



## Research Article

# Interaction of Dust-Acoustic Shock Waves in a Magnetized Dusty Plasma under the Influence of Polarization Force

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The interaction of dust-acoustic (DA) shock waves in a magnetized dusty plasma under the influence of nonextensively modified polarization force is investigated. The plasma model consists of negatively charged dust, Maxwellian electrons, nonextensive ions, and polarization force. In this investigation, we have derived the expression of polarization force in the presence of nonextensive ions and illustrated the head-on collision between two DA shock waves. The extended Poincaré–Lighthill–Kuo (PLK) method is employed to obtain the two-sided Korteweg–de Vries–Burgers (KdVB) equations and phase shifts of two shock waves. The trajectories and phase shifts of negative potential dust-acoustic shock waves after collision are examined. The combined effects of various physical parameters such as polarization force, nonextensivity of ions, viscosity of dust, and magnetic field strength on the phase shifts of DA shock waves have been studied. The present investigation might be useful to study the process of collision of nonlinear structures in space dusty plasma such as planetary rings where non-Maxwellian particles such as nonextensive ions, negatively charged dust, and electrons are present.

## 1. Introduction

Dusty plasma has fascinated the attention of many researchers for the study of different kinds of linear as well as nonlinear structures over the past many years. The presence of dust in an electron-ion plasma modifies the behaviour of nonlinear structures and generates new modes such as dust ion-acoustic waves [1] and dust-acoustic waves [2], which have been frequently studied theoretically and experimentally [3–6]. Dust-acoustic waves are low-frequency waves in which the inertia is provided by the dust mass and the restoring force is provided by the inertialess electrons and ions. Numerous investigations have reported the prevalent role of dust encountered in noctilucent clouds, planetary rings, comets, and plasma crystals [7–12] as well as in semiconductor manufacturing and ion implantation [13, 14]. Dust particulates in a dusty plasma are affected by various forces due to the large amount of dust charge on them, among which electrostatic force  $F_e = q_d E$  ( $q_d = -Z_d e$  is dust charge,  $Z_d$  is the number of electrons residing on dust

grain surface, and  $E$  is the electric field) has a significant influence on dust. As the bulk plasma is quasineutral, the electric field there is usually much weaker than the electric field in the sheath region. Hamaguchi Farouki [15, 16] reported that polarization force acts on the dust in the background of nonuniform plasma as a result of deformation of Debye sheath around the particulates. Spatial nonuniform dust density means that the dust charge is nonuniformly distributed in plasmas. The polarization force acts in the direction in which Debye length decreases and also acts in the direction opposite to the electrical force independent of the sign of dust charge [17]. Thus, the polarization force has a significant influence on the propagation characteristics of low-frequency linear DA waves. Various researchers have reported the effect of polarization force on different nonlinear structures (solitons and shocks) in the framework of Maxwellian and non-Maxwellian distribution in various plasma environments [17–25]. Sethi et al. [23] studied DA solitary and shock waves under the influence of superthermally modified polarization force in an unmagnetized dusty

plasma comprising of negatively charged dust grains, superthermally distributed ions, and Maxwellian electrons. The superthermally modified polarization force was found to be greater for more superthermal particles. Singh et al. [24] illustrated that the superthermality of ions has a significant influence on polarization force which further altered the dynamics of DA cnoidal waves in dusty plasma. In the recent past, Singh and Saini [25] examined the impact of superthermal polarization force on the modulational instability of DA waves and transition of rogue wave triplets to super rogue waves in space dusty plasmas.

Wave-wave interaction is a very fascinating phenomenon apart from the wave propagation in plasmas that can modify the properties of waves. This unique phenomenon of wave-wave interaction can be described by the use of the extended PLK method and has been investigated by numerous authors [26–32]. The interaction between waves can take place as overtaking collision or as head-on collision. It is well acknowledged that solitons preserve their asymptotic form while undergoing collision. During the before and after collisions, the solitary waves retain their shape, and also they have a novel impact on phase shifts. In this investigation, we have focused our study on the head-on collision of two DA shock waves under the effect of polarization force and their trajectories after the collision. The investigations of the head-on collision of nonlinear shock waves under the influence of polarization force have been reported by numerous researchers in the framework of Maxwellian and non-Maxwellian distributions in different plasma systems [26, 27, 31]. Head-on collision of modulated dust-acoustic waves in strongly coupled dusty plasma is investigated by El-Labany et al. [26]. They considered negatively charged dust grains and Maxwellian electrons and ions and studied modulational instability and effect of polarization force on the phase shifts of dust-acoustic solitary waves. Head-on collision of dust-acoustic shock waves in strongly coupled dusty plasmas was investigated by El-Shamy and Al-Asbali [27]. In the recent past, Singh et al. [31] investigated the head-on collision in a dusty plasma among DA multisolitons with ions following the hybrid distribution under the effect of the polarization force. It was found that rarefactive DA multisolitons are formed and phase shifts are strongly influenced by the polarization force.

