CORRESPONDENCE.

MR. SCRATCHLEY ON POST OBITS.

To the Editor of the Assurance Magazine.

Sir—Under the title of "Remarks on the Valuation of Post Obits and Contingent Reversions," Mr. Scratchley investigates, by means of "some simple new formulæ," "the fair amount for a Post Obit or charge to be made on Contingent Reversionary Property"; and shows, by the aid of a new table, the consideration to be paid for such post obits, by way of annuity, during the joint lives of the life tenant and the reversioner. The publication is intended for the especial benefit of solicitors and others "not conversant with the principles of calculation involved" in such inquiries; so that they may have no excuse, in future, for charging excessive amounts by way of post obit.

It is certainly very desirable to establish the correct principles which should govern all pecuniary transactions; but this, in my judgment, has not been done by Mr. Scratchley in the case of post obits; and however commendable the design of his work may be, I think that the task he has assigned to himself will be found to be very imperfectly performed.

I apprehend that the object of every person who lends money is not only to obtain a fair rate of interest, but to secure the repayment of his capital; and it is in these respects that Mr. Scratchley's pamphlet is defective, because he fails to show how these conditions can be fulfilled.

The subject of post obits, Mr. Scratchley observes, has become more especially interesting of late, in consequence of some recent actions at law brought against three or four Insurance Companies, to recover the amount of certain policies effected on the life of an individual who seems to have had a violent fancy for anticipating his inheritance by raising money in this particular way.

One such transaction he notices, where a post obit for £14,000 was given in consideration of the grant of an annuity of £330;* and, in his opinion, the former should have been reduced to £4,708, or the latter increased to £1,020.

Without pausing to inquire how far Mr. Scratchley is right and the actuary referred to is wrong, I must express my surprise, and I do so with deep regret, that any actuary should give an opinion founded on such erroneous assumptions.

The case put is—"What amount should a post obit be for, to be paid by a gentleman aged 35, if he outlive his father, aged 74? The consideration to be, an annual payment of £330, together with the necessary insurance on the younger life, which can be done at 5 per cent., he not being a very good one; the old life to be calculated to live ten years." And the answer, or opinion, if it can be so called, was—"Assuming that the older life will live ten years, and that the younger will survive the elder, I am of opinion that the amount of the post obit, to cover the necessary insurance, should not be less than £12,832."

Mr. Scratchley points out that two arbitrary assumptions are here laid down as conditions of the question—first, that the elder life would live ten years; and, second, that the younger life would certainly survive him—and upon these assumptions he proceeds to verify the actuary's valuation. But

* Not, as Mr. Scratchley has it, "an annuity of £330 a year."

surely, upon the assumption that the younger life would certainly survive the elder, there was no need for any insurance, and the question would have resolved itself into finding the amount of £330 per annum in ten years, at 5 per cent.

I wonder that Mr. Scratchley, in his evident anxiety to cut down the actuary's estimate, should have failed to notice this unnecessary piece of extravagance, particularly as this assumption of survivorship was a gratuitous one on the part of the actuary, and not insisted upon in the case submitted for his opinion.

In dealing with this particular question, Mr. Scratchley also makes an assumption, and, as it appears to me, an erroneous one. He takes for granted that at the death of the elder life the younger would have been in a position to meet his obligations. Now it is well known that all he would succeed to, on the happening of that event, would be a life estate, and his post obits must have been converted into equivalent annuities for the remainder of his life; the lenders were consequently quite right in protecting themselves by effecting *absolute* instead of *contingent* insurances.

It is evident that the exact nature of the charge granted by the borrower on his inheritance is not fully disclosed, and that Mr. Scratchley has pronounced an opinion on imperfect data. As regards the opinion of the actuary here referred to, it is the first time I have met with so curious an exhibition of actuarial judgment: I hope it is the only specimen of the kind on record. Mathematics are taught to very little purpose in the present day, if so little use is made of their application to life contingencies.

