabla\zeta \\ &= \bar{\mathrm{u}}_i \ , \end{aligned}$$

where $P_\zeta = P(x, y, \zeta)$. From equation (13),

(15) $$\begin{aligned} \bar{P} &= \frac{1}{(\zeta - z_i)} \int_{z_i}^{\zeta} P dz \\ &= P_\zeta + \tfrac{1}{2}\left(\rho_i g - \nabla z_i \cdot \left(\nabla P_\zeta + \bar{\rho}_i g \nabla \zeta\right)\right)(\zeta - z_i) \ . \end{aligned}$$

As our Equations of State we take

(16) $$\frac{\partial \bar{\rho}_i}{\partial t} = \gamma_i \bar{\rho}_i \frac{\partial \bar{P}_i}{\partial t} \ ,$$

(17) $$\nabla \bar{\rho}_i = \gamma_i \bar{\rho}_i \nabla \bar{P}_i \ ,$$

where γ_i are the respective fluid compressibilities. Equations (5), (14) to (17), together with the assumption

(18) $$\mathrm{v}_i = \bar{\mathrm{u}}_i \ ,$$

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give the required differential equations for P_ζ and ζ .

However, as we are primarily interested in horizontal boundaries $(\nabla z_i = 0)$ in this paper, we shall summarise the equations above for this case:

$$(19) \qquad (\zeta - z_i)\gamma_i\left[\bar{u}_i \cdot \nabla\bar{P} + \varepsilon \frac{\partial\bar{P}}{\partial t}\right] + \varepsilon \frac{\partial\zeta}{\partial t} + \nabla \cdot \left((\zeta - z_i)\bar{u}_i\right) = 0 \ ,$$

$$(20) \qquad \bar{u}_i = -\lambda_i\left(\nabla P_\zeta + \bar{\rho}_i g \nabla\zeta\right) \ ,$$

$$(21) \qquad \bar{P} = P_\zeta + \tfrac{1}{2}\bar{\rho}_i g(\zeta - z_i) \ .$$

Finally, since equations (8) and (9) contain $\bar{\rho}_i$ rather than ρ_i , we have assumed that density variations in the vertical are small relative to the mean density. However, from equations (16), (17) and (21), as $\delta\rho_i/\bar{\rho}_i \simeq \gamma_i \delta P \simeq \gamma_i \bar{\rho}_i g \delta z$, equations (8) and (9) will be adequate approximations whenever

$$(22) \qquad \bar{\rho}_i g \gamma_j (z_2 - z_1) << 1 \ , \quad i = 1, 2 \ , \quad j = 1, 2 \ ,$$

where $\delta\rho_i$, δP and δz are respectively increments to ρ_i, P and z . Equation (22) allows $\bar{P}$ in equation (19) to be replaced by P_ζ , and we have done this in the remainder of this paper.

We assume that equation (22) holds for the rest of this paper. If fluids 1 and 2 are respectively water and a perfect gas, then equation (22) reduces to $\bar{\rho}_1 g\gamma_2(z_2 - z_1) = \bar{\rho}_1 g(z_2 - z_1)/P << 1$, where P is reservoir pressure. But it is reasonable to expect that before exploitation $P \gtrsim \rho_1 g(z_0 - z_2)$, where $(z_0 - z_2)$ is reservoir depth below sea, or ground level. Then equation (22) holds provided $(z_2 - z_1)/(z_0 - z_2) << 1$, or the ratio of reservoir height to reservoir depth below sea level, say, is small relative to unity.

3. Linearisation

In this section we shall linearise the equations in order to discuss some relevant properties of the system, before we treat the non-linear equations numerically in the next section.

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To begin we state that in many practical situations $|\bar{u}_i \cdot \nabla P_\zeta| \ll \varepsilon(\partial P_\zeta/\partial t)$, and in this section we shall assume this to be so. Further, linearising P_ζ and ζ about some constant initial values,

$$P_\zeta = P_0 + P'_\zeta \ , \quad \zeta = \zeta_0 + \zeta' \ , \quad \lambda_i = \lambda_i(P_0) \ ,$$

$$\bar{\rho}_i = \bar{\rho}_i(P_0) \ , \text{ and so on,}$$

and then dropping primes yields the linearised equations

$$\varepsilon\gamma_i(\zeta_0-z_i)\frac{\partial P_\zeta}{\partial t} + \varepsilon\frac{\partial\zeta}{\partial t} = \lambda_i(\zeta_0-z_i)\left[\nabla^2P_\zeta+\bar{\rho}_ig\nabla^2\zeta\right] \ . \tag{23}$$

