ACCRETION PROCESSES IN CLOSE BINARIES

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Abstract. Accretion processes and the associated emission mechanisms are discussed, with special reference to binary X-ray sources.

1. Introduction

The physics of accretion is relevant to several classes of close binary systems. In these remarks, I shall focus attention on systems containing a compact component and therefore a deep potential well: the accretion-powered luminosity can then be very high. Although the resulting luminosity is then generally concentrated in the X-ray band, and can account for the properties of binary X-ray sources, it is important to emphasise that analogous processes can occur in other types of binary which are not yet known to be X-ray sources. Indeed, many aspects of accretion – discs, hot spots, etc. – can perhaps be investigated best by optical and ultraviolet observations of dwarf novae and similar systems.

The reader is referred to earlier reviews (e.g. Novikov and Thorne, 1973; Rees 1974a, b) for background material. An up-to-date and fuller discussion of the theoretical aspects of accretion is given by Lightman *et al.* (1976).

2. Accretion Discs

When the accretion flow is spherically symmetrical, the gravitational energy lost by infalling matter goes mainly into internal or kinetic energy. The efficiency for conversion of rest mass into radiation can be high if the material impacts onto a hard surface such as a neutron star. Spherical accretion onto black holes is relatively inefficient despite the deep potential well. Precisely *how* inefficient it is, however, is an open question: the well-known calculations by Schwartzman (1971) and by Shapiro (1973a, b) assumed laminar inflow, but Meszaros (1975) has recently argued that turbulent dissipation is likely to occur, especially when the accreted material contains random magnetic fields, and this can enhance the amount of emission.

For accretion on to a compact object in a binary system, the accretion luminosity tends to be high not only because the companion can be a prolific source of material but also because the captured material generally has too much angular momentum to fall radially inwards. In this situation we get accretion discs.

2.1. STEADY NEWTONIAN ACCRETION DISCS: GENERAL OUTLINE

The basic theory has been repeatedly reviewed elsewhere, so I shall give only a brief summary here, giving references where appropriate to the fuller treatment. It is assumed in the simplest versions of the theory that the gas moves in more or less Keplerian orbits, i.e.

$$r\Omega = \left(\frac{GM}{r}\right)^{\frac{1}{2}}, \tag{1}$$

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but spirals slowly inward at a rate governed by the efficiency with which viscosity transports angular momentum outwards. The disc radiates the energy supplied by viscous friction. The efficiency is always high for disc accretion, essentially because the gas has more time to radiate and cannot be swallowed until it has moved into a tightly-bound close orbit. If mass flows in at a steady rate $\dot{M}$, angular momentum flows in also at a corresponding rate $\beta \dot{M}(GMr_1)^{\frac{1}{2}}$, where r_1 is the inner boundary of the disc and β ($\leqslant 1$) depends on the nature of the inner boundary (see the discussion in Novikov and Thorne 1973).

If the characteristic disc thickness $h(r)$ is $\ll r$, which can be checked a posteriori, and if we can legitimately average over the vertical structure, then conservation of angular momentum and energy give the two following equations:

$$t_{\phi r} \,.\, r \,.\, 2\pi r \,.\, 2h = \dot{M}\,[(GMr)^{\frac{1}{2}} - \beta (GMr_1)^{\frac{1}{2}}], \tag{2}$$

where $t_{\phi r}$ denotes the viscous stress; and

$$\epsilon = -2t_{\phi r}\,\sigma_{\phi r} = \frac{3}{2}\,t_{\phi r}\,\Omega, \tag{3}$$

where ϵ is the energy liberated per unit volume, averaged over the vertical coordinate, and $\sigma_{\phi r}$ is the shear. The power per unit area, $F(r)$, from each side of the disc is then

$$F(r) = \epsilon h = \frac{3}{8\pi}\left(\frac{GM}{r^3}\right)\dot{M}\left[1 - \beta\left(\frac{r_1}{r}\right)^{\frac{1}{2}}\right]. \tag{4}$$