Although prolific literature on the interaction of DA waves has been reported, yet the combined effects of polarization force and nonextensivity of ions on the interaction of DA shock waves have not been investigated by researchers in dusty plasmas with application to planetary rings and comet tails. The manuscript is structured as follows. Section 2 presents the fluid model for a given plasma system. Derivations of two different types of KdV–Burgers (KdVB) equations and their corresponding phase shifts are displayed in Section 3. Numerical analysis and discussion are presented in Section 4. The last Section 5 is devoted to the conclusions.

## 2. The Fluid Equations

In statistical mechanics, there exist generally two types of systems such as extensive and nonextensive systems, depending on the range of interparticle forces. The extensive systems are considered for short-range interparticle forces and need Boltzmann–Gibbs (B-G) statistics. On the other hand, nonextensive systems hold for long-range interparticle forces, need generalized B-G statistics, and also include dissipative systems having nonvanishing thermodynamic currents. Tsallis [33] introduced  $q$ -nonextensive distribution which is used to model astrophysical regions (i.e., gravitational systems and stellar polytropes) [34, 35] where Maxwellian distribution might become inadequate. Tsallis  $q$ -entropy and the probabilistically independent postulate are known to be the basis of the nonextensive statistics. The principle of maximum  $q$ -entropy gives the power-law  $q$ -distribution which is called  $q$ -exponentials in nonextensive statistics. The nonextensivity is also known as pseudoadditivity or nonadditivity which gives both the  $q$ -entropy and the energy (for more details, see [36]). The parameter  $q$  measures the degree of nonextensivity of the system. The  $q$ -nonextensive number density of ions is given as follows [37]:

$$n_i = n_{i_0} \left[ 1 - (q-1) \frac{e\psi}{K_B T_i} \right]^{(q+1/2)(q-1)}. \quad (1)$$

We consider a magnetized dusty plasma comprising of inertial negatively charged dust, nonextensive ions, and Maxwellian electrons to study the head-on collision and phase shifts of two DA shock waves under the influence of nonextensively modified polarization force. Consider charge on negatively charged dust grain be  $q_d = -Z_d e$ , and  $n_{e_0}$ ,  $n_{i_0}$ , and  $n_{d_0}$  are the unperturbed electron, ion, and dust number densities, respectively.  $Z_d$  is the dust grain charge. The charge neutrality condition at equilibrium yields  $n_{e_0} + Z_d n_{d_0} = n_{i_0}$  which can be expressed as  $\mu_e + 1 = \mu_i$ , where  $\mu_e = n_{e_0}/Z_d n_{d_0} = 1/\rho - 1$  and  $\mu_i = n_{i_0}/Z_d n_{d_0} = \rho/\rho - 1$  and  $\rho = n_{i_0}/n_{e_0}$ . The electrons are considered Maxwellian with density  $n_e = n_{e_0} \exp(e\psi/K_B T_e)$ . After Taylor expansion of the RHS of equation (1) for  $e\psi/K_B T_i \ll 1$ , we have

$$n_i = n_{i_0} \left[ 1 - h_1 \frac{e\psi}{K_B T_i} + h_2 \left( \frac{e\psi}{K_B T_i} \right)^2 \right], \quad (2)$$

where  $h_1 = q + 1/2$  and  $h_2 = (q+1)(3-q)/8$ . The expression for polarization force  $F_{pol}$  is given by [15]

$$F_{pol} = \frac{q_d^2}{2} \frac{\nabla \lambda_D}{\lambda_D^2}. \quad (3)$$

We have now derived the polarization force (i.e., deformation of Debye sphere around the dust grains) acting on dust grains modified by nonextensive ions in a nonuniform dusty plasma comprising of nonextensive ions and Maxwellian electrons where the expression for modified Debye length  $\lambda_D$  for negative dust (i.e.,  $n_e T_i \ll n_i T_e$ ) is

$\lambda_D \approx \sqrt{K_B T_i / 4\pi e^2 n_i h_1}$  in the presence of nonextensive ions. The expression for nonextensively modified polarization force is obtained as

$$\mathbf{F}_{\text{pol}} = -Z_d e R h_0 \left( \frac{n_i}{n_{i_0}} \right)^{1/2} \nabla \psi, \tag{4}$$

where  $R = Z_d e^2 / 4K_B T_i \lambda_{Di_0}$  measures the polarization force effect, i.e., interaction of highly negatively charged dust with nonextensive ions with  $\lambda_{Di_0} = \sqrt{K_B T_i / 4\pi e^2 n_{i_0} h_1}$  and  $h_0 = (h_1 - (2h_2 - (h_1^2/2)(e\psi/K_B T_i)))$ . Thus, nonextensivity of ions (via  $q$ ) notably alters the polarization force  $\mathbf{F}_{\text{pol}}$  and also polarization parameter  $R$ . When  $q \rightarrow 1$ , the polarization force expression equation (4) agrees well with the expression for the case of Maxwellian ions and electrons derived by Asaduzzaman et al. [18].

