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HYPERSPACES OF H-CLOSED SPACES

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A space is $H(i)$ $[R(i)]$ if every open [regular] filter base has a cluster point and $H(ii)$ $[R(ii)]$ if every open [regular] filter base with a unique cluster point converges. This terminology is due to C. T. Scarborough and A. H. Stone [**11**]; $H(i)$ spaces have been studied as quasi-H-closed spaces in [**10**] and as generalized absolutely closed spaces in [**6**]. Hausdorff $H(i)$ $[H(ii)]$ spaces are called *H-closed* [*minimal Hausdorff*] and regular T_1 $R(i)$ $[R(ii)]$ spaces are called *R-closed* [*minimal regular*]. For a space X, 2^X is the set of all non-empty closed subsets of X with the finite topology [**8**]. The present study was motivated by the long-standing problem of whether or not a T_3 space with every closed subset R-closed is compact, and also by the well-known result ([**8**] and [**14**]) that X is compact if and only if 2^X is compact. We show that a T_1 space X is $H(i)$ if and only if 2^X is $H(i)$, and that if 2^X is $H(ii)$ $[R(i)$, $R(ii)$, feebly compact] then X is $H(ii)$ $[R(i)$, $R(ii)$, feebly compact]. We cannot expect X to be H-closed if and only if 2^X is H-closed since 2^X is Hausdorff if and only if X is T_3 [**8**, Theorem 4.9.3], and a T_3 H-closed space is compact; however, we do prove that $H(X)$, the set of all non-empty θ-closed subsets of a Hausdorff space X is H-closed (in the relative topology inherited from 2^X) if and only if X is H-closed and Urysohn.

For $A_1, \ldots, A_n$ subsets of X, let

$$\langle A_1, \ldots, A_n\rangle = \{F \in 2^X : F \subset \bigcup A_i,\ F \cap A_i \neq \emptyset \text{ for all } i\}.$$

The *finite topology* on 2^X is the topology with base $\{\langle U_1, \ldots, U_n\rangle : U_i$ open in X, $i = 1, \ldots, n\}$.

For a space X and $A \subseteq X$, the *θ-closure* of A, denoted $\mathrm{Cl}_\theta A$, is $\{x$: every closed neighborhood of x meets $A\}$. A is *θ-closed* if $\mathrm{Cl}_\theta A = A$. $\mathrm{int}_\theta A$ is defined analogously. $\mathrm{Cl}_\theta A$ is closed and $\mathrm{int}_\theta A$ is open. These concepts were first defined by Velicko [**13**]. For a Hausdorff space X, let $H(X)$ denote the collection of all *θ-closed* subsets of X with the topology $H(X)$ inherits as a subset of 2^X.

The following facts are easily verified.

1.1 $\mathrm{Cl}\langle A_1, \ldots, A_n\rangle = \langle \mathrm{Cl}\, A_1, \ldots, \mathrm{Cl}\, A_n\rangle$ [**8**, Lemma 2.3.2]

1.2 $\langle A_1, \ldots, A_n\rangle = \langle X, A_1\rangle \cap \ldots \cap \langle X, A_n\rangle \cap \langle \bigcup A_i\rangle$

1.3 $\mathrm{int}\langle A\rangle = \langle \mathrm{int} A\rangle$ [**7**, p. 161, Vol. I]

1.4 $\mathrm{int}\langle X, A\rangle = \langle X, \mathrm{int}\, A\rangle$

1.5 $\langle \mathrm{int}\, A_1, \ldots, \mathrm{int}\, A_n\rangle \subset \mathrm{int}\langle A_1, \ldots, A_n\rangle$

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1.6 $\operatorname{int}\mathrm{Cl}\langle A_1, \ldots, A_n\rangle = \operatorname{int}\mathrm{Cl}\langle X, A_1\rangle \cap \ldots \cap \operatorname{int}\mathrm{Cl}\langle X, A_n\rangle \cap \operatorname{int}\mathrm{Cl}\langle \cup A_i\rangle = \langle X, \operatorname{int}\mathrm{Cl}\, A_1\rangle \cap \ldots \cap \langle X, \operatorname{int}\mathrm{Cl}\, A_n\rangle \cap \langle \operatorname{int}\mathrm{Cl} \cup A_i\rangle$

1.7 For $A \subseteq X$, $\mathrm{Cl}\, A \subseteq \mathrm{Cl}_\theta A$.

1.8 For U open in X, $U \subseteq \operatorname{int}_\theta \mathrm{Cl}_\theta U$.

1.9 If $\mathscr{U} = \langle U, U_1, U_2, \ldots, U_n\rangle, \mathscr{V} = \langle V, V_1, \ldots, V_m\rangle$ where

$$\bigcup_{i=1}^{n} U_i \subseteq U \quad \text{and} \quad \bigcup_{i=1}^{m} V_i \subseteq V \quad \text{and} \quad U \cap V = \emptyset,$$

then either some U_i fails to meet V, $1 \leqq i \leqq n$, or some V_j fails to meet U, $1 \leqq j \leqq m$.