Note the non-intuitive factor 3 in Equation (4). It is important to realise that $F(r)$ is independent of the magnitude of the viscosity $t_{\phi r}$, and depends only on $\dot{M}$. However the *spectrum* of the emerging radiation does depend on the density in the disc which, for a given $\dot{M}$, depends on viscosity. All that can definitely be said is that the disc temperature T(r) must exceed the black body temperature $T_{\rm bb}(r)$ that corresponds to the energy flux $F(r)$:

$$T_{\rm bb} \simeq 7 \times 10^6 \left(\frac{\dot{M}}{10^{16}\ {\rm gm\,s^{-1}}}\right)^{\frac{1}{4}} \left(\frac{r}{10^6\ {\rm cm}}\right)^{-\frac{3}{4}} \left(\frac{M}{M_\odot}\right)^{\frac{1}{4}} \tag{5}$$

Relation (5) guarantees that the energy resulting from accretion onto a neutron star or stellar-mass black hole will be predominantly in the X-ray band if the luminosity is $\gtrsim 10^{36}$ erg s^{-1} ($\dot{M} \gtrsim 10^{16}$ gm s^{-1}).

The equation of state of the disc material is approximately

$$p = 2\,\rho\,\frac{kT}{m_p} + P_{\rm rad}, \tag{6}$$

although in reality there may of course be extra contributions to p due to turbulence and magnetic fields. Hydrostatic equilibrium in the z-direction then gives

$$-\frac{{\rm d}p}{{\rm d}z} = \rho z \left(\frac{GM}{r^3}\right),$$

or, approximately,

$$p \simeq \rho h^2\,(GM/r^3). \tag{7}$$

Note that self-gravitation of the disc material is neglected – this is under almost all circumstances a good assumption. The above relations, together with equations for radiative transfer and energy balance, in principle 'close' the problem, though one must afterwards check that $h \ll r$, because otherwise Equation (7) would break down, and even the assumption of Keplerian orbits (Equation (1)) would be invalid because radial pressure gradients would be significant. In practice, however, uncertainty about viscosity renders elaborate model-building premature. Most features of the vertical structure are sensitive to viscosity, and therefore so is the spectrum of the emergent radiation. One general result, however, is that radiation pressure prevents the inner region of the disc from being thinner than $h_{min} \simeq \frac{L}{L_{edd}} r_1$, where L is the total disc luminosity and

$$L_{edd} = \frac{4\pi\, GMm_p c}{\sigma_T} \simeq 1.3 \times 10^{38} \left(\frac{M}{M_\odot}\right) \text{erg s}^{-1}. \tag{8}$$

2.2. THE INNER BOUNDARY

(a) *When the compact object is a black hole,* r_1 is conventionally taken to be the radius of the innermost stable circular orbit, and $\beta \simeq 1$. For a Schwarzschild black hole, $r_1 = 3\, r_{Sch}$, where $r_{Sch} = 2GM/c^2 \simeq 3\ (M/M_\odot)$ km; and $L \simeq 0.06 \dot{M}c^2$. For a 'maximal Kerr' black hole (a = m) whose angular momentum vector is aligned with that of the disc, the efficiency is ~42% (Bardeen, 1970). However, Page and Thorne (1974) have shown that, even in the context of an ideal thin disc model, the maximum realistic efficiency is ~30%. This is because the hole tends preferentially to swallow radiation with angular momentum opposite to that of the disc. This prevents a/m from exceeding a 'critical' value ~0.998, and would reduce it to this value on a timescale $\ll M/\dot{M}$ if it started off closer to unity. The relativistic corrections are rather more interesting and significant in the Kerr case than for the Schwarzschild metric, because r_1 is closer to the horizon.

One should in any case, however, be cautious about applying the standard disc model as discussed by, for instance, Novikov and Thorne (1973) to real systems. Apart from the likelihood of instabilities and the complications which arise when $L \simeq L_{edd}$ (which I shall return to later) it is by no means clear that the viscous torques at the innermost stable orbit are zero, nor that radiation emitted from $r < r_1$ is necessarily negligible (Stoeger 1976).

(b) *For accretion onto an unmagnetised neutron star or white dwarf* we have $r_1 = r_*$ (the stellar radius) and $\beta \leqslant 1$. The efficiency is $\sim GM/c^2 r_*$, and unless the star is spinning almost at break-up speed (in which case $\beta \simeq 1$) we expect comparable luminosities from the disc and from an equatorial boundary layer on the star itself.