The dynamics of DA shocks waves in polarized dusty plasma is characterized by the following set of normalized fluid equations (continuity, momentum, and Poisson):

$$\frac{\partial n_d}{\partial t} + \nabla \cdot (n_d \mathbf{u}_d) = 0, \tag{5}$$

$$\frac{\partial \mathbf{u}_d}{\partial t} + (\mathbf{u}_d \cdot \nabla) \mathbf{u}_d = \chi_p \nabla \psi + \eta_l \nabla^2 \mathbf{u}_d - \Omega (\mathbf{u}_d \times \hat{z}), \tag{6}$$

$$\nabla^2 \psi = n_d + n_e - n_i, \tag{7}$$

where  $\nabla = \hat{x}(\partial/\partial x) + \hat{y}(\partial/\partial y) + \hat{z}(\partial/\partial z)$  and  $\mathbf{u}_d = \hat{x}u_{dx} + \hat{y}u_{dy} + \hat{z}u_{dz}$ .  $n_d$ ,  $\mathbf{u}_d$ , and  $\psi$  are the number density, velocity, and potential of dust fluid,  $\sigma = T_i/T_e$ , and  $\chi_p = 1 - R(h_1 - (2h_2 + (h_1^2/2)))$ . Density  $n_j$  ( $j = d, e, i$ ) is normalized by unperturbed density of dust  $n_{d_0}$ , dust velocity  $\mathbf{u}_d$  by  $\sqrt{Z_d K_B T_i / m_d}$ ,  $\psi$  by  $K_B T_i / e$ , time  $t$  by  $\omega_{pd}^{-1}$ , where  $\omega_{pd} = (4\pi(Z_d e)^2 n_{d_0} / m_d)^{1/2}$ , and  $x$  by  $\lambda_{Dd}$ , where  $\lambda_{Dd} = (K_B T_i / 4\pi n_{d_0} Z_d e^2)^{1/2}$ .  $\eta_l$  is the dust viscosity normalized by  $\omega_{pd} \lambda_{Dd}^2$ . The magnetic field is assumed to be along the  $z$ -axis (i.e.,  $\mathbf{B} = B_0 \hat{z}$ ), and the dust cyclotron frequency  $\Omega (= Z_d e B_0 / m_d)$  is normalized by  $\omega_{pd}$ .

### 3. Derivation of KdVB Equations and Phase Shifts

We employ an extended PLK perturbation method to investigate the collision between two DA shock waves in a magnetized dusty plasma. The perturbed quantities are described as follows:

$$Y = Y_0 + \sum_{r=1}^{\infty} \epsilon^{r+1} Y_r, \quad \Gamma = \sum_{r=1}^{\infty} \epsilon^{r+2} \Gamma_r, \tag{8}$$

where  $Y = (n_d, u_{dz}, \psi)$ ,  $Y_0 = (1, 0, 0)$ , and  $\Gamma = (u_{dx}, u_{dy})$ . The scaling variables  $x$  and  $t$  are stretched by the new coordinate system  $\xi$ ,  $\eta$ , and  $\tau$  in multiple scale variables:

$$\xi = \epsilon(l_x x + l_y y + l_z z - v_0 t) + \epsilon^2 M_0(\eta, \tau) + \dots, \tag{9}$$

$$\eta = \epsilon(l_x x + l_y y + l_z z + v_0 t) + \epsilon^2 N_0(\xi, \tau) + \dots, \tag{10}$$

$$\tau = \epsilon^3 t, \tag{11}$$

where  $\xi$  and  $\eta$  denote the opposite side trajectories of DA shock waves and  $\epsilon$  is a small parameter determining the strength of nonlinearity. The functions  $M_0(\eta, \tau)$  and  $N_0(\xi, \tau)$  are determined later.  $l_x, l_y$ , and  $l_z$  represent the direction cosines along the  $x, y$ , and  $z$  directions, respectively, such that  $l_x^2 + l_y^2 + l_z^2 = 1$ . The value of dust viscosity is considered small and its value is set as  $\eta_l = \epsilon \eta_{l_0}$ , where  $\eta_{l_0}$  is  $O(1)$ . Making use of equations (8)–(11) into (5)–(7), at the lowest orders of  $\epsilon$ , we obtain the following first-order equations:

$$v_0 \left( -\frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \right) n_{d_1} + l_z \left( \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \right) u_{dz_1} = 0, \tag{12}$$

$$-v_0 \left( \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \right) u_{d_1} + (1 - R h_1) \left( \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \right) \psi_1 = 0, \tag{13}$$

$$(1 - R h_1) l_x \left( \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \right) \psi_1 - \Omega u_{dy_1} = 0, \tag{14}$$

$$(1 - R h_1) l_y \left( \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \right) \psi_1 + \Omega u_{dx_1} = 0, \tag{15}$$

