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A UNIFIED APPROACH TO CONTINUOUS AND CERTAIN NON-CONTINUOUS FUNCTIONS

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Abstract

A unified theory of continuous and certain non-continuous functions is proposed and developed. The proposed theory encompasses in one the theories of continuous functions, upper (lower) semicontinuous functions, almost continuous functions, c-continuous functions, c^*-continuous functions, s-continuous functions, 1-continuous functions, H-continuous functions, and the ε-continuous functions of Klee.

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1. Introduction

In recent years there has been an awakening of interest in results dealing with discontinuous functions and operations. Moreover, the notions of certain non-continuous functions seem natural in connection with the mathematical modelling of certain physical problems, where there is strong empirical evidence of a weaker form of the continuity of a function, though its full-continuity cannot be tested experimentally (see [1], [8]).

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Various types of non-continuous functions occur in the literature which have been extensively studied by several authors from different viewpoints. Certain of these non-continuous functions have properties similar to those of continuous functions and their theories run, either in part or in whole, parallel to the theory of continuous functions. Further, analogies inherent in their definitions, as well as the nature of results obtained in the process of their study, suggest the need of formulating a unified theory, including the theory of continuity and its generalizations. The purpose of this paper is an attempt leading towards the fulfillment of this need.

In Section 2 of this paper we introduce the notion of a P-continuous function, which leads to the formulation of a coherent unified theory of continuity and certain variants of continuity. Basic properties of P-continuous functions are studied in Section 4. In Section 5, sufficient conditions on domain and/or range are obtained which imply the continuity of P-continuous functions. The results obtained in the process unify and improve scores of results in the literature pertaining to continuous functions and certain classes of non-continuous functions.

2. A unified theory

Throughout the paper, P will denote a property, not necessarily topological, possessed by certain subsets of a topological space. In particular, for example, P may denote any one of the following properties: the property of being a (regularly) closed set; the property of being a δ-closed set ([28], [40]); the property of being a (regularly) open set; the property of being a regularly closed set; the property of being a zero set; the property of having complement of diameter at least ε ($\varepsilon > 0$ is a real number and the space under consideration is assumed to be a metric space); the property of being H-closed (referred to as absolutely closed in [2]) or quasi H-closed ([32], [35]); compactness; countable compactness; the Lindelöf property; and connectedness. It may be observed that the first five properties mentioned in the above list are not necessarily preserved under topological embeddings and the sixth is not a topological invariant, while the last four properties are well-known continuous invariants.

In the sequel that follows, we shall note that corresponding to each property mentioned in the above paragraph, there corresponds a concept like continuity in the literature, which in general is weaker than the usual concept of continuity.

DEFINITIONS 2.1. Let X be a topological space and let $A \subset X$. We say that

(i) A is a *P-set* if A possesses property P, and

(ii) A has *P-complement* if $X - A$ possesses property P.

DEFINITIONS 2.2. Let $f: X \to Y$ be a function from a topological space X into a topological space Y. Then f is said to be

(i) *P-continuous* if for each $x \in X$ and each open set V containing $f(x)$ and having P-complement there is an open set U containing x such that $f(U) \subset V$;

(ii) a *semi P-function* if for each closed P-set $K \subset Y$, $f^{-1}(K)$ is a closed P-set; and

(iii) a *weak semi P-function* if for each closed P-set $K \subset Y$, $f^{-1}(K)$ is closed.

Several authors have studied semi P-functions for a particular property P. For example, while Lee [14], Jones [6] and Long [15] have studied semi P-functions for P-equal to connectedness, Mathur [24] and Noiri [30] introduced semi P-functions for P equal to the property of being a δ-closed set and when P is the property of being a zero set the corresponding semi P-functions are considered in [38]. Moreover, it turns out that in general the concepts of a continuous function and a semi P-function are independent of each other and either of them is a weak semi P-function. However, a weak semi P-function need not be either continuous or a semi P-function (see [9, page 175]). Further, we shall see that the concepts of a P-continuous function and a weak semi P-function coincide (Theorem 3.1).