1.10 If $A, B \in 2^X$ and $A \not\subseteq \mathrm{Cl}_\theta B$, then there exist disjoint open sets U and V of 2^X containing A and B respectively.

1.11 The arbitrary union of θ-open subsets is θ-open [**13**].

1.12 For U open in X, $\mathrm{Cl}\, U = \mathrm{Cl}_\theta U$ [**13**], so that

$$\mathrm{Cl}\, U = \mathrm{Cl}(\operatorname{int}_\theta \mathrm{Cl}_\theta U).$$

1.13 $\mathrm{Cl}_{H(X)}\langle X, U\rangle = \langle X, \mathrm{Cl}\, U\rangle \cap H(X), \mathrm{Cl}_{H(X)}\langle V\rangle = \langle \mathrm{Cl}\, V\rangle \cap H\langle X\rangle$, and $\operatorname{int}_{H(X)}\mathrm{Cl}_{H(X)}(\langle U\rangle \cap H(X)) = (\operatorname{int}_{2^X}\mathrm{Cl}_{2^X}\langle U\rangle) \cap H(X) = \langle \operatorname{int}\mathrm{Cl}\, U\rangle \cap H(X)$.

1.14 If X is H-closed and Urysohn and A is a regularly closed subset of X, then $\mathrm{Cl}_\theta A = A$, i.e., A is θ-closed [**13**].

The reader is referred to [**7**] and [**8**] for other facts about the finite topology and to [**1**] and [**11**] for the standard characterizations of $H(i)$, $R(i)$, $H(ii)$, and $R(ii)$ spaces. Whenever we consider 2^X, we shall assume X is T_1.

Definitions. A collection of subsets of X is *inadequate* [**16**, Exercise 175] if it fails to cover X. A collection of subsets of X is *proximately finitely inadequate* if no finite subcollection is a proximate cover of X.

Notice that the standard covering characterization for $H(i)$ spaces may be stated: a space X is $H(i)$ if and only if every proximately finitely inadequate collection of open sets is inadequate.

LEMMA 1 [**4**, p. 15]. *Let* $U_1, \ldots, U_n$ *be open in* X *and let* $U = U_1 \cap \ldots \cap U_n$. *Then* $\operatorname{int}\mathrm{Cl}\, U = \operatorname{int}\mathrm{Cl}\, U_1 \cap \ldots \cap \operatorname{int}\mathrm{Cl}\, U_n$.

THEOREM 1. *A space* X *is* $H(i)$ *if and only if every subbasic open cover has a finite proximate subcover.*

Proof. To prove the sufficiency, let $\mathscr{S}$ be a subbase for X and let β be a proximately finitely inadequate family of open sets in X. By maximality, $\beta \subseteq \alpha$, where α is a maximal family with this property. We shall show α, and thus β, is inadequate. First, notice that for any open set A, $A \in \alpha$ if and only if $\operatorname{int}\mathrm{Cl}\, A \in \alpha$. Now, the family $\mathscr{S} \cap \alpha$ is a proximately finitely inadequate collection of subbasic open sets and so is inadequate.

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We claim

$$\bigcup \{A : A \in \alpha\} = \bigcup \{A : A \in \mathscr{S} \cap \alpha\}.$$

If $x \in A$, $A \in \alpha$, there exist $U_1, \ldots, U_n$ in $\mathscr{S}$ such that $x \in U_1 \cap \ldots \cap U_n \subset A$. Since int Cl $A \in \mathscr{S}$, by maximality so is int Cl$(U_1 \cap \ldots \cap U_n)$, and this set is equal to int Cl $U_1 \cap \ldots \cap$ int Cl U_n by Lemma 1. We claim that for some i, $i = 1, \ldots, n$, int Cl $U_i \in \alpha$. For suppose not. Then, for each i there are sets $A_{1i}, \ldots, A_{mi}$ in α such that

$$X = \text{Cl } A_{1i} \cup \ldots \cup \text{Cl } A_{mi} \cup \text{Cl}(\text{int Cl } U_i) = \text{Cl } A_{1i} \cup \ldots \cup \text{Cl } A_{mi} \cup \text{Cl } U_i.$$