$$n_{d_1} = -a_1 \psi_1, \tag{16}$$

where  $a_1 = \mu_e \sigma + \mu_i h_1$ . On simplifying equations (12)–(16), the relations of different first-order physical quantities are obtained as follows:

$$\psi_1 = \psi_{1\xi}(\xi, \tau) + \psi_{1\eta}(\eta, \tau), \tag{17}$$

$$n_{d_1} = -\frac{(1 - R h_1) l_z^2}{v_0^2} (\psi_{1\xi}(\xi, \tau) + \psi_{1\eta}(\eta, \tau)), \tag{18}$$

$$u_{dx_1} = -\frac{(1 - R h_1) l_y}{\omega_{cd}} \left( \frac{\psi_{1\xi}(\xi, \tau)}{\partial \xi} + \frac{\psi_{1\eta}(\eta, \tau)}{\partial \eta} \right), \tag{19}$$

$$u_{dy_1} = \frac{(1 - R h_1) l_x}{\omega_{cd}} \left( \frac{\psi_{1\xi}(\xi, \tau)}{\partial \xi} + \frac{\psi_{1\eta}(\eta, \tau)}{\partial \eta} \right), \tag{20}$$

$$u_{dz_1} = -\frac{(1 - R h_1) l_z}{v_0} (\psi_{1\xi}(\xi, \tau) - \psi_{1\eta}(\eta, \tau)), \tag{21}$$

$$n_{d_1} = -a_1 (\psi_{1\xi}(\xi, \tau) + \psi_{1\eta}(\eta, \tau)). \tag{22}$$

From equation (17), one predicts that there are two DA shock waves in which  $\psi_{1\xi}(\xi, \tau)$  is propagating towards the right and  $\psi_{1\eta}(\eta, \tau)$  is propagating towards the left. On solving equations (17)–(22), the expression for phase velocity in a magnetized dusty plasma under the effect of nonextensively modified polarization force is obtained as  $v_0 = l_z \sqrt{(1 - R h_1) / a_1}$ .

Finally, for the next higher orders of  $\epsilon$ , we obtain

$$\begin{aligned}
 -4l_z \frac{\partial^2 u_{dz_3}}{\partial \xi \partial \eta} = & \int \left[ \frac{\partial \psi_{1\xi}(\xi, \tau)}{\partial \tau} + A \psi_{1\xi}(\xi, \tau) \frac{\partial \psi_{1\xi}(\xi, \tau)}{\partial \xi} + B \frac{\partial^3 \psi_{1\xi}(\xi, \tau)}{\partial \xi^3} - C \frac{\partial^2 \psi_{1\xi}(\xi, \tau)}{\partial \xi^2} \right] d\eta \\
 & + \int \left[ \frac{\partial \psi_{1\eta}(\eta, \tau)}{\partial \tau} - A \psi_{1\eta}(\eta, \tau) \frac{\partial \psi_{1\eta}(\eta, \tau)}{\partial \eta} - B \frac{\partial^3 \psi_{1\eta}(\eta, \tau)}{\partial \eta^3} + C \frac{\partial^2 \psi_{1\eta}(\eta, \tau)}{\partial \eta^2} \right] d\xi \\
 & + \int \int \left[ 2\nu_0 \frac{\partial M_0}{\partial \eta} + D \psi_{1\eta}(\eta, \tau) \right] \frac{\partial^2 \psi_{1\xi}(\xi, \tau)}{\partial \xi^2} d\xi d\eta \\
 & + \int \int \left[ 2\nu_0 \frac{\partial N_0}{\partial \xi} + D \psi_{1\xi}(\xi, \tau) \right] \frac{\partial^2 \psi_{1\eta}(\eta, \tau)}{\partial \eta^2} d\xi d\eta.
 \end{aligned} \tag{23}$$

From equation (23), it is noticed that the first (second) term of the right side is proportional to  $\eta$  ( $\xi$ ), and thus, the integrating function is independent of  $\eta$  ( $\xi$ ). These two secular terms in the above equation must be removed to avoid spurious resonance. Hence, we obtain the following nonlinear equations:

$$\frac{\partial \phi_\xi}{\partial \tau} + A \phi_\xi \frac{\partial \phi_\xi}{\partial \xi} + B \frac{\partial^3 \phi_\xi}{\partial \xi^3} - C \frac{\partial^2 \phi_\xi}{\partial \xi^2} = 0, \tag{24}$$

$$\frac{\partial \phi_\eta}{\partial \tau} - A \phi_\eta \frac{\partial \phi_\eta}{\partial \eta} - B \frac{\partial^3 \phi_\eta}{\partial \eta^3} + C \frac{\partial^2 \phi_\eta}{\partial \eta^2} = 0. \tag{25}$$

In the above equations, we have considered  $\psi_{1\xi} = \phi_\xi$  and  $\psi_{1\eta} = \phi_\eta$  for simplicity. The nonlinear, dispersion, and dissipation coefficients, respectively, are given as

$$A = \frac{-3l_z^2(1 - Rh_1)}{2\nu_0} + \frac{\nu_0 b}{a_1} + \frac{\nu_0(R(2h_2 + (h_1^2/2)))}{2(1 - Rh_1)}, \tag{26}$$

$$B = \frac{(1 - l_z^2)(1 - Rh_1)\nu_0}{2a_1\Omega^2} + \frac{\nu_0}{2a_1}, \tag{27}$$

$$C = \frac{\eta_{l_0}}{2}, \tag{28}$$

where  $b = \mu_e \sigma^2/2 - \mu_i h_2$ . Equations (24) and (25) are the KdV–Burgers equations of two shock waves travelling towards each other in the reference frame of  $\xi$  and  $\eta$ .