Rayburn [33] has studied continuous mappings between Tychonoff spaces which pull back hard sets to hard sets. Continuous mappings, which pull back compact sets to compact sets, occur frequently in the literature and are of significance in both topology and analysis. Yet another point which may be advanced in favour of semi P-functions is that, in contrast to P-continuous functions, the composition of two semi P-functions is again a semi P-function, and hence their study from a categorical viewpoint seems useful.

Tables 1 and 2 below illustrate the type of P-continuity and the type of semi P-function induced by a property P. References are quoted as an aid to the literature of the corresponding non-continuous function. However, no claim is made to completeness or originality of the source.

3. Basic properties of P-continuous functions

In this section we study basic properties of P-continuous functions. It turns out that the proofs of the main results pertaining to P-continuous functions are not so very different from those in the classical case of continuity. Hence, in the sequel, we shall omit the proofs of some results and include only those which are necessary for the clarity and continuity of the presentation.

TABLE 1

	Property P	Type of P-continuous function	References
1.	the property of being a closed set	continuous function	
2(a).	the property of being a regularly closed set	almost continuous function	[16, 29, 25, 26, 36]
2(b).	the property of being a δ-closed set	almost continuous function	[16, 19, 25, 26, 36]
3.	compactness	c-continuous function	[3, 18, 21, 27]
4.	countable compactness	c^*-continuous function	[21, 31]
5.	the property of being quasi H-closed	H-continuous function	[17, 29]
6.	the property of being a regularly closed compact set	almost c-continuous function	[5, 27]
7.	connectedness	s-continuous function	[9, 12, 34]
8.	the Lindelöf property	l-continuous function	[13]
9.	the property of having complement of diameter $\geq \varepsilon$ (here the space is assumed to be a metric space)	ε-continuous function	[7, 8, 29]
10.	the property of being an open set	mildly continuous function	[37]
11.	the property of being a zero set	z-continuous function	[38]
12.	being a ray $(-\infty, a]$, $a \in \mathbf{R}$ (here $Y = \mathbf{R}$)	lower semicontinuous function	

TABLE 2

Property P	Type of semi P-function	References
1. connectedness	semiconnected function	[6, 14, 15]
2. the property of being a δ-closed set	δ-continuous function	[24, 30]
3. the property of being a closed set	continuous function	
4. the property of being a zero set	z-continuous function	[38]

THEOREM 3.1. *Let $f: X \to Y$ be a function from a topological space X into a topological space Y. The following statements are equivalent.*

(a) *f is P-continuous.*

(b) *If V is an open subset of Y having P-complement, then $f^{-1}(V)$ is an open subset of X.*

(c) *f is a weak semi P-function.*

(d) *For each $x \in X$ and each net $\{x_\alpha\}$ which converges to x, the net $\{f(x_\alpha)\}$ is eventually in each open set containing $f(x)$ and having P-complement.*

COROLLARY 3.2. *Let P denote a property possessed by all singletons in a topological space and let $f: X \to Y$ be P-continuous and injective. If Y is T_1, then so is X.*

REMARK 3.1. Since every singleton in a T_1-space is closed, compact and connected, Corollary 3.2 includes [18, Theorem 3] and [9, Corollary 2.2].

THEOREM 3.3. *Suppose P is a property possessed by all singletons in a topological space and let $f: X \to Y$ be a P-continuous, closed function from a normal space X onto a space Y. If either of the spaces X and Y is T_1, then Y is Hausdorff.*

PROOF. In view of closedness of the function f, in either case, we may assume that the space Y is T_1. Let y_1 and y_2 be any two distinct points in Y. Then $\{y_1\}$ and $\{y_2\}$ are closed P-sets in Y so that by Theorem 3.1, $f^{-1}(y_1)$ and $f^{-1}(y_2)$ are closed subsets of X. By normality of X, there are disjoint open sets U_1 and U_2 containing $f^{-1}(y_1)$ and $f^{-1}(y_2)$, respectively. Since f is closed, the sets $V_1 = Y - f(X - U_1)$ and $V_2 = Y - f(X - U_2)$ are open in Y. It is easily verified that V_1 and V_2 are disjoint and contain y_1 and y_2 respectively. Thus Y is Hausdorff.

