pairs of Hermitian and quadratic forms, some extremal properties of eigenvalues, etc. The way is paved for a discussion of tensors by the instructor if he so desires, but tensors themselves are not mentioned.

There is an unusually extensive bibliography devoted almost exclusively to books and arranged by subject matter. Answers are provided to most of the problems, including some theoretical ones. There is an index of symbols in addition to the regular index.

Professor Ficken has provided a usable and unusually versatile text.

Martin H. Pearl, University of Maryland

Notes on Logic, by Roger C. Lyndon. Van Nostrand Mathematical Studies No. 6. vi + 97 pages. New York, 1966. $2.50.

This book is a remarkable example of selection and distillation. Assuming only a slight acquaintance with Zorn's Lemma, cosets and equivalence classes, definition by induction, homomorphism and infinite cardinals, the author presents in 90 pages a highly readable and self-contained account, complete with proofs (reliance on intuitive plausibility does not appear until page 84) and a valuable set of exercises, of a closely knit and highly significant portion of mathematical logic. In fact, a strong case could be made for claiming that the totality of what these 90 pages leave out is less important than what they contain.

The central theme is "the semantic connection between a formal language and a mathematical system serving as a model", and the topics treated include the Deduction Theorem, The Consistency Theorem, The Adequacy Theorem and Compactness Theorem, The Löwenheim-Solem Theorem, Gentzen's Natural Inference, The Herbrand Gentzen Theorem, Tarski's Theorem (on the inexpressibility of the property of being a true formula), Gödel's Incompleteness Theorem and Church's Theorem. Other topics are the usual preliminaries to these.

To pack so much into so small a space something obviously has to be sacrificed: the sacrifice is astonishingly small. It is assumed that the reader can handle routine chores on his own and does not need the ministrations of pedagogical virtuosity in designing carefully graduated approaches to everything. Toward the end, some of the really long proofs are of course omitted, but the reader is succinctly made aware of what is missing. The reward of such a lean, but careful, treatment is a sense of pace and purpose, and clarity of structure which is very difficult to achieve otherwise. This book should be an invaluable aid to the student as a complement to most of the classics in this field.

Excellence notwithstanding, the reviewer feels that some further polishing would be well worth the trouble.

A slightly fuller exposition is desirable in a few places. On page

10, at the bottom, the definition of "free" will be mysterious to readers not already familiar with the notion: No. 1, of the same series (Halmos), has an excellent and brief explanation on page 41. The phrase "the easiest way out" on page 13, line 14, covers far too much. On pages 14, 15 too much reliance is put on the explanatory powers of algebraic symbolism: the alternative definition is intended to explain away the apparent circularity of "all interpretations" appearing in the definition of an interpretation. But "all ϕ" appears in the alternative definition and the improvement is not completely obvious. The proposition on page 16 motivates most of the preceding few pages: an earlier mention of it would be helpful, and also perhaps the suggestion that the reader try to provide counter-examples and see where they fail. Similar comments apply to the Corollary on page 18. In the definition of semantic implication on page 18, something should be said to the effect that "implies" refers to material implication, and a brief discussion of this latter notion would not be out of place, considering the central importance of this section. On page 39 the fact that "p is evidently equivalent to p' " needs some elaboration, although the expository difficulties here are considerable. Some examples of interesting theorems concerning dense linear order and identity would clarify matters considerably in a small space. "A moments reflection shows" on page 41, line 3, is a little too quick for most readers. The section on the Deduction Theorem, while very brief, is uncommonly good: in most treatments its motivation and relationship with other results are well-kept secrets. The arguments on page 52 would be clarified if an extra type font were used to distinguish different types of sets. Most readers (even algebraists) will have some difficulty in accepting A+ and E- without explanation (page 68). On page 70 it is nearly, but not quite, obvious that the alterations do not essentially change the system. Note should be taken that the last modification of E+ actually violates the scheme for the other rules. Some care should be taken that the implications of x not occurring free in T are fully grasped. An example such as

$$\frac{p(x) \vee p(\xi)}{p(x) \vee A\xi\, p(\xi)}$$

would be helpful (showing what must be avoided). On page 73 it would help to observe that in each rule the operative part is the first (from the left) non-simple formula occurring, and that in going from conclusion to premise its length is always shortened. From page 84 on, the use of "arithmetically expresses" for "expresses" would make for easier reading. On page 84, line 10, "corresponding" needs to be explained. In line 11, "clear" should be replaced by "plausible" to guard against misunderstanding: the considerations are not at all trivial! On page 84, "whose number" should be "whose Gödel number" to prevent confusion between α and γ. Concerning lines -13 to -11, the more usual definition is that $s(\overline{m}, \overline{n}) = \overline{q}$ if and only if there exists a t for which $m = \gamma[t]$ and there exists a $p(v_1)$ such that $n = \gamma[p(v_1)]$ and $q = \gamma[p(\overline{m})]$. The definition in the text is less specific, and this might cause difficulties later on. From page 86 on, the sketch of the proof of Church's Theorem is a helpful one, but the extent of its sketchiness might have been more

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clearly acknowledged. The "punch line" on page 86 is "should then be expressible" in line -7: beneath this simple phrase there lurks nothing less than Church's Thesis!

A few misprints were detected, as well as some slips, but none likely to cause difficulty.

R.A. Staal, The University of Waterloo

Report on Injective Modules, by Tsai Chi - Te. Queen's papers in pure and applied mathematics, No. 6, Kingston, 1966. 243 pages. $3.00.

This useful report includes many results on injective modules which have not previously appeared in book form. Proofs are given in great detail and there are few misprints. The bibliography includes ninety entries.

The book is divided into two parts, the first entitled "general properties of injective modules" and includes the proof of the existence of injective hulls, and the structure theory of the injective hull of a finite dimensional module and its ring of endomorphisms. The second part is called "injective modules over various rings" and these rings include principal ideal domains, Dedekind domains, Prüfer rings, integral domains, semi-simple (completely reducible), finite dimensional algebras (over a field) and rings with chain conditions.

W.D. Burgess, McGill University

Abstract Theory of Groups, by O.U. Schmidt. Translated from the Russian by F. Holling and J.B. Roberts; edited by J.B. Roberts. W.H. Freeman and Co., San Francisco, 1966. vii + 174 pages. $5.00.

The Russian original of this book was published in a small edition in 1916, at a time when Burnside's treatise was the only large scale work on group theory in existence. An apparently unchanged second edition appeared in 1933. It is truly regrettable that this work had not been made generally available much earlier. But it seems that few copies, if any, have ever left the USSR. An unchanged reprint appeared in 1959 in a small volume of Selected Works on Mathematics of Otto Schmidt (together with his original papers on group theory, all translated into Russian). Thus the book became accessible.

The first part, Chapters 1 - 4 (about one third of the whole work) includes the definition and the simplest consequences, as well as most general theorems concerning invariance, homomorphism, automorphisms together with a good deal of information on finite groups and permutation groups. Most proofs are clearly given with finite groups in mind; notions of infinite set theory are not introduced. The second part, Chapters 5 - 10, deals with finite groups only; it includes Landau's theorem, several