**3.1. Solution of KdVB Equation and Phase Shifts.** The steady-state solutions of equations (24) and (25) are obtained as follows [27]:

$$\phi_\xi = \phi_m \left[ 1 - \tanh \left[ w \left( \xi - \left( \frac{6C^2}{25B} \right) \tau \right) \right] \right]^2, \tag{29}$$

$$\phi_\eta = \phi_m \left[ 1 + \tanh \left[ w \left( \eta + \left( \frac{6C^2}{25B} \right) \tau \right) \right] \right]^2. \tag{30}$$

The electrostatic potential for weak head-on collision is given as

$$\begin{aligned}
 \Phi = & \phi_m \left[ 1 - \tanh \left[ w \left( \xi - \left( \frac{6C^2}{25B} \right) \tau \right) \right] \right]^2 \\
 & + \phi_m \left[ 1 + \tanh \left[ w \left( \eta + \left( \frac{6C^2}{25B} \right) \tau \right) \right] \right]^2,
 \end{aligned} \tag{31}$$

where  $\phi_m = 3C^2/25AB$  is the maximum amplitude and  $w = 10B/C$  is the width of DA shock waves.

In the next higher order, the third and fourth terms on the right-hand side of equation (23) become secular and we obtain [27]

$$2\nu_0 \frac{\partial M_0}{\partial \eta} = -D \phi_\eta, \tag{32}$$

$$2\nu_0 \frac{\partial N_0}{\partial \xi} = -D \phi_\xi, \tag{33}$$

where

$$D = \frac{\nu_0 b}{a_1} + \frac{(1 - Rh_1)l_z^2}{2\nu_0} + \frac{\nu_0(R(2h_2 + (h_1^2/2)))}{2(1 - Rh_1)}. \tag{34}$$

Equations (32) and (33) are solved using appropriate initial and boundary conditions to determine the phase shifts as

$$\Delta M_0(\eta, \tau) \approx -e^2 \frac{3\eta_{l_0}}{50\nu_0} \frac{CD}{AB}, \tag{35}$$

$$\Delta N_0(\xi, \tau) \approx e^2 \frac{3\eta_{l_0}}{50\nu_0} \frac{CD}{AB}. \tag{36}$$

These phase shifts are functions of different coefficients that are dependent on various physical parameters. Therefore, any variation in physical parameters makes a significant change in the phase shifts.

### 4. Numerical Analysis and Discussion

In this investigation, the head-on collision of DA shock waves in a magnetized dusty plasma consisting of non-extensive ions and Maxwellian electrons using the extended PLK perturbation method is presented. The numerical values of the typical physical parameters of planetary rings [20, 24],  $n_{i_0} = 5 \times 10^7 \text{ cm}^{-3}$ ,  $n_{e_0} = 4 \times 10^7 \text{ cm}^{-3}$ ,  $Z_d =$

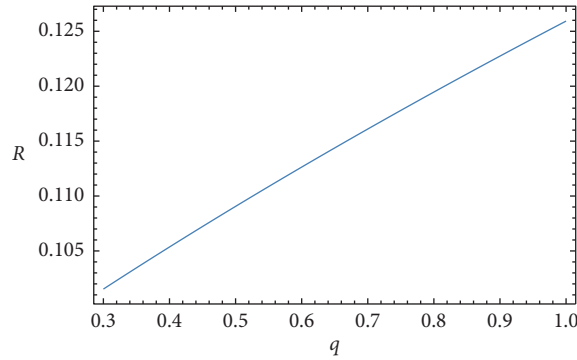


FIGURE 1: The variation of polarization force parameter ( $R$ ) with nonextensivity of ions (via  $q$ ).