REMARK 3.2. The substitution of compactness for P in Theorem 3.3 yields [18, Theorem 4] on c-continuous functions and the substitution of connectedness for P gives [9, Theorem 2.3] pertaining to s-continuous functions.

REMARK 3.3. There is a continuous open mapping of a subset of the plane onto a non-Hausdorff T_1-space (see [9, Remark 2.1]). Thus, Theorem 3.3 is false with 'closed' replaced by 'open'. Moreover, the identity mapping of an infinite indiscrete space onto the same set endowed with the cofinite topology shows that the hypothesis of P-continuity is essential in Theorem 3.3.

DEFINITION 3.1 [20]. A function $f: X \to Y$ is said to have a *strongly closed graph* in case for each $(x, y) \notin G(f)$ there are open sets U and V containing x and y respectively, such that $U \times \overline{V}$ is disjoint from $G(f)$.

THEOREM 3.4. *Let $f: X \to Y$ be P-continuous and let Y be a Hausdorff space such that Y possesses a base of closed P-neighbourhoods. Then f has a strongly closed graph.*

PROOF. Let (x, y) be any point of $X \times Y$ which does not lie in the graph of f. Then $f(x) \neq y$. Since Y is Hausdorff, there are disjoint open sets V_1 and V_2 containing $f(x)$ and y, respectively. By hypothesis on Y, there exists a closed P-neighbourhood V of y which is contained in V_2. By Theorem 3.1, $f^{-1}(V)$ is closed in X and does not contain x. Since f is P-continuous, there is an open set U such that $x \in U \subset X - f^{-1}(V)$ and such that $f(U) \subset Y - V$. Then $U \times W$ contains (x, y), where $W = \operatorname{int} V$ and $U \times \overline{W}$ is disjoint from $G(f)$. Thus $G(f)$ is strongly closed in $X \times Y$.

REMARK 3.4. Theorem 3.4 contains several known results in the literature. For example, with P being countable compactness, it gives an improved version of [21, Theorem 3.2]; with P being H-closed it yields an improved version of [17, Theorem 8] and for P being connectedness it reduces to [9, Theorem 2.6]. Similarly, for P being compactness it yields [27, Theorem 3.4], and if P is the property of being regular closed, it gives a sufficient condition for an almost continuous function to have a strongly closed graph.

THEOREM 3.5. *Let P denote a finitely productive property and let $f: X \to Y$ be a function from a topological space X into a topological space Y such that the graph function is P-continuous. If X possesses property P, then f is P-continuous.*

Proof. Let $x \in X$ and let V be an open set containing $f(x)$ such that $Y - V$ is a P-set. Then $p_y^{-1}(V)$ is open in $X \times Y$. Since each of the spaces X and $Y - V$ have property P and since P is finitely productive, $X \times (Y - V) = X \times Y - p_y^{-1}(V)$ possesses property P. Thus $p_y^{-1}(V)$ is an open set in $X \times Y$ having P-complement. Therefore, by P-continuity of g, there is an open set U containing x such that $g(U) \subset p_y^{-1}(V)$. It follows that $p_y(g(U)) = f(U) \subset V$, so f is P-continuous.

Remark 3.5. Theorem 3.5 contains several known results in the literature. For example, with P being compactness (respectively connectedness) it gives [18, Theorem 9] (respectively [9, Theorem 2.7]). Similarly, the substitution of quasi H-closed for P in Theorem 3.5 yields an improved version of Theorem 3.12 of Noiri [29] where X is required to be compact.

Remark 3.6. We do not know whether the converse of Theorem 3.5 is true. In particular, it is not known whether the converse of Theorem 3.5 holds for P being compactness or connectedness (see [9], [18]).