But then it is easily shown that

$$X = (\text{Cl } A_{11} \cup \ldots \cup \text{Cl } A_{m1}) \cup \ldots \cup (\text{Cl } A_{1n} \cup \ldots \cup \text{Cl} A_{mn}) \cup \text{int}(\text{Cl } U_1 \cap \ldots \cap \text{Cl } U_n)$$

and since

$$\text{int}(\text{Cl } U_1 \cap \ldots \cap \text{Cl } U_n) = \text{int Cl } U_1 \cap \ldots \cap \text{int Cl } U_n,$$

int Cl $U_1 \cap \ldots \cap$ int Cl U_n is not in α, which is not possible. It follows that int Cl U_i, and hence U_i, is in α for some i, $i = 1, \ldots n$. Therefore, $x \in U_i$, $U_i \in \mathscr{S} \cap \alpha$, and α is inadequate.

LEMMA 2. *Let $\mathscr{F}$ be an open filter base on X and let $\mathscr{F}' = \{\langle U \rangle : U \in \mathscr{F}\}$. Then:*

(a) *$\mathscr{F}'$ is an open filter base on 2^X; $\mathscr{F}'$ is regular if $\mathscr{F}$ is regular and countable if $\mathscr{F}$ is countable;*

(b) *if x_0 is a cluster point of $\mathscr{F}$, $\{x_0\}$ is a cluster point of $\mathscr{F}$;*

(c) *if A is a cluster point of $\mathscr{F}'$ and $y \in A$, then y is a cluster point of $\mathscr{F}$;*

(d) *if x_0 is the unique cluster point of $\mathscr{F}$, then $\{x_0\}$ is the unique cluster point of $\mathscr{F}'$;*

(e) *if $\mathscr{F}' \to \{x_0\}$, then $\mathscr{F} \to x_0$.*

Proof. We prove only (c). If $y \in A \in \text{Cl}\langle V \rangle = \langle \text{Cl } V \rangle$ for all $V \in \mathscr{F}$, then $y \in A \subseteq \text{Cl } V$. So, $Y \in \cap \hat{\mathscr{F}} \text{Cl } V$ and y is a cluster point of $\mathscr{F}$.

Definition. X is *feebly compact* if every countable open filter base has a cluster point.

PROPOSITION 1. *If 2^X is $H(i)$ [or $R(i)$ or feebly compact] then X is $H(i)$ [or $R(i)$ or feebly compact, respectively].*

Proof. Let 2^X be $H(i)$ [$R(i)$, feebly compact] and let $\mathscr{F}$ be an open [regular, countable open] filter base on X. By Lemma 2(a), $\mathscr{F}'$ is an open [regular, countable open] filter base on 2^X and hence has a cluster point F. But if $y \in F$, then by Lemma 2 (c), y is a cluster point of $\mathscr{F}$.

THEOREM 2. *X is $H(i)$ if and only if 2^X is $H(i)$.*

Proof. The sufficiency follows immediately from Proposition 1. For the necessity, by Theorem 1 it is enough to show that every subbasic open cover of 2^X has a finite proximate subcover. Let

$$2^X = \bigcup_\alpha \langle X, U_\alpha \rangle \cup \bigcup_\beta \langle V_\beta \rangle$$

and let

$$F = X - \bigcup_\alpha U_\alpha.$$

Then F is closed. If $F = \emptyset$, then $X = \bigcup_\alpha U_\alpha$ and since X is $H(i)$ there is a finite sub-collection $U_{\alpha_1}, \ldots, U_{\alpha_n}$ such that $X = \bigcup \text{Cl } U_{\alpha_i}$. But then

$$2^X = \text{Cl}\langle X, U_{\alpha_1} \rangle \cup \ldots \cup \text{Cl}\langle X, U_{\alpha_n} \rangle.$$