I now proceed to notice some of Mr. Scratchley's "simple new formulæ;" and in his first "elegant proof," we recognize "an old friend in a new face." Adopting, for the sake of uniformity, Mr. Scratchley's notation, which is peculiar, we produce his first new formula:—

If $\pi_{x,y}$ = net annual premium to produce £1 at death of either x or y, and $p_{x,y \atop (1)}$ = annual Office premium to secure £1 in case of x dying before y, then $\pi_{x,y} - p_{x,y \atop (1)}$ is the annuity which should be paid to life x for a post-obit of £1.

In other words, $p_{x,y \atop (1)} + p_{y,x \atop (1)} = p_{x,y}$, which is to be found in all the text-books, and in the prospectus of every Insurance Company whose tables are properly constructed, Mr. Scratchley coolly asserts is not to be found in works on life contingencies.

It is true that, by putting $p_{x,y}$ under the form of $\pi_{x,y}$, and using a different rate of interest, he may consider himself entitled to claim a discovery; but is he not aware that other writers have done the like? and that, even in the pages of your *Magazine*, the same thing has been mentioned more than once?

Will Mr. Scratchley point out the difference between Milne's formula of

$$\mathfrak{AB}_1 + \mathfrak{BA}_1 = \mathfrak{AB},$$

and his own of

$$S_{x,y \atop (1)} + S_{y,x \atop (1)} = S_{x,y}?$$

or between

$$\frac{\mathfrak{AB}_1}{1 + AB} + \frac{\mathfrak{BA}_1}{1 + AB} = \frac{\mathfrak{AB}}{1 + AB}$$

and

$$\frac{S_{x,y}}{\underset{(1)}{*}a_{x,y}} + \frac{S_{y,x}}{\underset{(1)}{a_{x,y}}} = \frac{S_{x,y}}{a_{x,y}},$$

or

$$p_{x,y} + p_{y,x} = p_{x,y}?$$
$$\underset{(1)}{} \quad \underset{(1)}{}$$

$\pi_{x,y}$ or $\dfrac{\mathfrak{AB}}{1 + AB}$ may be taken at any rate of interest, without disturbing the principle or establishing a claim to novelty.

The next claim to novelty is to be found in the expression for the value of a post obit after n years: $O\left(1^{i} - \dfrac{a_{x+n,\, y+n}}{a_{x,y}}\right)$; but as Mr. Scratchley merely remarks incidentally that the coefficient of O is a neat formula for the value of a policy, I am in doubt whether he considers this in the light of a discovery. To remove all doubt upon the point, I would also remark incidentally that this formula is identical with the $1 - \dfrac{1 + {}^{n}A}{1 + A}$ of Mr. Griffith Davies, and the $1 - \dfrac{1 + a_{m+n}}{1 + a_{m}}$ of Mr. David Jones; making, of course, the necessary alteration in these to include two lives, and bearing in mind that $a_{x,y}$ is used to represent the value usually denoted by $1 + a_{x,y}$ or $1 + AB$.

Then follows a table containing the values of $\pi_{x,y}$ at 5, 6 and 7 per cent., and $p_{x,y}$ according to the premiums charged by the Western Life Assurance Society. I think it would be much more useful to have given a table of the present value of a post obit, instead of its equivalent in the shape of a joint life annuity, as persons who raise money in this way generally do it to meet the wants of the moment, and not to increase their income. There is another objection, which may be mentioned at once— and it is, that the lender or grantee of the post obit is left to the uncertain and consequently unsatisfactory remuneration of an extra 1 or 2 per cent. for the risk he runs of the joint existence of the two lives being prolonged beyond their average duration, instead of being protected or being shown how he may protect himself from this contingency, and realize the rate of interest he stipulates for. This question has been so fully explained in the pages of the *Assurance Magazine*, that it is only necessary to allude to it here for the purpose of observing that Mr. Scratchley, in his remarks upon the plan recommended, contents himself with stating that it cannot be supported by any satisfactory reasoning, without pointing out any particular defect in it except that at the early ages of life it gives negative results.