Theorem 3.6. *Let $f: X \to Y$ be any function.*
(a) *If f is P-continuous and $A \subset X$, then $f|A: A \to Y$ is P-continuous.*
(b) *If $\{U_\alpha: \alpha \in A\}$ is an open cover of X and if for each α, $f_\alpha = f|U_\alpha$ is P-continuous, then f is P-continuous.*
(c) *If $\{F_\beta: \beta \in B\}$ is a locally finite closed cover of X and if for each β, $f_\beta = f|F_\beta$ is P-continuous, then f is P-continuous.*

Remark 3.7. Theorem 3.6 includes several known results in the literature. For example, with P being connectedness it gives [9, Theorem 2.8], for P being compactness it yields an assertion which improves Theorems 2 and 4 of Gentry and Hoyle [3], and the substitution of the property of being regular closed for P yields Theorems 2.6, 3.7 and 2.8 of Singal and Singal [36]. Similarly, with P the property of being quasi H-closed we get Theorems 4 and 6 of Long and Hamlett [17] and with P the property of being a closed set, we get well-known classical results pertaining to continuous functions.

Theorem 3.7. *If $f: X \to Y$ is continuous and $f: Y \to Z$ is P-continuous, then $g \circ f: X \to Z$ is P-continuous.*

Proof. Let K be a closed P-subset of Z. By Theorem 3.1 $g^{-1}(K)$ is closed and since f is continuous, $(g \circ f)^{-1}(K) = f^{-1}(g^{-1}(K))$ is closed in X.

Remark 3.8. In general the composition of a P-continuous function $f: X \to Y$ and a continuous function $g: Y \to Z$ need not be P-continuous

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(see for example [9, Remark 2.2], [3, Example 3] and [17, Example 2]). Thus, in particular, the composition of P-continuous functions may fail to be P-continuous.

REMARK 3.9. Theorem 3.7 yields [3, Theorem 3], [9, Theorem 2.9] and [17, Theorem 5] respectively on substituting compactness, connectedness and the property of being a quasi H-closed set, for P, respectively. Further, the substitution of the property of being a closed set in Theorem 3.7 for P yields the fact that the composition of continuous functions is continuous and the substitution of the property of being a regularly closed set for P gives the corresponding result for almost continuous functions.

THEOREM 3.8. *Let $f: X \to Y$ be a quotient mapping. Then a function $g: Y \to Z$ is P-continuous if and only if $g \circ f$ is P-continuous.*

REMARK 3.10. The substitution of connectedness for P in Theorem 3.8 yields [9, Theorem 2.10] pertaining to s-continuous functions. If P is the property of being a closed set, then one gets the well-known result that a function out of a quotient space is continuous if and only if its composition with the quotient mapping is continuous. Similarly, the substitution of the property of being a regularly closed set for P yields an assertion which generalizes Theorem 2.5 of Singal and Singal on almost continuous functions [36].

4. Continuity of P-continuous functions

In this section we obtain sufficient conditions on the domain or the range (or both) to imply continuity of P-continuous functions, which in particular yields sufficient conditions for continuity of almost continuous functions, c-continuous functions, c^*-continuous functions, H-continuous functions, s-continuous functions and many other non-continuous functions. First we quote the following definition from [22].

DEFINITION 4.1. A topological space X is called a *saturated space* if any intersection of open sets in X is itself an open set equivalently, every point of X possesses a minimum neighbourhood.

THEOREM 4.1. *Let X be a saturated space and suppose Y possesses a base of closed P-neighbourhoods. If $f: X \to Y$ is P-continuous, then f is continuous.*

PROOF. Let $x \in X$ and let V be an open subset of Y containing $f(x)$. By hypothesis on Y, there is a P-neighbourhood U of $f(x)$ such that $U = \overline{U} \subset V$. Let $y \in Y - U$. Again, by hypothesis on Y, there is a closed P-neighbourhood U_y of y such that $U \cap U_y = \varnothing$. Thus $Y - U_y$ is an open set containing $f(x)$ and having P-complement. Since f is P-continuous, there is an open set N_y containing x such that $f(N_y) \subset Y - U_y$. Let $N = \bigcap\{N_y : y \in Y - U\}$. Now, N contains x and since X is a saturated space, N is open. Clearly, $f(N) \subset U \subset V$ and hence f is continuous.

REMARK 4.1. The substitution of compactness (respectively connectedness) for P in Theorem 4.1 gives [3, Theorem 7] (respectively [9, Theorem 2.11]). Similarly, the substitution of the property of being quasi H-closed for P in Theorem 4.1 yields the corresponding result pertaining to H-continuous functions.