If $F \neq \emptyset$, then $F \in 2^X$ so $F \in \langle V_{\beta_0} \rangle$ for some β_0; that is,

$$F \subset V_{\beta_0} \subset \text{int Cl } V_{\beta_0}$$

and so

$$X - \text{int Cl } V_{\beta_0} \subset X - F = \bigcup_\alpha U_\alpha.$$

Since $X - \text{int Cl } V_{\beta_0}$ is regularly closed, it is $H(i)$ [**8**, 2.2], and hence there exists a subcollection $U_{\alpha_1}, \ldots, U_{\alpha_n}$ such that

$$X - \text{int Cl } V_{\beta_0} \subset \text{Cl } U_{\alpha_1} \cup \ldots \cup \text{Cl } U_{\alpha_n}.$$

We claim

$$2^X = \text{Cl}\langle X, U_{\alpha_1} \rangle \cup \ldots \cup \text{Cl}\langle X, U_{\alpha_n} \rangle \cup \text{Cl}\langle V_{\beta_0} \rangle.$$

For if $G \in 2^X$ and $G \subset \text{Cl } V_{\beta_0}$ then

$$G \in \langle \text{Cl } V_{\beta_0} \rangle = \text{Cl}\langle V_{\beta_0} \rangle.$$

If G is not contained in $\text{Cl } V_{\beta_0}$, then G is not contained in $\text{int Cl } V_{\beta_0}$, so that $G \cap \text{Cl } U_{\alpha_k} \neq \emptyset$ for some k, $k = 1, \ldots, n$, and then

$$G \in \langle X, \text{Cl } U_{\alpha_k} \rangle = \text{Cl}\langle X, U_{\alpha_k} \rangle.$$

We omit the easy proofs of the corollaries below.

COROLLARY 1. *For a Hausdorff space X the following are equivalent:*
(a) *X is compact;*
(b) *2^X is compact;*
(c) *2^X is minimal Hausdorff;*
(d) *2^X is H-closed;*
(e) *2^X is minimal regular;*
(f) *2^X is R-closed.*

COROLLARY 2. *A Hausdorff space X is H-closed if and only if 2^X is $H(i)$.*

THEOREM 3. *A Hausdorff space X is H-closed and Urysohn if and only if $H(X)$ is H-closed.*

Proof. Suppose X is H-closed and Urysohn. Then, by 1.10, $H(X)$ is a Hausdorff space. Also, if

$$H(X) = \bigcup_\alpha(\langle X, U_\alpha\rangle \cap H(X)) \cup \bigcup_\beta(\langle V_\beta\rangle \cap H(X)),$$

then

$$H(X) = \bigcup_\alpha (\langle X, \operatorname{int}_\theta \operatorname{Cl}_\theta U_\alpha\rangle \cap H(X)) \cup \bigcup_\beta (\langle V_\beta\rangle \cap H(X)).$$

Now, if $F = X - \bigcup_\alpha \operatorname{int}_\theta \operatorname{Cl}_\theta U_\alpha$,

$$F = \bigcap_\alpha (X\backslash\operatorname{int}_\theta \operatorname{Cl}_\theta U_\alpha) = \bigcap_\alpha (X\backslash\operatorname{int} \operatorname{Cl} U_\alpha)$$

and by 1.14, each $X\backslash\operatorname{int} \operatorname{Cl} U_\alpha$ is θ-closed. Thus F is θ-closed.

Using 1.7 through 1.13 above, the remainder of the demonstration that $H(X)$ is $H(i)$ is essentially as in the proof of Theorem 1. Thus, $H(X)$ is $H(i)$ and Hausdorff, so H-closed.

Now suppose $H(X)$ is H-closed. It follows almost exactly as in Proposition 1 that X is $H(i)$, and thus H-closed since X is Hausdorff. We will now show that X is Urysohn. Suppose to the contrary, that X is not Urysohn. Then there exist $x, y \in X$ such that $\operatorname{Cl} N_x \cap \operatorname{Cl} N_y \neq \emptyset$ for every $N_x \in \mathscr{N}_x$ and $N_y \in \mathscr{N}_y$. (Here $\mathscr{N}_p$ denotes the set of all open sets containing $p \in X$.) Let N_x and N_y be chosen so that $N_x \cap \operatorname{Cl} N_y = \emptyset$. Now since X is Hausdorff, and $\mathscr{F}(X)$, the set of all non-empty finite subsets of X is dense in 2^X and $\mathscr{F}(X) \subseteq H(X)$, it follows from 1.9 that

$$\mathscr{F} = \{\langle N_y, N_y \cap N_z\rangle \cap H(X) : z \in \operatorname{Cl} N_x \cap \operatorname{Cl} N_y, N_z \in \mathscr{N}_z\}$$

is an open filter base on $H(X)$. Thus, since $H(X)$ is $H(i)$, there exists a cluster point T of $\mathscr{F}$ and $T \in H(X)$. It then follows from 1.13, that

$$(*) \quad T \in \bigcap \{\langle \operatorname{Cl} N_y, \operatorname{Cl}(Nz \cap N_y)\rangle : z \in \operatorname{Cl} N_y \cap \operatorname{Cl} N_x, N_z \in \mathscr{N}_z\}.$$