Now, if this proves anything, it simply shows that it is impossible to invest money in a Government security for any very lengthened term at so high a rate of interest as 5 or 6 per cent., and that in such cases a reduced rate of something like 4 per cent. can only be looked for. A reversion, in order to be saleable in the market, must be well secured, either in Consols or on land; and it is unreasonable for a purchaser or a lender to expect to realize 5 or 6 per cent. for thirty or forty years, upon a security paying little more than 3 per cent. For example: at age 20, the expectation of life by the Carlisle Table is rather more than forty years; and according to Mr. Scratchley and the antiquated mode of valuation, the present value of

* The $a_{x,y}$ of Mr. Scratchley being the $1 + a_{x,y}$ or $1 + AB$ of other writers.

£1 at death would be $1-d_5\{1+(a_{20})_5\}=\cdot19919$; by the modern plan recommended in this *Magazine*, the value would be

$$1-d_4\{1+(a_{20})_{3\cdot5}\}=\cdot19569.$$

While therefore Mr. Scratchley leads his client to believe that he would realize 5 per cent. upon his investment, the plan which he condemns would show him that he was only making 4 per cent.

Although the results of the two modes of valuation are almost identical, even at an extreme age (for a man cannot well begin to borrow before he is twenty), the principle involved is as different as truth is from error. It was never contended (so long, at least, as the Carlisle or Equitable Experience Tables are used) that the cost of an annuity during the continuance of the reversion would exceed the sum indicated by a $3\frac{1}{2}$* per cent. value; and Mr. Scratchley, by reducing this to 3 per cent., to support his argument, has exhibited the same unfairness as when he attempts to appropriate as his own, under a change of symbols, the formulæ of other writers.

The annual survivorship premiums, $p_{x,y}$, given in the table, are not from any standard table, but appear to be those of the Western Life Assurance Company, the mode of constructing which is not explained.

Another novelty appears in Art. 23, in which a claim to a discovery is made in even stronger terms than before. Mr. Scratchley says, "$(\pi_x)_5$, the net annual premium, or sinking fund to realize £1 at death of x, at 5 per cent., is a much more satisfactory form than any other, and is worthy of consideration, as it does not appear to be noticed by any other writer." I have really no wish to impute dishonesty to Mr. Scratchley, by supposing that he would make a statement knowing it to be false, and I am unwilling to give the only other reason that can be assigned for such an observation; but I should like to ask him what is the difference between his

$$(\pi_x)_5=\frac{(S_x)_5}{(a_x)_5}, \text{ and the } \frac{(\mathfrak{A}_x)_5}{1+(a_x)_5} \text{ of Milne and other writers?}$$

The value of a post obit being $\mathfrak{A}_x$ or S_x, I apprehend that any student of ordinary capacity and a moderate amount of reading would at once know that the annual premium or equivalent annuity to be granted was $\dfrac{S_x}{a_x}$; and because Mr. Scratchley chooses to disguise this under the form of π_x, he pronounces it worthy of consideration, and a fact which other writers have failed to observe. This is really too bad.

I have not the patience to pursue this inquiry further. Apart from its supposed novelties and its undeserved claim to discoveries, I should have pronounced this work to be a respectable production for anyone not supposed to be acquainted with the practical details of the questions treated of; and I am inclined to suspect that the author had some misgivings as to the real value of his labours when he omitted to state, on his title page, the nature of his present pursuits, which might have inspired confidence in the readers of a publication emanating from one supposed to be familiar with his subject.

I certainly admire "neat formulæ" and "elegant solutions;" and I am pleased with novelties, properly so called, if they possess any merit. Moreover, I believe that the discoveries in life contingencies are not so

---

* At the present price of Consols, 94, the rate of annuity granted at age twenty, by the Government Tables, is £5. 0s. 7d. per cent.; the value, therefore, is 19·884 years' purchase, and differs from the Carlisle $3\frac{1}{2}$ per cent. by ·028 only.

completely exhausted that we must resort to the very questionable expedient of disguising old expressions under new symbols, in order to establish the semblance of a claim to be a discoverer. I hope, therefore, that when Mr. Scratchley next favours us with any remarks, I may not be called upon to say of them, what I feel obliged to state of these, that " what is new in them is not true," and " what is true is not new."