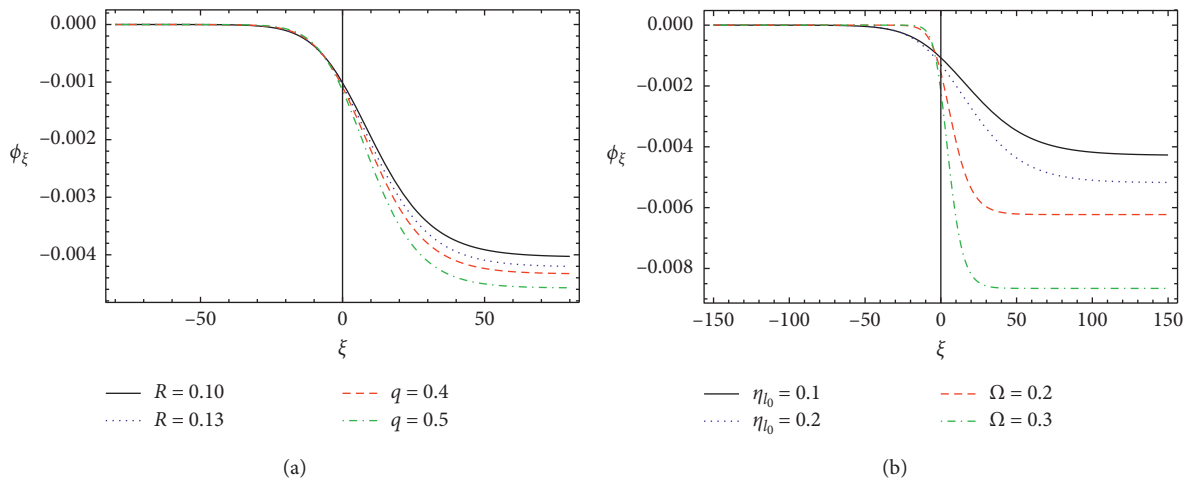


FIGURE 2: The variation of rarefactive DA shocks profile with respect to  $\xi$  (a) for different values of polarization parameter ( $R$ ) and nonextensivity parameter ( $q$ ) and (b) for different values of viscosity  $\eta_{i0}$  and magnetic field strength whereas other parameters  $\rho = 1.3$ ,  $l_z = 0.8$ ,  $\sigma = 0.25$ ,  $\Omega = 0.8$ , and  $\eta_{i1} = 0.35$  are fixed.

$3 \times 10^3$ ,  $n_d = 10^7 \text{ cm}^{-3}$ ,  $T_e = 50 \text{ eV}$ ,  $T_i = 0.05 \text{ eV}$ , and  $R = 0 - 0.14$ , are used for analysis. The shock waves are formed as a result of balance of nonlinearity, dispersion, and dissipation. The KdVB equations have been derived in equations (24) and (25). Numerically, it is seen that the nonlinear coefficient  $A$  (equation (26)) is negative (i.e.,  $A < 0$ ) and the dispersion coefficient  $B$  (equation (27)) is positive (i.e.,  $B > 0$ ) for the chosen set of parameters. It implies that  $AB < 0$ ; as a result, negative potential shock structures are formed. Since the various physical parameters such as nonextensivity of ions, polarization force, obliqueness, magnetic field strength, and other plasma parameters have a profound influence on the characteristics of DA shock waves, it is of paramount importance to carry out numerical analysis.

**4.1. Influence of Nonextensivity of Ions (via  $q$ ) on the Polarization Force.** Figure 1 depicts the variation of the polarization force parameter  $R$  with nonextensive parameter ( $q$ ). Since the parameter  $R$  is a function of nonextensivity of ions (via  $q$ ), any variation in  $q$  will change the value of  $R$ . It is observed that as the nonextensive parameter  $q$  is increased, the polarization force parameter  $R$  increases. Thus,

nonextensivity of ions has a profound influence on the polarization force parameter  $R$  (i.e., polarization force).

**4.2. The Variation of Profile of Negative Polarity DA Shock Waves.** Figure 2(a) depicts the variation of the amplitude of negative polarity DA shock waves in  $R - q$  plane. It is found that the amplitude of negative polarity DA shock waves is increased with rise in both polarization force parameter (via  $R$ ) and nonextensive parameter (via  $q$ ). Figure 2(b) shows that the amplitude of negative polarity shock waves is also enhanced with an increase in both magnetic field strength (via  $\Omega$ ) and obliqueness (via  $l_z$ ). It is remarked that the amplitude of DA shock waves is modified with the variation in plasma parameters due to the change in nonlinear as well as dispersive effects under the influence of polarization force in the presence of dissipation effect.

**4.3. Evolution and Phase Shift of Two Negative Polarity DA Shock Waves.** The collision process of DA shock waves in a magnetized dusty plasma for different time intervals is depicted in Figure 3. The mesial line indicated in the figure separates the before and after collision process. The upper

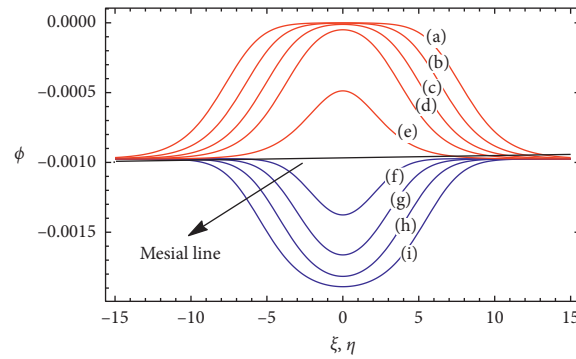


FIGURE 3: The collision of two DA shock waves at different time intervals for  $q = 0.33$ ,  $R = 0.1$ ,  $\rho = 1.3$ ,  $l_z = 0.8$ ,  $\sigma = 0.25$ ,  $\Omega = 0.8$ , and  $\eta_l = 0.2$ . (a)  $\tau = 50$ , (b)  $\tau = 40$ , (c)  $\tau = 30$ , (d)  $\tau = 20$ , (e)  $\tau = 0$ , (f)  $\tau = -50$ , (g)  $\tau = -40$ , (h)  $\tau = -30$ , and (i)  $\tau = -20$ .