(c) *When the central object is a strongly magnetised neutron star or white dwarf* the disc extends inward only as far as an Alfvén radius r_A. The theoretical value of r_A is uncertain (cf. Lamb and Pethick, 1974) but the best estimate is probably given by

$$\frac{(H(r_A))^2}{4\pi} \simeq t_{r\phi}, \tag{9}$$

i.e. r_A is the largest radius where the viscous stresses in the disc can be balanced by the stellar magnetic field. If $r_A \gg r_*$, radiation from the disc is unimportant relative to the contribution from material which eventually reaches the stellar surface.

2.3. VISCOSITY IN DISCS

Most authors have made some kind of simplifying assumptions about the viscosity. Shakura and Sunyaev (1973), in their very detailed discussion, assume $t_{\phi r} = \alpha p$ where α is a parameter less than unity. Pringle and Rees (1972) made a similar kind of ad hoc assumption. Lynden-Bell and Pringle (1974), who were concerned with the more general case of unsteady discs where $\dot{M}(r)$ is not constant, proposed that the viscosity was due to turbulence, whose amplitude adjusted itself so that the Reynolds number calculated with the inclusion of turbulent viscosity maintained itself at a value ~100. Stewart (1975) has recently attempted to consider turbulent viscosity from first principles. Eardley and Lightman (1975) have considered *magnetic* viscosity resulting from amplification and annihilation of chaotic magnetic flux. They find a viscosity which is equivalent to a value of Shakura and Sunyaev's α-parameter in the range 0.01–1.

The real viscosity is in any case likely to depend on the z-coordinate. It is therefore probably a very crude over-simplification to average over the vertical structure. The inward drift velocity may be z-dependent, and this would introduce a new set of terms into the disc structure equations.

2.4. RADIATIVE TRANSFER

Electron scattering provides the dominant opacity in the inner region of discs, from which most of the luminosity comes. Because of the high electron temperature, the process of 'Comptonization' is important. I shall not discuss this here, but the reader is referred to Lightman *et al.* (1976) for a detailed treatment. Another important effect, which has not been much considered in the published literature, is the heating of the outer parts of the disc by radiation from further in. This can thicken the outer parts of the disc and modify the integrated spectrum.

If there are large random motions in the disc, as seems inevitable if the viscosity is so large that $\alpha \simeq 1$, the energy may be deposited predominantly in a hot 'corona' away from the disc's plane of symmetry (Price and Liang, 1976, Icke, 1976). This would affect the spectrum, and would perhaps make it easier to explain the hard X-rays from Cygnus X-1, which pose difficulties for the standard disc model. If shock heating were intense, it could drive a wind-like outflow perpendicular to the plane of the disc. The value of $\dot{M}$ would then depend on r, which would fundamentally invalidate the standard model: for instance $F(r)$ would no longer vary as r^{-3} (Equation (4)), and we should have lost our one viscosity-independent deduction. Because of all these and other complications, it is perhaps not surprising that there has been little success in fitting the spectrum of Cyg X-1.

2.5. INSTABILITIES AND RAPID VARIABILITY

Thermal instabilities (Pringle *et al.* 1973) may occur in accretion discs; another possibility is the viscous instability discussed by Lightman and Eardley (1974) which occurs if $ht_{\phi r}$ increases as the surface density in the disc decreases; and there are no doubt many other possibilities. Moreover the material in the disc may be inhomogeneous – in particular, blobs of dimensions ~h would seem inevitable in any disc where $\alpha \simeq 1$ or where

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the magnetic pressure is comparable with the total pressure. There are thus many reasons for expecting irregular short-term X-ray variability from accretion discs around black holes.

The minimum characteristic timescale t_{min} for large-amplitude variations is of the order of the orbital period at r_1. The minimum epicyclic period in the disc will be comparable with this. For a Schwarzschild black hole $t_{min} \simeq 5 \times 10^{-4} (M/M_\odot)$ s; and t_{min} can be up to 8 times shorter for a Kerr hole than for a Schwarzschild hole of the same mass. There could be variations on any timescale longer than this. However any very slow variations are likely to be caused by alterations in $\dot{M}$ or in conditions at the outer boundary of the disc.

2.6. RELATIVISTIC EFFECTS

Relativistic effects are unimportant for discs around Schwarzschild black holes where $r_1 = 6\,GM/c^2$ (unless we receive detectable radiation from material in non-circular 'plunge orbits' with $r < r_1$). For a Kerr metric, however, there are some effects which have potentially observable consequences.