<div align="center">

I am, Sir,

Yours very truly,

AN ACTUARY.

</div>

<div align="center">

ON THE SAME SUBJECT.

To the Editor of the Assurance Magazine.

</div>

SIR—A tract has been recently issued by Mr. Scratchley, " On the Valuation of Post Obits and Contingent Reversions," which affords matter for a few useful remarks; and I shall be glad if you can make room for them in the pages of the *Magazine.*

Mr. Scratchley takes for his text the case submitted to an actuary in connection with the Joddrell Policies, which case is as follows:—

" What amount should a post obit be for, to be paid by a gentleman aged 35, if he outlives his father, aged 74 ?

" The consideration to be an annual payment of £330, together with the necessary insurance on the younger life, which can be done at 5 per cent., he not being a very good one; the old life to be calculated to live ten years."

Leaving out of view the last clause—" the old life to be calculated to live ten years"—which is here out of place, and on which I shall have more to say anon, the case is a sufficiently simple one. To solve it generally: calling the elder and the younger lives (y) and (x) respectively, and the amount to be assured on (x) against (y)—which will also be the amount of the post obit—A, then it is obvious that the seller of the annuity will have to pay annually, during the joint continuance of $(x.y)$, $a + \pi A$, where a is the annuity allowed to (x), and π the Office premium per £ for the survivorship assurance; and he will receive A on the failure of the joint lives, from the Office, if this failure take place by the death of (x), and from (x) if by the death of (y). Now the present value of the compound payment is $\dfrac{(a + \pi A) N_{x-1.y-1}}{D_{x.y}}$, and that of the benefit is $\dfrac{A(v N_{x-1.y-1} - N_{x.y})}{D_{x.y}}$. Hence, equating, and solving for A, we get,

$$A = \frac{a}{v - \pi - N_{x.y} \div N_{x-1.y-1}}.$$

In the case before us, $a = 330$ and $\pi = \cdot 05$. Hence, using Jones's Tables, we easily obtain,

<div align="center">

5 per Cent.   6 per Cent.

A=£4,564. 12s., or £4,715. 11s.

</div>

The opinion of the actuary consulted is stated to have been as follows:—

" Assuming that the older life will live ten years, and that the younger will survive the elder, I am of opinion that the amount of the post obit, to cover the necessary insurance, should not be less than £12,832."

The amount here assigned as that of the post obit is very different from that just found, being nearly three times as large. The difference originates, no doubt, somehow, in the actuary having not only adopted the gratuitous

assumption prescribed in the case, namely, that the elder life will survive ten years, but also added another at his own hand, namely, that the younger life will survive the elder. The actuary does not state, of course, how he arrives at his result; but Mr. Scratchley gives what he calls a solution of the problem, as modified by the two assumptions, in which he brings out the same result. The probability therefore is, that the actuary's process is the same as Mr. Scratchley's.

Now I believe I am warranted in saying, that the so called solution is not a solution of the problem as modified, but of a different problem altogether, namely, of this:—Required the amount, A, of an annuity certain, payable in advance, of $a + \pi A$, in ten years. And to reduce the problem as modified to this, a third assumption is necessary, which is tacitly made by Mr. S. in his solution—namely, that the joint lives will fail (of course by the death of the elder) *exactly* at the point of time which separates the tenth from the eleventh year!

The legitimate form of the problem, subject to the first two assumptions, would be this:—An annuity (in advance) of $a + \pi A$ is to be paid during the next ten years, and continued during the joint continuance of two lives, to be nominated at the end of that time, aged respectively 45 and 84. Required A, the amount of the equivalent assurance on the joint duration of the same two lives. It is likely enough that in this form the problem would be repudiated both by the propounders and by the actuary. It is not the less true, however, that this is the form to which their unwarranted assumptions reduce it. In this form the problem admits of a legitimate solution, which I do not stop to give.