Let $w \in \operatorname{Cl} N_x \cap \operatorname{Cl} N_y$ and $N \in \mathscr{N}_w$. Then $(T \cap \operatorname{Cl} N) \supseteq (T \cap \operatorname{Cl}(N \cap N_y))$ and by (*), this latter set is non-empty. This implies that $w \in \operatorname{Cl}_\theta T = T$. Hence

$$(\operatorname{Cl} N_x \cap \operatorname{Cl} N_y) \subseteq T.$$

Now if $M \in \mathscr{N}_x$,

$$(T \cap \operatorname{Cl} M) \supseteq (\operatorname{Cl} N_x \cap \operatorname{Cl} N_y \cap \operatorname{Cl} M)$$
$$\supseteq (\operatorname{Cl}(M \cap N_x) \cap \operatorname{Cl} N_y)$$

and this last set is non-empty, so that $x \in \operatorname{Cl}_\theta T = T$. However by (*),

$$T \subseteq \operatorname{Cl} N_y \cup \operatorname{Cl}(M \cap N_y) \subseteq \operatorname{Cl} N_y,$$

so $x \in T \subseteq \operatorname{Cl} N_y$ and this is impossible. Hence X must be Urysohn. This completes the proof.

Remark. Note that in the above proof we showed that if $H(X)$ is $H(i)$ and X is Hausdorff, then X must be Urysohn, i.e., we did not employ the Hausdorffness of $H(X)$ in this part of the proof.

Definition [**15**]. A space X is said to be *seminormal* (resp., *θ-seminormal*) if every closed (resp., θ-closed) subset has a neighborhood base consisting of regularly open sets.

LEMMA 3. *Let $U_1, U_2, \ldots, U_n$ be open subsets of X and let $\bigcup_{i=1}^n U_i \subseteq U$. Consider the following:*

(a) *each of $U, U_1, U_2, \ldots, U_n$ is regularly open in X;*
(b) $\mathscr{U} = \langle U, U_1, U_2, \ldots, U_n\rangle$ *is regularly open in 2^X;*
(c) $\mathscr{V} = \langle U, U_1, U_2, \ldots, U_n\rangle \cap H(X)$ *is regularly open in $H(X)$.*

Then (a) *implies* (b), (b) *implies* (c) *if X is Hausdorff, and either* (b) *or* (c) *implies U is regularly open in X.*

The proofs are elementary and are omitted.

THEOREM 4. *X is seminormal if and only if 2^X is semiregular.*

Proof. Let X be seminormal and let $F \in \langle V_1, \ldots, V_n\rangle \subseteq 2^X$. Let W be a regularly open subset of X with $F \subset W \subseteq \bigcup_{i=1}^n V_i$ and for each i, $1 \leqq i \leqq n$, let $x_i \in F \cap V_i$. Then for each i, $1 \leqq i \leqq n$, let W_i be a regularly open subset of X such that $x_i \in W_i \subseteq V_i \cap W$. It follows that

$$F \in \mathscr{W} = \langle W, W_1, W_2, \ldots, W_n\rangle \subset \langle V_1, V_2, \ldots, V_n\rangle$$

and so $\{F\}$ has a base of regularly open subsets of 2^X.

Now suppose 2^X is semiregular, F is a closed subset of X, and R is an open subset of X containing F. Since 2^X is semiregular there exists a regularly open subset $\mathscr{U}$ of 2^X with $F \in \mathscr{U} \subseteq \langle R\rangle$. Let

$$\langle U, U_1, U_2, \ldots, U_n\rangle$$

be a basic open subset of 2^X such that $U \supseteq \bigcup_{i=1}^n U_i$, $F \in \langle U, U_1, \ldots, U_n\rangle \subseteq \mathscr{U}$. Then

$$\mathrm{int}_{2^X}\mathrm{Cl}_{2^X}\langle U, U_1, \ldots, U_n\rangle = \langle \mathrm{int\,Cl}\,U, \mathrm{int\,Cl}\,U_1, \ldots, \mathrm{int.Cl}\,U_n\rangle \subseteq \mathscr{U}.$$

This implies $F \subseteq \mathrm{int\,Cl}\,U \subseteq R$ and X is seminormal. This completes the proof.