Without loss of generality we take P_0 and ζ_0 as zero, so that the Laplace transform of equation (23) yields

$$\varepsilon\gamma_i(\zeta_0-z_i)sLP_\zeta + \varepsilon sL\zeta = \lambda_i(\zeta_0-z_i)\left[\nabla^2LP_\zeta+\bar{\rho}_ig\nabla^2L\zeta\right] \ , \tag{24}$$

where

$$(LP_\zeta,\ L\zeta) = \int_0^\infty e^{-st}(P_\zeta,\ \zeta)dt \ .$$

Writing out equation (24) for each fluid, and eliminating either LP_ζ or $L\zeta$ yields

$$\left(a\nabla^4-bs\nabla^2+cs^2\right)\left(LP_\zeta,\ L\zeta\right) = 0 \ , \tag{25}$$

where

$$a = \lambda_1\lambda_2(z_2-\zeta_0)(\zeta_0-z_1)g(\bar{\rho}_1-\bar{\rho}_2) \ , \tag{26}$$

$$b = \varepsilon\left((z_2-\zeta_0)\lambda_2+(\zeta_0-z_1)\lambda_1\right) \ , \tag{27}$$

$$c = \varepsilon^2\left((z_2-\zeta_0)\gamma_2+(\zeta_0-z_1)\gamma_1\right) \ , \tag{28}$$

and equation (22) has been used.

The characteristic equation associated with equation (25) is

$$aD^{-2} - bD^{-1} + c = 0 \ ,$$

$$D^{-1} = (b/2a)\left(1\pm\left(1-\left(4ac/b^2\right)\right)^{\frac{1}{2}}\right) \ , \tag{29}$$

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where D has dimensions L^2T^{-1} , and so behaves as a diffusivity. Clearly, $|4ac/b^2| < 2|(\bar\rho_1-\bar\rho_2)|g(\gamma_2(z_1-\zeta)+\gamma_1(\zeta-z_1))$, which is less than unity when equation (22) holds. When $4ac/b^2 \ll 1$, the two roots of equation (29) are

$$(30) \qquad D \simeq \frac{b}{c} = \frac{\lambda_1(\zeta_0-z_1)+\lambda_2(z_2-\zeta_0)}{\varepsilon(\gamma_1(\zeta_0-z_1)+\gamma_2(z_2-\zeta_0))} = G ,$$

$$(31) \qquad D \simeq \frac{a}{b} = \frac{\lambda_1\lambda_2(\zeta_0-z_1)(z_2-\zeta_0)g(\bar\rho_1-\bar\rho_2)}{\varepsilon(\lambda_1(\zeta_0-z_1)+\lambda_2(z_2-\zeta_0))} = W ,$$

with G reducing to the well-known single compressible fluid diffusivity $(\lambda/\varepsilon\gamma)$ when $\zeta = z_i$, and W is analogous to a result of Buckley and Leverett [1, p. 535, eqn. 9.5.64]. From equations (30), (31) and (22), $W/G \ll 1$, since $2W/G = 2ac/b^2 < (\bar\rho_1-\bar\rho_2)g(\gamma_1(\zeta_2-z_1)+\gamma_2(z_2-\zeta_0))$. As ζ_0 varies from z_1 to z_2 , G varies monotonically from $\lambda_2/\varepsilon\gamma_2$ to $\lambda_1/\varepsilon\gamma_1$, whereas W has a maximum at $\zeta_0 = (\sqrt{\lambda_1}z_1 + \sqrt{\lambda_2}z_2)/(\sqrt{\lambda_1} + \sqrt{\lambda_2})$, and is zero when $\zeta = z_i$. An example of G and W are given in Figure 2 where we have chosen parameter values from the D sands of the Maui gas field. For this case the gas dominates G until the water level has risen to approximately 80% of the maximum height, $z_2 - z_1$; whereas the maximum of W occurs at 79% of (z_2-z_1) .

We shall end this section by discussing a simple solution to equation (25) in order to illustrate the roles of the diffusivities G and W .