Cunningham (1975) has considered the extent to which radiation emitted near r_1 is beamed into the equatorial plane by a combination of doppler and gravitational focusing effects. This effect is counteracted by the classical limb darkening effect which causes the surface brightness of a disc whose dominant opacity results from electron scattering to be greater *perpendicular to* the plane.

If the spin of the black hole is obliquely oriented relative to the plane of the disc, then the disc structure is more complicated (Bardeen and Petterson, 1975). The Lense-Thirring precession period for particle orbits out of the *hole's* equatorial plane is $P_{precession} \simeq (r/r_{Sch})^3\ (m/a)\ r_{Sch}/c$. The infall timescale for an element of gas is $P_{infall} \simeq P_{orbit}\ (v_\vartheta/v_r)$, where $P_{orbit} \simeq (r/r_{Sch})^{3/2}\ r_{Sch}/c$. The disc will be forced to align with the hole within a radius r_{align} where $P_{precession} \simeq P_{orbit}$. Even though this is an essentially relativistic effect, r_{align} can be $\gtrsim 100\,r_{Sch}$ if (as we generally expect) $v_r \ll v_\vartheta$. This kind of effect could perhaps be discerned with X-ray polarimetry, since the polarization vector is determined by the disc orientation (Rees, 1975; Lightman and Shapiro, 1975).

In cases of wind accretion where the existence of a disc is marginal, Shapiro and Lightman (1975) have argued that sometimes the specific angular momentum of the captured material may briefly reverse, leading to a retrograde disc. The likelihood of this depends on the type of fluctuations that occur in radiation-driven stellar winds. For a Kerr metric, r_1 is larger and the efficiency correspondingly lower for retrograde orbits, so one would expect a consequent change in the X-rays if $a/m \simeq 1$. One suspects, however, that when the specific angular momentum of accreted material is low, the angular momentum vector will vary wildly in direction rather than switching clearly between two states. This interesting effect will therefore be mixed up with that considered by Bardeen and Petterson (1975). The specific angular momentum may sometimes get so small that the accretion resembles the dissipative spherical inflow considered by Mészáros (1975).

It would obviously be very important if, by X-ray observations, one could diagnose the metric around a black hole candidate and thereby test theories of gravitation in the strong field regime beyond the post-Newtonian approximation – indeed it is because of these hopes that the topic has attracted such interest from beyond the traditional con-

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fines of the binary star community. There seems, however, little immediate prospect of this.

2.7. X-RAY OBSERVATIONS OF DISCS

The expected X-ray spectrum is unfortunately very sensitive to uncertainties about viscosity, etc. Meszáros (1974) calculates that broadening due to doppler effects and electron scattering is likely to preclude detection of X-ray line features from discs. Of potentially greater importance for elucidating disc structure are polarimetric observations – up to 10% linear polarization is predicted on the basis of standard disc models (Rees, 1975; Lightman and Shapiro, 1975).

3. Modes of Mass Transfer: what is the Outer Radius of the Disc?

It is conventional to distinguish two modes of mass transfer: Roche lobe overflow, and accretion from a stellar wind. This distinction is convenient for the present exposition, but one should bear in mind the possibility of intermediate cases. For instance, a star nearly filling its Roche lobe may have an enhanced stellar wind because of the reduced gravity. Also, the surface of the companion star, on the wind itself, may be affected by X-ray emission from the compact object.

3.1. ROCHE LOBE OVERFLOW

This is likely to be occurring in the HZ Her – Her X-1 system, though perhaps not in the other well-known X-ray binaries which involve more massive stars. It has recently been proposed (Fabian *et al.*, 1975) that the X-ray emission from some globular clusters may involve Roche lobe overflow in low-mass binary systems formed by tidal capture.

In this situation, the outer radius r_2 of the disc is comparable with the size of the Roche lobe around the compact object, and therefore very much larger than any reasonable value of r_* or r_A (i.e. $r_2 \gg r_1$). Other contributors will be describing calculations of the structure and size of such discs. The motions at $r \simeq r_2$ will be significantly non-circular. Also, probably not all of the gas transferred in the stream finds its way onto the compact star: a fraction may be expelled from the system, or returned to the companion, carrying the angular momentum transported out through the disc. Note, however, that this conclusion is not inevitable if the angular momentum in the disc is transported away via tidal effects or shocks instead of by conventional viscous processes.

