The Pedal Triangle.

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The area of the pedal triangle of a given triangle is easily shown by trilinear co-ordinates to bear to that of the original triangle the ratio $R^2 - S^2 : 4R^2$ where S is the distance of the point from the circumcentre of the triangle. A proof, by purely geometrical methods, of this theorem was read before the Society (*Proceedings*, Vol. III., pp. 78–79) by Mr Alison.

The following geometrical proof proceeds on somewhat different lines.

FIGURE 16.

Let ABC be the triangle, P′ the point, P′L′, P′M′, P′N′ the perpendiculars; P the point where AP′ meets the circumcircle, PL, PM, PN its perpendiculars, AK perpendicular to BC, OY perpendicular to AP.

Let AL cut M′N′ in L″; then from similarity, P′L″ is parallel to PL, and therefore collinear with P′L′.

$$\text{Area } L'M'N' : \text{area } P'M'N' = L'L'' : L''P'$$

$$\text{area } P'M'N' : \text{area } PMN = L''P'^2 : LP^2$$

$$= AP' . L''P' : AP . LP$$

$$\text{area } PMN : \text{area } PBC = PM^2 : PC^2$$

$$= PY^2 : PO^2$$

$$= PA^2 : 4PO^2$$

because $\angle PCM = PBA = POY.$

$$\text{Area } PBC : \text{area } ABC = PL : AK$$

$$\therefore \quad \text{area } L'M'N' : \text{area } ABC = L'L'' . AP' . AP : 4OP^2 . AK$$

$$\text{But} \quad L'L'' : AK = LL'' : AL = PP' : AP$$

$$\therefore \quad L'L'' . AP = AK . PP'$$

$$\therefore \quad \text{area } L'M'N' : \text{area } ABC = AK . AP' \cdot PP' : 4OP^2 . AK$$

$$= PP' . AP' : 4OP^2$$

$$= OP^2 - OP'^2 : 4OP^2$$

which is the theorem required.

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