Consider the one-dimensional problem in the halfspace $x > 0$, with a constant velocity withdrawal from $x = 0$. From equation (20),

$$\frac{Q_i}{\lambda_i} = \frac{\partial P_\zeta}{\partial x} + \bar\rho_i g \frac{\partial \zeta}{\partial x} \quad \text{at} \quad x = 0 ,$$

for some constants Q_i , and so

$$(32) \qquad \frac{\partial \zeta}{\partial x} = \left(\frac{Q_1}{\lambda_1} - \frac{Q_2}{\lambda_2}\right) / (\bar\rho_1-\bar\rho_2)g , \text{ at } x = 0 ,$$

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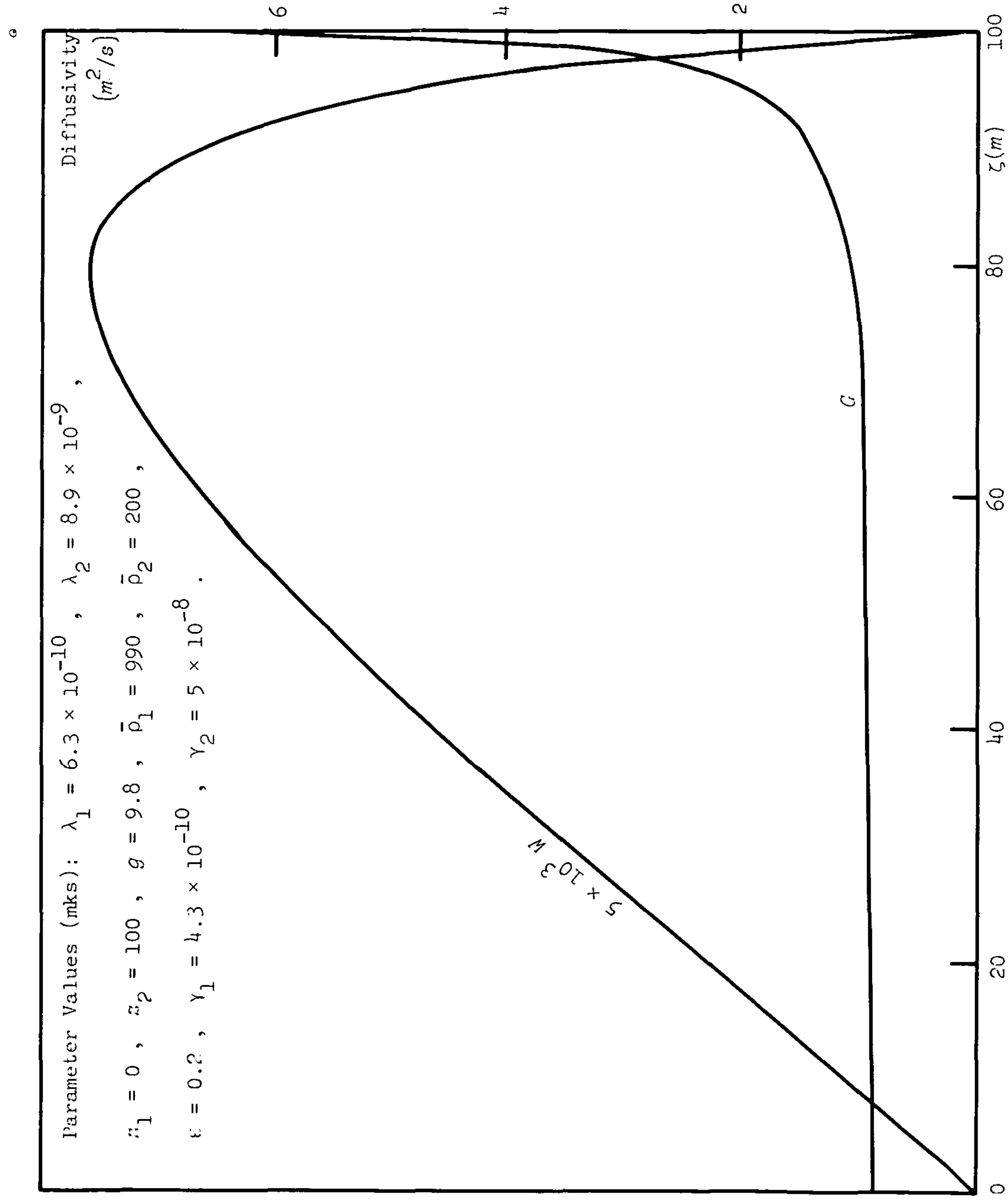

FIGURE 2. Apparent diffusivities versus ζ

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(33) $$\frac{\partial P_\zeta}{\partial x} = \left(\frac{\bar\rho_1 Q_2}{\lambda_2} - \frac{\bar\rho_2 Q_1}{\lambda_1}\right) \Big/ \left(\bar\rho_1 - \bar\rho_2\right) \ , \ \text{at} \quad x = 0 \ .$$