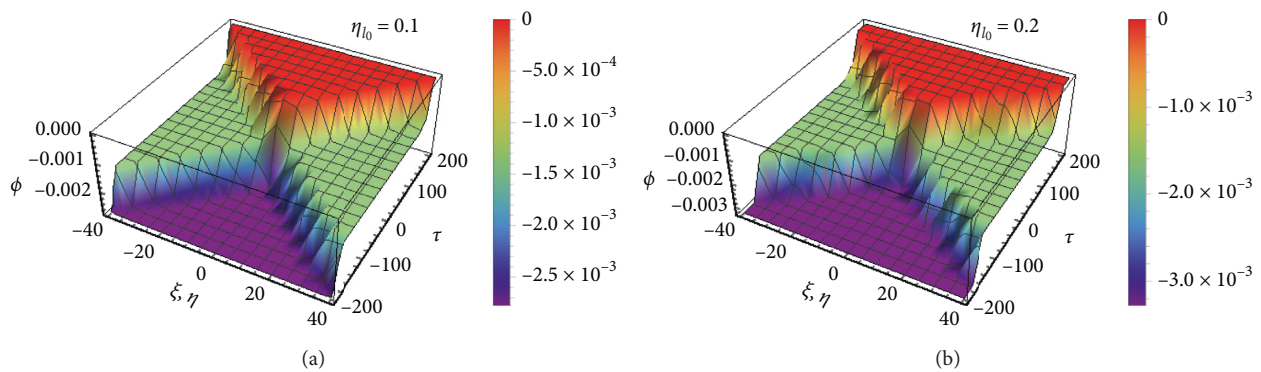


FIGURE 4: Collision process of two DA shock waves for (a)  $\eta_{l_0} = 0.1$  and (b)  $\eta_{l_0} = 0.2$  and  $\rho = 1.3$ ,  $l_z = 0.8$ ,  $\sigma = 0.25$ ,  $\Omega = 0.8$ ,  $R = 0.11$ , and  $q = 0.33$  are fixed.

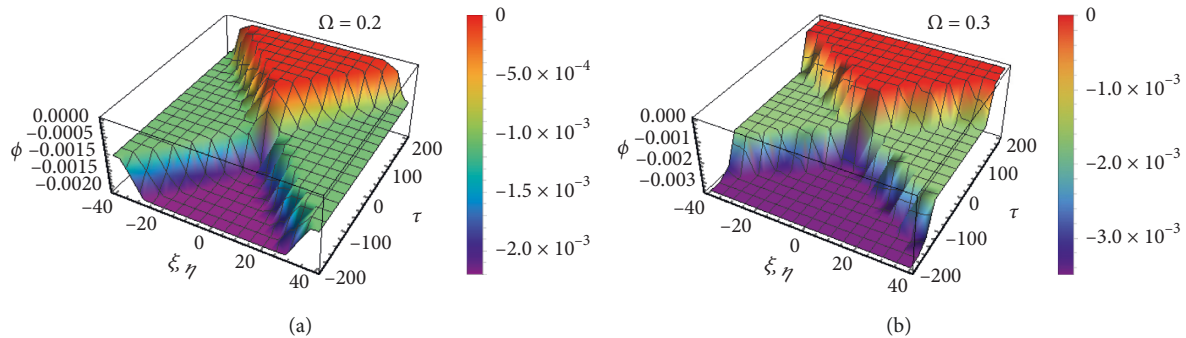


FIGURE 5: Collision process of two DA shock waves for (a)  $\Omega = 0.2$  and (b)  $\Omega = 0.3$  and  $\rho = 1.3$ ,  $l_z = 0.8$ ,  $\sigma = 0.25$ ,  $R = 0.11$ ,  $\eta_l = 0.2$ , and  $q = 0.33$  are fixed.

red curves illustrate the profiles of DA shocks before collision and lower blue curves for after collision. This whole process illustrates that two opposite directional propagating negative polarity DA shock waves come closer, interact, and then depart (for more clarity, see animation of collision in the supplementary section (available here)). Figures 4(a) and 4(b) depict the variation of phase shift occurred during the collision process between two negative polarity DA shock waves for different values of dust fluid viscosity ( $\eta_{l_0}$ ). It is recognised that the magnitude of phase shift escalates with rise in viscosity (clearly seen in color bar). Figures 5(a) and

