

Abstracts of Australasian PhD theses

On multifunctions and multimeasures

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In this thesis, the theories of integration of perfectly measurable multifunctions and of multimeasures with values in locally convex spaces are studied. In Chapter I, we extend the selection theorems of Castaing [1] and Robertson [5]. For this purpose, the basic notions of Ω -Polish coverings and Ω -Polish spaces are introduced, and we study in particular how a Banach space (or more generally a linear subspace of a Banach space) can be given a useful Ω -Polish covering. We then prove the existence of a measurable selector for a multifunction taking values in an Ω -Polish space or in a space which is the continuous image of an Ω -Polish space.

In Chapter II, we introduce perfectly measurable multifunctions, which can be regarded as well enough behaved for their integrals to correspond satisfactorily with their ranges of values. We then study the properties of perfectly measurable multifunctions and find conditions for a multifunction to be perfectly measurable. We also examine the relationship between perfect and 0-perfect measurabilities and the perfect measurability of a multifunction related to its graph.

Chapter III is concerned with defining and elucidating the basic properties of integrals of perfectly measurable multifunctions. We give sufficient conditions for a multifunction to be integrable and study operations on integrable multifunctions. We also use a result of Kingman and Robertson [2] to obtain the convexity of the integrals (this generalizes a result of Richter [4]). A theorem concerning the compactness of integrals is also proved.

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In Chapter IV, we develop the theory of multimeasures with values in locally convex spaces. We generalize the concept of "thinness" introduced by Kingman and Robertson in [2] and apply it to observe the convexity of the range of a multimeasure; the result obtained partly generalizes Theorem 1 of [3] which is itself an extension of the well-known theorem of Lyapunov. It is also shown that under some circumstances the range of a multimeasure has the same weak closure as its convex hull. We also investigate the existence of control measures for a multimeasure, and extend the classical Radon-Nikodym theorem for multimeasures.

References

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