A tedious calculation using equations (24), (22), (30) to (33), and the assumption $\bar\rho_1 > \bar\rho_2$ gives

$$LP_\zeta \simeq \frac{-\sqrt{G}}{s^{3/2}\left(\lambda_1\left(\zeta_0 - z_1\right) + \lambda_2\left(z_2 - \zeta_0\right)\right)} \left(\left(Q_1\left(\zeta_0 - z_1\right) + Q_2\left(z_2 - \zeta_0\right)\right)e^{-\sqrt{(s/G)}x} + \sqrt{W/G}\,\frac{\left(\rho_1\lambda_1\left(\zeta_0 - z_1\right) + \rho_2\lambda_2\left(z_2 - \zeta_0\right)\right)}{\left(\bar\rho_1 - \bar\rho_2\right)}\left(\frac{Q_2}{\lambda_2} - \frac{Q_1}{\lambda_1}\right)e^{-\sqrt{(s/W)}x}\right) ,$$

$$L\zeta \simeq \frac{\sqrt{W}}{g\left(\bar\rho_1 - \bar\rho_2\right)s^{3/2}} \times \left(\left(\frac{Q_2}{\lambda_2} - \frac{Q_1}{\lambda_1}\right)e^{-\sqrt{(s/W)}x} - \frac{\left(\lambda_1\gamma_2 - \lambda_2\gamma_1\right)\sqrt{W}\left(Q_1\left(\zeta_0 - z_1\right) + Q_2\left(z_2 - \zeta_0\right)\right)e^{-\sqrt{(s/G)}x}}{\lambda_1\lambda_2\sqrt{G}\left(\gamma_1\left(\zeta_0 - z_1\right) + \gamma_2\left(z_2 - \zeta_0\right)\right)}\right) ,$$

whose inverse Laplace transform is well documented. For exploitation of fluid 2 alone $\left(Q_1 = 0 \ , \ Q_2 > 0\right)$ and for the numerical values in Figure 2, the initial pressure drop and interface rise at the origin are determined largely from uncoupled linear diffusion equations with respective diffusivities G and W and satisfying the boundary conditions in equations (31) and (32). Other linearised solutions can be obtained from superposition of solutions from the diffusion equation with the respective diffusivities in equations (29).

4. Numerical solutions

The dependence of mobilities, densities and compressibilities $\left(\lambda_i, \bar\rho_i, \gamma_i\right)$ on the interfacial pressure, P_ζ, is required in order to solve the full non-linear equations (19) to (21). In this section we shall assume that fluid 1 and 2 behave respectively as a perfect liguid and gas. Then

(34) $$\lambda_i, \ \bar\rho_1, \ \bar\rho_2/\bar P_\zeta, \ \gamma_1 \ \text{ are constant, and } \ \gamma_2 P_\zeta = 1 \ .$$

If equations (20) and (21) are substituted into equation (19), two equations result, the appropriate weighted sums of which yield

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(35) $$\varepsilon(\gamma_1(\zeta-z_1)+\gamma_2(z_2-\zeta))\frac{\partial P_\zeta}{\partial t}$$
$$= \nabla\cdot(\lambda_1(\zeta-z_1)+\lambda_2(z_2-\zeta))\nabla P_\zeta + \nabla\cdot(\lambda_1\bar\rho_1 g(\zeta-z_1)+\lambda_2\bar\rho_2 g(z_2-\zeta))\nabla\zeta$$
$$+ (\lambda_1\gamma_1(\zeta-z_1)+\lambda_2\gamma_2(z_2-\zeta))\nabla P_\zeta\cdot\nabla P_\zeta$$
$$+ (\lambda_1\gamma_1\bar\rho_1 g(\zeta-z_1)+\lambda_2\gamma_2\bar\rho_2 g(z_2-\zeta))\nabla P_\zeta\cdot\nabla\zeta ,$$