It is very evident that the case, with its slipshod English, is the production of a non-professional person, and there is therefore some excuse for the introduction in it of the first assumption. The idea was probably to furnish the actuary with what was considered a necessary datum for the solution of the problem, by intimating that the elder life was of such a goodness that his duration might be considered as ten years certain. But surely the duty of an actuary in such circumstances was, instead of adopting an unwarranted assumption, and adding to it another equally unwarranted, to discard all such, and solve the problem on correct principles; explaining to his client, *if necessary*, that in this mode of proceeding due account was taken of the contingencies, not only of the next ten years, but also of every subsequent year during which the two lives could jointly subsist. Especially the anomaly ought to have been avoided, of making an assumption at variance with the conditions of the problem. If the younger life is to survive the elder, why assure against the contingency of the predecease of the former?

Mr. Scratchley says of the first two assumptions spoken of (both of which he seems to father on the framer of the case), that they "are erroneous, as by the Carlisle Law of Mortality the joint existence of two lives aged 35 and 74 is only worth, at 5 per cent., 5·881, or less than six years' purchase." *Erroneous* is clearly not the term by which to describe them. They are unwarranted, unnecessary—altogether out of place. However, to pass from this, are we to understand from the language just quoted that Mr. S.'s only or chief objection to them rests on the term, ten years, to which they extend? Would his objection have been weakened, had the term been less; and would it have been removed had the term been reduced to that of the mean duration, *discounted at five per cent.?* All this the language quoted seems to imply. And yet this cannot be his view, as

he discards the assumptions from the amended statement of the case, which he gives preparatory to a correct solution of it. The language quoted must therefore, I suppose, just be looked upon as an illustration of the vagueness of expression one is apt to fall into who commences writing on a subject which he has not fully considered in all its bearings.

Mr. Scratchley gives two correct solutions of the problem, as divested of the extraneous assumptions imported into it. The first, he says, is in accordance with "the ordinary mode of calculation." I suppose it is, where the use of Barrett's columns, with the facilities they afford, is eschewed. But I doubt whether one person in ten of those into whose hands the tract will fall will understand that, by his periphrastic definition of "A"—namely, the "accumulated amount, with interest (by the end of the year when one [?] has died), of £1 a year, payable in advance, during the joint existence of x, y"—he means simply, the assurance on $(x.y)$ that an annual premium of £1 will provide.

Of his second solution Mr. S. says, that it is in a form which he believes is "not to be found in works on life contingencies." It may be so. Works on life contingencies do not profess to solve all the problems that may arise in practice, nor to give forms for their solution. What such works generally profess is, to lay down and exemplify principles; but the practical application of these is necessarily left to the judgment and intelligence of their readers.

The tract under consideration affords matter for further remark; but having exhausted my own time, your space, and, I fear, the patience of your readers, I must for the present forbear.

I am, Sir,

Your most obedient Servant,

London, 13th February, 1857.   G.

ON THE GRANT OF POLICIES WITHOUT FURTHER PAYMENT, IN CONSIDERATION OF PREMIUMS RECEIVED.

To the Editor of the Assurance Magazine.

Sir—It has been the practice of late years, with some of the Life Insurance Offices, when the assured is unable to continue his payments on an ordinary policy, to offer him, in exchange, a policy of a smaller amount, free from all future payments—or, a "paid-up" policy. More recently, the amount of this paid-up policy has been fixed by a new Office (the Unity) as equal to the amount of the premiums that have been paid on the policy, so that the representatives of the assured will receive back the amount that has been paid to the Office. As this is a plan that is likely to be pleasing to the public, from the circumstances of its being so readily understood, and carrying with it the appearance of equity, it will be interesting to examine in some detail its operation in practice.* We can very readily find a simple expression for the amount of the paid-up policy that may be given in exchange for the surrender of an ordinary policy after the lapse of any number of years. Thus, suppose that a policy, taken out at the age m, has been in force n years, the premium being just due and not paid; also, as usual, let a_m, A_m, denote the values of an annuity and an assurance of £1 on a life aged m; also, P_m the annual premium for an ordinary assur-

* There are two letters on this subject in the *Post Magazine* for October, 1855, but they take only a partial view of the question.