# CORRESPONDENCE.

## THE ASSES' BRIDGE.

To the Editor of the *Mathematical Gazette.*

DEAR SIR,—Didasculus in the December *Gazette* (XXII, p. 487) asks the following question: Given two triangles in a plane which are indirectly congruent, is it possible to split them up into parts so that each part of one triangle is directly congruent to the corresponding part of the other ?

That the answer is " Yes " is seen by putting together the facts: (i) two congruent isosceles triangles are both directly and indirectly congruent; (ii) a right-angled triangle can be split into two isosceles triangles; (iii) any triangle can be split into two right-angled triangles.

This argument holds for euclidean geometry only. To see the same thing for elliptic geometry or on a sphere, we note that if the circumcentre of a triangle be joined to the vertices we have, in general, three isosceles triangles. Thus if the circumcentre is inside the triangle or on a side, the proof can be finished; if it lies outside the triangle, split the triangle up into right-angled triangles and use the fact that the circumcentre of a right-angled triangle now lies within it, since the angle-sum of a triangle is now greater than two right angles.

But in hyperbolic geometry, unless the circumcentre lies inside the triangle, our methods so far fail, since the circumcentre of a right-angled triangle lies outside it, if it exists. We now invoke the incentre $I$; let $P$, $Q$, $R$ be the feet of the perpendiculars from $I$ to the sides; joining the points $I$, $P$, $Q$, $R$ we have the dissection needed. This last method holds in all three geometries for all cases.

The other question raised by Didasculus, whether in this way the congruence axioms could be replaced by axioms involving direct congruence only, is much deeper. If the isosceles triangle theorem is used, as above, nothing is gained; for it is easy to see that the full congruence axioms imply this theorem, while that theorem with the restricted congruence axioms gives the full axioms. What can be done, and what cannot be done, with the restricted axioms, has engaged the attention of Hilbert and his disciples.

Yours truly,

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## REFERENCE FOR SIMILAR TRIANGLES.

To the Editor of the *Mathematical Gazette.*

SIR,—I think many teachers will agree with Mr. Siddons in his contention, in the February *Gazette*, that when quoting a reference such as SAS it is unnecessary to differentiate it according as it is being used for congruence or for similarity. But, still in the cause of simplicity, may I ask Mr. Siddons why, in quoting the equiangular reference for similarity, he prefers AAA to AA?

Probably there will be a widespread welcome for his suggestion that some accepted symbol for " is similar to " would now be useful. From the time of Leibniz onwards, various attempts have, of course, been made to introduce such a symbol. An account of the chief proposals in this direction is given in Cajori's *History of Mathematical Notations.* According to Cajori, Leibniz, in 1679, invented $\sim$ or $\backsim$ for the purpose, because " the sign is the letter S (first letter in *similis*) placed horizontally " But it would seem that the example of Leibniz did not commend itself to his successors and, at any rate in this



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