DEFINITION 4.2. A topological space X is called a *semilocally P-space* if for each $x \in X$ and each open set U containing x there is an open set V such that $x \in V \subset U$ and $X - V$ is the union of finitely many disjoint closed P-sets.

It is obvious that the notion of a semilocally P-space is a simultaneous abstraction of the concepts of a semilocally connected space and a semiregular space.

THEOREM 4.2. *If $f: X \to Y$ is P-continuous and if Y is a semilocally P-space, then f is continuous.*

PROOF. Let $x \in X$ and let V be an open neighbourhood of $f(x) = y$ in Y. Since Y is a semilocally P-space, there is an open set $N_y \subset V$ containing y and such that $Y - N_y$ consists of a finite number of closed P-sets $S_1, S_2, \dots, S_n$. For each $k = 1, 2, \dots, n$, $f^{-1}(S_k)$ is a closed set by Theorem 3.1. Therefore, $\bigcup_{k=1}^{n} f^{-1}(S_k) = A$ is a closed subset of X and does not contain any point of $f^{-1}(y)$. So, $U = X - A$ is an open set containing x and $f(U) = N_y \subset V$. Thus f is continuous.

REMARK 4.2. Theorem 4.2 includes several results in the literature and has many important implications. For example, with P being connectedness Theorem 4.2 and Theorem 3.1 give Theorem 9 of Sanderson [34] and Theorem 2.12 of Kohli [9], which in turn includes a result of Lee [14] (and Long [15]) pertaining to semiconnected functions. Similarly, the substitution of regular closed for P in Theorem 4.2 yields Theorem 2.4 of Singal and Singal [36].

Theorem 4.3 [11]. *Let $f: X \to Y$ be a closed (or an open) connected monotone function into a T_1-semilocally connected space Y. Then f is continuous.*

Proof. It is easily verified that for a closed connected subset K of Y, $f^{-1}(K)$ is closed and connected. Now, with P being connectedness and from Theorem 3.1, we have f is P-continuous and so in view of Theorem 4.2, f is continuous.

Remark 4.3. Theorem 4.3 generalizes Theorems 1 and 7 of Hagan [4] and also includes Corollary 2 of Long [15].

Theorem 4.4. *Let $f: X \to Y$ be a bijection such that both f and f^{-1} are P-continuous. If X and Y are semilocally P-spaces, then f is a homeomorphism.*

Theorem 4.5. *Let $f: X \to Y$ be a P-continuous function from a first countable space X into a countably compact Hausdorff space Y which possesses a base of closed P-neighbourhoods. Then f is continuous.*

Proof. Suppose f is not continuous at a point $x \in X$. Then there is an open neighbourhood V of $f(x)$ such that, for every open set U containing x, $f(U)$ is not contained in V. Let $U_1 \supset U_2 \supset U_3 \supset \cdots$ be a countable base at x and choose $x_n \in U_n$ such that $f(x_n) \notin V$. Then $x_n \to x$ and since Y is countably compact and since countable compactness is closed hereditary, the sequence $\{f(x_n)\}$ has a cluster point $y \in Y - V$. Since Y is Hausdorff, there are disjoint open sets V_1 and V_2 such that $f(x) \in V_1 \subset V$ and $y \in V_2$. Since there is a closed P-neighbourhood W of y such that $W \subset V_2$, $Y - W$ is an open set having a P-complement. Now, if U is any open set containing x, there is $U_n \subset U$ and a point $x_n \in U_n$ such that $f(x_n) \in W$, due to the fact that the sequence $\{f(x_n)\}$ clusters at y. Consequently $f(U) \not\subset Y - W$. This contradicts the fact that f is P-continuous.

Remark 4.4. The above theorem yields [12, Theorem 2.1] and [18, Theorem 12] respectively, on substituting connectedness and compactness for P, respectively.

Finally, we suggest that the approach of this paper be compared with the work of Sanderson [34] on non-continuous functions and Mägrel's paper [23] on continuous and measurable selections.

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