3.2. STELLAR WIND

If the stellar wind velocity is v_{wind} and the orbital velocity v_{orb}, then a compact object exposed to the wind accretes from a cylindrical wake of cross-section $\pi r_{\mathrm{wake}}{}^2$ swept back by an angle $\sim \tan^{-1}(v_{\mathrm{orb}}/v_{\mathrm{wind}})$, where

$$r_{\mathrm{wake}} \simeq \frac{2\,GM}{(v_{\mathrm{orb}}{}^2 + v_{\mathrm{wind}}{}^2)}. \tag{10}$$

In most situations $v_{\mathrm{wind}} > v_{\mathrm{orb}}$.

The typical specific angular momentum of an accreted particle is $\sim v_{\mathrm{wind}}\, r_{\mathrm{wake}}{}^2$; but contributions from opposite sides of the whole wake tend to cancel out, except for a fraction $\sim (r_{\mathrm{wake}}/r_{\mathrm{orb}})(v_{\mathrm{orb}}/v_{\mathrm{wind}})$. The *net* specific angular momentum of accreted material is thus $\sim(r_{\mathrm{wake}}/r_{\mathrm{orb}})^2 \sim(v_{\mathrm{orb}}/v_{\mathrm{wind}})^4$ times smaller than in the Roche-lobe overflow case. The outer radius r_2 of the disc, defined as the radius of the Keplerian orbit with the same specific angular momentum, is then very sensitive to v_{wind}:

$$r_2 \sim v_{\mathrm{wind}}^{-8}. \tag{11}$$

However, things are not quite as sensitive as Equation (11) implies, because the accretion rate $\dot{M}$, proportional to $\rho_{\mathrm{wind}}\, v_{\mathrm{wind}}^{-3}$, is constrained in any system by the observed X-ray luminosity. Nevertheless the existence of discs around black holes is marginal in some cases (Illarionov and Sunyaev, 1975; Shapiro and Lightman, 1975). For magnetized neutron stars exposed to stellar winds, it is unlikely that r_2 would be as large as the Alfvén radius. Therefore we would not expect a disc at all in these circumstances: the situation would instead be analogous to the flow of the solar wind around the Earth's magnetosphere.

Gas in the wind would join the accretion wake after passing through a shock. Illarionov and Sunyaev (1975) suggest that the shape of this shock – e.g. whether it is approximately conical or paraboloidal in shape – depends on the efficiency of cooling in the wake.

The ionisation structure of the wind will be governed by a balance between X-ray photoionization and recombination, and will therefore change in response to variations in the wind strength or X-ray source strength (Pringle, 1973; Buff and McCray, 1974; McCray and Hatchett, 1975). This type of effect may provide an explanation for the behaviour of Cen X-3 after emergence from 'extended lows' (Giacconi, 1975). Enhanced absorption by denser or cooler material in the accretion wake may be the cause of absorption dips seen at various orbital phases in X-ray binaries (Jackson, 1975).

4. Accretion onto Magnetised Neutron Stars: Inflow from the Alfvén Surface

The standard model for accretion onto magnetised neutron stars was originally outlined by Pringle and Rees (1972), Davidson and Ostriker (1973) and Lamb *et al.* (1973) with application to Her X-1 and Cen X-3. There have subsequently been a number of attempts to discuss the physical processes and radiation mechanisms at the base of the accretion column (e.g. Davidson, 1973; Basko and Sunyaev, 1976; Shapiro and Salpeter, 1975). These studies are relevant to the spectrum and pulse shape of the X-rays from Her X-1 and Cen X-3.

If the opacity of the accretion column is small, we may get *either*: (a) a basically thermal spectrum, modified by the effects of the strong magnetic field, and a pencil beam (Basko and Sunyaev 1975); *or* (b) cyclotron radiation and emission at the first few harmonics of the Larmor frequency ν_{L}, yielding a fan beam (Gnedin and Sunyaev, 1973).

If electron scattering in the accretion column *is* significant, we tend to expect a fan beam at $\nu \gtrsim \nu_{\mathrm{L}}$, but a pencil beam at $\nu \ll \nu_{\mathrm{L}}$, because then $\sigma \ll \sigma_{\mathrm{T}}$ for radiation propagating along the field.