(36)
$$\varepsilon(\gamma_1(\zeta-z_1)+\gamma_2(z_2-\zeta))\frac{\partial\zeta}{\partial t}$$
$$= \nabla\cdot((\lambda_1\gamma_2-\lambda_2\gamma_1)(\zeta-z_1)(z_2-\zeta)\nabla P_\zeta) + \nabla\cdot((\lambda_1\gamma_2\bar\rho_1-\lambda_2\gamma_1\bar\rho_2)g(\zeta-z_1)(z_2-\zeta)\nabla\zeta)$$
$$+ (\lambda_1\gamma_2\bar\rho_1 g(\zeta-z_1)+\lambda_2\gamma_1\bar\rho_2 g(z_2-\zeta))\nabla\zeta\cdot\nabla\zeta$$
$$+ (\lambda_1\gamma_1+\lambda_1\gamma_2-\lambda_2\gamma_1)\gamma_2(\zeta-z_1)(z_2-\zeta)\nabla P_\zeta\cdot\nabla P_\zeta$$
$$+ [\lambda_1\gamma_2(\zeta-z_1)+\lambda_2\gamma_1(z_2-\zeta)+(\zeta-z_1)(z_2-\zeta)g\gamma_2[\gamma_1\lambda_1\bar\rho_1-\gamma_1\lambda_2\bar\rho_2+\gamma_2\lambda_1\bar\rho_1]]\nabla P_\zeta\cdot\nabla\zeta .$$

Introducing the definitions

$$P_\zeta(x,\ t) = P_\zeta(x,\ 0)P'(x,\ t) , \qquad \nabla = L^{-1}\nabla' ,$$
$$\frac{\partial}{\partial t} = T^{-1}\frac{\partial}{\partial\tau} , \qquad \zeta = (z_2-z_1)\zeta' ,$$
(37) $$z_i = (z_2-z_1)z_i' , \qquad \bar\rho_2(x,\ t) = \bar\rho_2(x,\ 0)P'(x,\ t) ,$$

where L and T are respectively length and time scales, allows equations (35) and (36) to be written in non-dimensional form. The resulting equations (which we omit) were solved numerically using the NAG package DO3PGF, for one-dimensional Cartesian coordinates.

Our boundary conditions were that fluid 1 had zero velocity at $x = 0$, while both fluids had zero velocity at $x = L$. Fluid 2 is withdrawn at $x = 0$ at a constant rate (by mass) exhausting the reservoir in time T. We chose $L = 4000$, $T = 2\times10^8$, $\zeta'(x,\ 0) = 0.7$, $P_\zeta(x,\ 0) = 3\times10^7$, and the numerical values used in Figure 2.

At least three different processes can be inferred from the numerical results. Firstly, Figure 3 (page 470) shows that initially substantial upconing is confined about the sink, where large spatial gradients at ζ are required to satisfy equation (31). Initially the diffusivity W should be relevant near $x = 0$ and the numerical values in Figure 3 tend

