Further $E(x^{(i)}y^{(j)}) = a_{ij}$ and x and y are independent only if all the a_{ij} vanish. Theorem 1 evidently requires that each of the members of a complete set on the first distribution is uncorrelated with each of the members of a complete set on the second distribution. This example can be extended to show that the orthonormal sets chosen must be complete in the enunciation of Theorem 2.

Much has been written on the relation between independence and zero correlation but it appears that these theorems are more general than any previously proved. They are more general than that of Sarmanov [4] because they hold for several dimensions and avoid making the bounded ϕ^2 restriction namely that dP/dP^*, the Radon-Nikodym derivative of P with regard to P^*, is square summable on P^*. The probability function in multivariate ϕ^2 bounded distributions can be written as a product of the marginal probabilities by a series in products of orthonormal functions, in which the coefficient of each product is simply its expectation. Such expansions are given in Lancaster [1] and [3]. Dependencies in multivariate distributions are classified in Lancaster [3] by the vanishing of various classes of the generalised correlation coefficients.

It is evident from these papers that the ordinary Pearson χ^2 chooses the indicator variables on the marginal distributions as the set of functions to be normalised. However, the method can be generalised to use, for example, the Hermite-Chebyshev polynomials in the joint normal case. The χ^2 in either of these cases tests whether the sum of the squares of the generalised coefficients of correlation can be all considered to be zero.

References

[1] Lancaster, H. O., The structure of bivariate distributions, Ann. Math. Statist. 29 (1958), 719–736.

[2] Lancaster, H. O., Zero correlation and independence, Aust. J. Statist. 1 (1959), 53–56.

[3] Lancaster, H. O., On tests of independence in several dimensions, J. Aust. Math. Soc. 1 (1960), 241–254.

[4] Sarmanov, O. V., Maximum correlation coefficient (non-symmetrical case), Dokl. Akad. Nauk S.S.S.R. 121 (1958), 52–55.

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CORRECTIONS

to H. O. Lancaster: "On tests of independence in several dimensions". This Journ. 1 (1960), 241–254.

At the top of page 244, equation (3) should read

$$\phi^2_{xy} + \phi^2_{xz} + \phi^2_{yz} + \phi_{xyz} = \phi^2 \tag{3}$$

The statement in the lines following (3) that "each ρ is less in absolute value than unity" is not universally true.

In the statement of Theorem 8, an integral sign has been omitted after "expression".

In (18) for $\pm$ read $-$.

On page 253, in lines 14 and 15 in place of "$N^{\frac{1}{2}}$ times square roots of the ϕ^2_{xy}, ϕ^2_{xz} and ϕ^2_{yz}", read "$N\phi^2_{xy}$, $N\phi^2_{xz}$ and $N\phi^2_{yz}$".