5(b) depict the variation of phase shift occurred during the collision process between two negative polarity DA shock waves for different values of magnetic field strength (via  $\Omega$ ). It is found that the magnitude of phase shift enhances as the value of  $\Omega$  increases (see side color bar). Figure 6(a) illustrates the variation of phase shift with nonextensive parameter ( $q$ ) for different values of polarization parameter ( $R$ ) in the presence of magnetic field. It is seen that magnitude of phase shift escalates with rise in nonextensive parameter ( $q$ ) and suppresses with increase in polarization force (via  $R$ ). Figure 6(b) shows the same variation in the

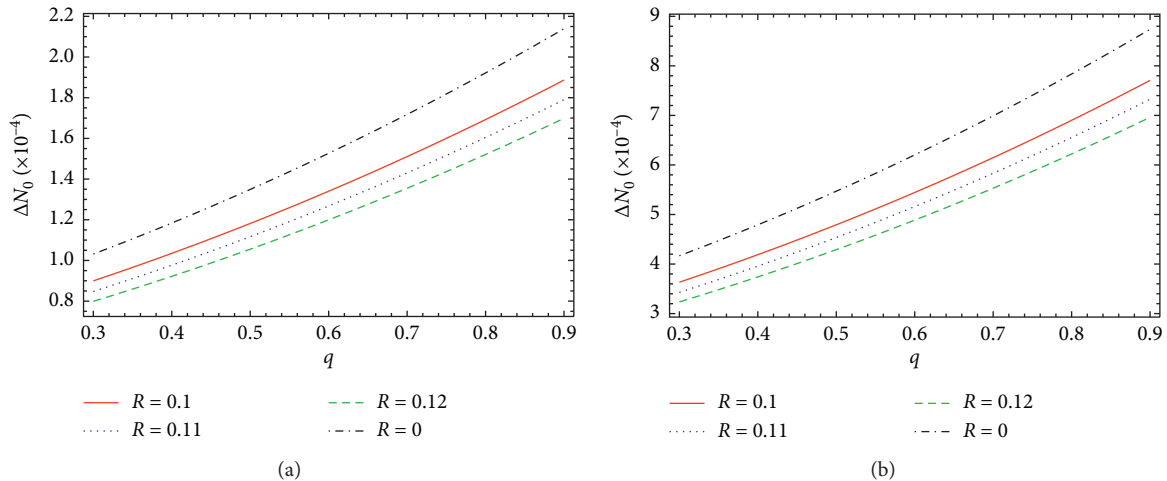


FIGURE 6: The variation of phase shift of two DA shock waves vs.  $q$  for different values of  $R$  (a) with magnetic field and (b) without magnetic field and  $\rho = 1.3$ ,  $l_z = 0.8$ ,  $\sigma = 0.25$ , and  $\eta_l = 0.2$  are fixed.

absence of magnetic field. It is found that magnitude of phase shift is more (almost four times greater) than the case in the absence of magnetic field. It is remarked that magnetic field strength has a significant influence on the profile of DA shocks. It is concluded that the phase shift of DA shock waves is significantly affected under the influence of polarization force (via  $R$ ), nonextensivity parameter (via  $q$ ), strength of magnetic field (via  $\Omega$ ), and other plasma parameters.

## 5. Conclusions

In the present investigation, we have studied the effects of nonextensively modified polarization force on the head-on collision between two DA shock waves in a magnetized dusty plasma composed of negatively charged dust, Maxwellian electrons, and  $q$ -nonextensive ions. The extended PLK method is employed to derive the two opposite directional KdVB equations and the expressions for phase shifts after collision. The  $q$ -nonextensive ions yield to escalate the polarization force parameter ( $R$ ). Only negative polarity DA shock waves are formed under the influence of polarization force. The amplitude of DA shock waves and their phase shifts are significantly modified by the physical parameters such as nonextensivity of ions (via  $q$ ), modified polarization force parameter (via  $R$ ), viscosity (via  $\eta_l$ ), obliqueness (via  $l_z$ ), and magnetic field strength (via  $\Omega$ ). The time evolution of rarefactive DA shock waves under the influence of polarization force has been illustrated. The physical parameters have an emphatic effect on the characteristics of DA shock waves and their phase shifts occurred due to head-on collision among DA shock waves. It is important to conclude that the results of our present investigation might be useful for understanding the nonlinear features of disturbances in space/astrophysical dusty plasma systems (e.g., Saturn's rings, interstellar clouds, and cometary environment) where negatively charged dust and nonextensive ions are present.

## Data Availability

The derived data generated in this research will be shared on reasonable request to the corresponding author.

## Disclosure

This paper was presented as an invited talk at the 12th International Conference on Plasma Science and Applications, held from 11 to 14 November 2019, Lucknow, India.

## Conflicts of Interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as potential conflicts of interest.

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## Supplementary Materials

Shocks-collision video. (*Supplementary Materials*)

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