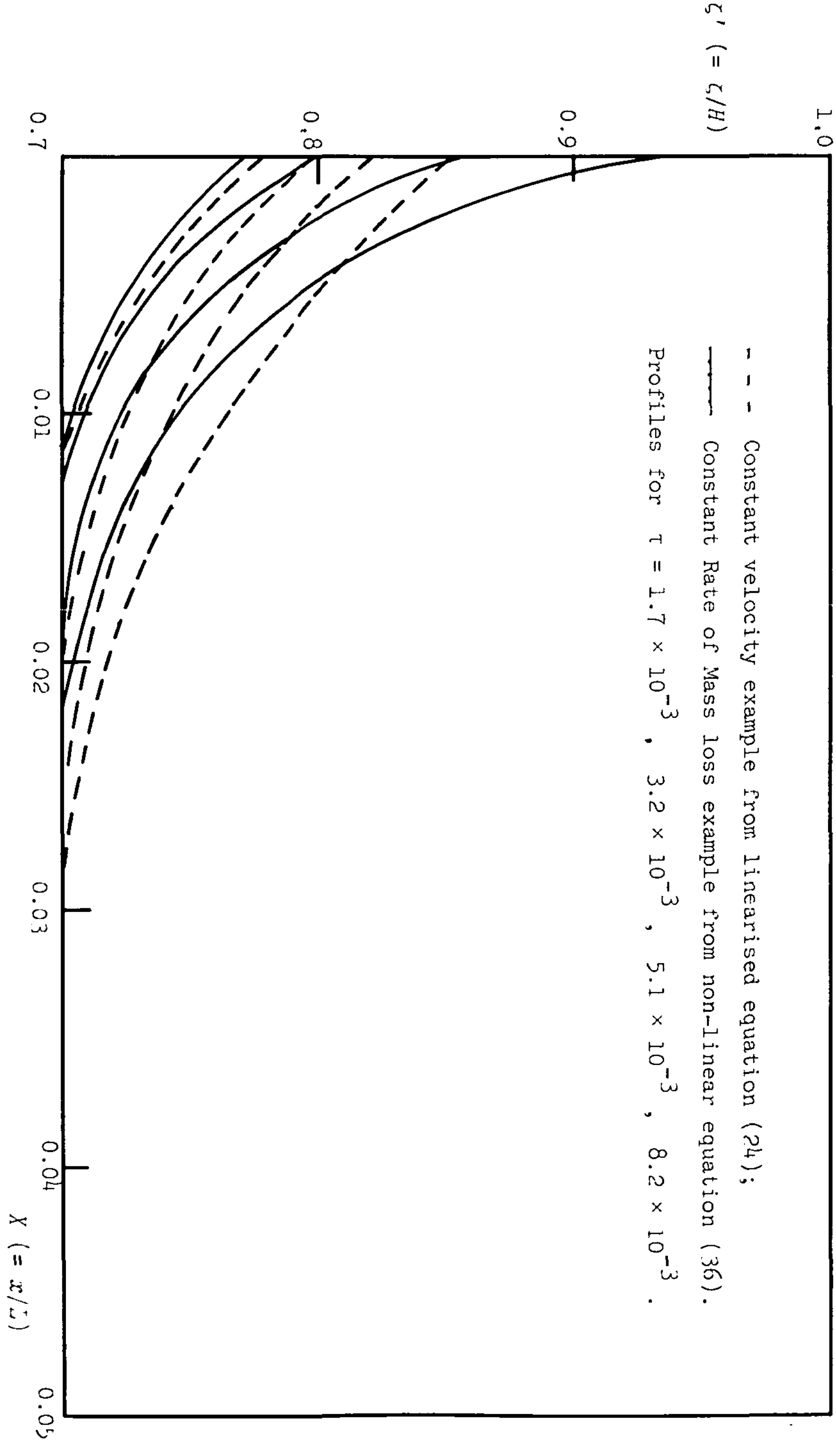

FIGURE 3. Upconing about a sink

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TABLE 1

Pressure drawdown without upconing ($\zeta' = 0.7$)

τ \ X	0.0	0.1	0.33	0.5	0.73	0.9	1.0
0.00323	0.894	0.944	0.981	0.993	0.998	1.00	1.00
0.0112	0.798	0.857	0.915	0.943	0.966	0.974	0.975
0.0236	0.692	0.759	0.827	0.862	0.891	0.902	0.904
0.0350	0.600	0.675	0.749	0.787	0.819	0.831	0.833
0.0477	0.496	0.583	0.665	0.706	0.741	0.753	0.755
0.0594	0.394	0.496	0.588	0.633	0.670	0.683	0.686
0.0707	0.282	0.410	0.514	0.564	0.604	0.618	0.620

TABLE 2

Pressure drawdown with upconing

τ \ X	0.0	0.1	0.33	0.5	0.73	0.9	1.0
0.00588	0.897	0.937	0.970	0.984	0.994	0.997	0.997
0.0222	0.795	0.844	0.887	0.910	0.930	0.937	0.938
0.0413	0.699	0.752	0.799	0.824	0.845	0.583	0.854
0.0636	0.591	0.649	0.700	0.727	0.750	0.759	0.760
0.0850	0.491	0.555	0.610	0.639	0.664	0.673	0.674
0.111	0.381	0.451	0.511	0.542	0.568	0.577	0.579
0.139	0.279	0.354	0.417	0.449	0.475	0.485	0.487

to support this. Secondly, beyond the sink region the interface drops slightly below its original height and the diffusivity, G (equation (30)), should be relevant here. Thirdly, for long times, the depletion time, T, will be important.

Finally, Tables 1 and 2 show that in the presence of a mobile surface it takes longer for a fixed pressure drop to occur at a sink and that a

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smaller pressure drop exists across the reservoir.

5. Concluding remarks

The pressure and interface response in an idealised, horizontal, two-fluid reservoir under exploitation has been discussed. Although equations were derived for almost horizontal boundaries, we quickly restricted discussion to exactly horizontal boundaries. The two main geometric assumptions were that the ratios of reservoir height to both reservoir length and reservoir depth below sealevel, say, were small relative to unity. The former assumption led to Boussinesq averaging over the reservoir height, and the latter to a characteristic reservoir pressure P_ζ as well as implying that the interface diffusivity W was much smaller than the pressure diffusivity, G.

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