

A SYSTEM OF NONLINEAR
REACTION DIFFUSION EQUATIONS

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This thesis investigates the behaviour of the solutions of a system of reaction-diffusion equations. These equations model an exothermic chemical reaction that takes place within a catalyst and which is sustained by a single diffusing reactant. In dimensionless coordinates these equations have the form

$$\left. \begin{aligned} \nabla^2 u - \frac{\partial u}{\partial t} + \lambda(1+v)e^u &= 0 \\ \nabla^2 v - \beta \frac{\partial v}{\partial t} - \lambda\alpha(1+v)e^u &= 0 \end{aligned} \right\} \quad x \in \Omega, \quad t > 0$$

(P)

$$\frac{\partial u}{\partial n} + \mu u = 0, \quad \frac{\partial v}{\partial n} + \nu v = 0, \quad \text{on } \partial\Omega$$

$$u(x, 0) = \phi(x), \quad v(x, 0) = \psi(x), \quad x \in \Omega$$

where u corresponds to the temperature, $1+v$ to the concentration of the reactant, β is the Lewis number and λ is the Frank-Kamenetskii parameter.

The major part of this study is concerned with the solution of the steady state system associated with problem (P). Using the maximum principle, methods of upper and lower solutions and standard topological results from nonlinear analysis, existence, uniqueness and multiplicity results are obtained. In particular it is shown that the steady state

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equations have a solution for all values of the parameters except for the special case $\mu < \nu = \infty$; and when the parameter λ is sufficiently large or small this solution is unique.

The final part of this thesis considers the question of the existence of solutions to problem (P) which are defined for all time. Such existence results are derived for some special cases; but for the general case only partial answers, such as a local existence theorem, are obtained.

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