THEOREM 5. *$H(X)$ is semiregular if and only if X is θ-seminormal.*

Proof. Suppose $H(X)$ is semiregular, A is a θ-closed subset of X, and G is an open subset of X containing A. Then $\langle G\rangle \cap H(X)$ is open in $H(X)$ and contains $A \in H(X)$. Hence there exists a regular open set $\langle U, U_1, U_2, \ldots, U_n\rangle \cap H(X))$ such that

$$U \supseteq \bigcup_{i=1}^n U_i,$$

$$A \in (\langle U, U_1, \ldots, U_n\rangle \cap H(X)) \subseteq \langle G\rangle \cap H(X).$$

Then, by Lemma 3, U is regularly open in X and $A \subseteq U \subseteq G$. This implies that X is θ-seminormal.

Now suppose X is θ-seminormal. Let $A \in H(X)$ and let $\mathscr{U} = \langle U, U_1, \ldots, U_n \rangle$ be a basic open subset of 2^X where $U \supseteq \bigcup_{i=1}^n U_i$ and $A \in \mathscr{U}$. Since X is θ-seminormal, there exist regularly open subsets R, $R_1, R_2, \ldots, R_n$ of X such that $A \subseteq R \subseteq U$ and $\emptyset \neq R_i \cap A \subseteq U_i \cap R$ for $1 \leqq i \leqq n$. Then

$$\mathscr{R} = \langle R, R_1, \ldots, R_n \rangle \cap H(X)$$

is regularly open in $H(X)$ and $A \in R \subseteq \mathscr{U}$. Thus $H(X)$ is semiregular.

COROLLARY 4. *If X is seminormal and $H(i)$ then 2^X is $H(ii)$.*

Proof. If X is seminormal and $H(i)$, then by Theorems 2 and 4 2^X is $H(i)$ and semiregular, hence $H(ii)$ by [**10**, 2.11].

COROLLARY 5. *A Hausdorff space X is compact if and only if $H(X)$ is minimal Hausdorff.*

Proof. If X is compact and Hausdorff, $2^X = H(X)$ and 2^X is compact. Thus $H(X)$ is minimal Hausdorff.

On the other hand, if $H(X)$ is minimal Hausdorff, then $H(X)$ is semiregular and $H(i)$. But by Theorems 2, 3 and 5, X is Urysohn and minimal Hausdorff. Such spaces are compact.

PROPOSITION 2. *If 2^X is $H(ii)$ [$R(ii)$] then X is $H(ii)$ [$R(ii)$].*

Proof. Let 2^X be $H(ii)$ [$R(ii)$] and let $\mathscr{F}$ be an open [regular] filter base on X with a unique cluster point x_0. By Lemma 2 (d), $\{x_0\}$ is the unique cluster point of the open [regular] filter base $\mathscr{F}'$ on 2^X. Since 2^X is $H(ii)$ [$R(ii)$], $\mathscr{F}'$ converges to $\{x_0\}$ and so, by Lemma 2 (e), $\mathscr{F}$ converges to x_0.

Remarks. (1) Since every countably compact space is feebly compact and every feebly compact space is pseudocompact [**12**, Theorem 2.6], J. Keesling's example in [**5**, p. 765] is a feebly compact space whose hyperspace is not feebly compact. J. Ginsburg [**3**] has shown that if 2^X is feebly compact then X is feebly compact and that if X is regular and 2^X is feebly compact then all finite powers of X are feebly compact. (He calls feebly compact spaces *$\mathscr{G}$-pseudocompact.*) Ginsburg has also considered the problems of characterizing those spaces whose hyperspace is countably compact and those spaces whose hyperspace is pseudocompact.

(2) Dix Pettey has reported, in a private communication, that he has constructed an R-closed space X such that 2^X is not $R(i)$.

(3) If the empty set is included as an isolated point in 2^X, as is done in [**5**], then whenever 2^X is $H(ii)$ it is nonvacuously $H(ii)$. (See [**11**].)

(4) It follows, from Theorem 3, that if κX denotes the generalized, H-closed Katetov extension of a space X [**6**], then, in general, $\kappa(H(X)) \neq H(\kappa(X))$. For if X is a non-Urysohn H-closed space, $H(\kappa(X)) = H(X)$ and so if $\kappa(H(X)) = H(X)$, X must be Urysohn.

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