In either of the above cases, one might expect up to ~50 per cent linear polarization (Rees, 1975). There are two factors which might complicate the interpretation of the observed pulse shapes. First, there may be absorption and/or reflection of radiation emitted downward from the accretion column by parts of the neutron star surface away from the magnetic polar caps. Second, we may be wrong to attribute the observed pulse shape to a constant beam pattern scanning the Earth: the infall time from r_A is less than the spin period, so the accretion onto the stellar surface may itself be modulated within each spin period (cf. Feigelson, 1975). This is a specially attractive possibility in the case of long-period sources (e.g. Ariel 1118–61) with complex and energy-dependent pulse shapes.

It has conventionally been assumed that the area of the regions near the magnetic infalling polar caps where the gas hits the surface is bounded by the field lines which, if undistorted by the effects of current at the Alfven radius, would have penetrated out to ν_A. But it is still unclear how good this assumption is: the complicated processes whereby material penetrates into the magnetosphere and crosses field lines are currently being investigated by Arons and Lea, Elsner and Lamb, and Michel.

4.1. SPIN-UP OR SPIN DOWN?

Processes at the magnetosphere are also responsible for determining the rate at which the accretion process alters a spinning neutron star's angular momentum. Infalling plasma carries a specific angular momentum $\sim(r_A/r_*)^2$ times that of the star itself ('lever-arm' effect). However, torques at R_A may tend to speed up or slow down the star according as $\Omega_* \lessgtr$ the mean angular velocity of material just outside r_A. If there is a disc outside r_A, we expect Ω_* to stabilise at a value $\sim(GM/r_A{}^3)^{\frac{1}{2}}$, the Keplerian angular velocity at $r \simeq r_A$. This may be the situation in Her X-1. Accretion perhaps cannot occur at all if $\Omega_* \gg (GM/r_A{}^3)^{\frac{1}{2}}$.

A new problem is posed by Vela X-1 (3U 0900–40) and the transient sources, which have periods of a few minutes, if these involve spinning neutron stars. If a neutron star forms with a rapid period, it can be quickly slowed down by the 'propeller effect' (Illarionov and Sunyaev, 1975; Fabian, 1975). However the observed periods are longer than the Keplerian period at any plausible value of r_A. This means that *either* the accretion has just switched on and they are currently being spun up, or else the equilibrium value of Ω_* is much lower than $(GM/r_A{}^3)^{\frac{1}{2}}$ because the accretion is from a wind of very low specific angular momentum and there is no disc outside r_A.

5. Accretion onto White Dwarfs

The theory of accretion onto white dwarfs is of interest for two distinct reasons. Firstly, it is possible that some of the transient sources involve spinning white dwarfs rather than neutron stars; and, secondly, dwarf nova systems such as DQ Her apparently contain accreting white dwarfs, and it is therefore relevant to estimate the expected properties of the emission.

If the accreted material is of 'ordinary' composition, then – in contrast to the situation with neutron stars – the *nuclear* energy overwhelms the gravitational, i.e. $\langle L_{\text{nuclear}}\rangle > \langle L_{\text{acc}}\rangle$. However, the nuclear energy may be released in explosive outbursts, so that for most of the time the gravitational output dominates. In any case (as we see below) the accretion process is likely to be the only reasonably efficient way of producing hard X-rays from white dwarfs.

The temperature of a white dwarf surface could never (unless $L \gg L_{\text{edd}}$) be high enough that the luminosity emerged in the X-ray band. However if accretion occurs there will be a shock above the surface. Below the shock, the electron temperature can be $\gtrsim 50$ keV, and hard X-rays can be emitted via optically thin bremsstrahlung (Hoshi, 1973; Aizu, 1973).

If $L_{\rm acc} \gtrsim 10^{36}$ erg s^{-1}, the cool infalling matter *above* the shock will be opaque enough to absorb and degrade most of the X-rays, except that if nuclear burning is going on concurrently there might be a large enough flux of far-UV thermal photons from the stellar surface to maintain the level of ionisation above the shock. Compton cooling by these photons would then however prevent the shocked gas from getting hot enough to emit hard X-rays efficiently. If the magnetic field is strong enough to channel the inflow, i.e. to make $r_{\rm A} > r_*$ (as must be the case in DQ Her and in the periodic transient sources if these involve white dwarfs), then the X-ray efficiency is reduced because cyclotron cooling behind the shock can be competitive with bremsstrahlung. These and related matters are most fully discussed by Fabian *et al.* (1976).

6. The 'Eddington Limit'

Various ways of violating the condition $L \lesssim L_{\rm edd}$, where $L_{\rm edd}$ is given by Equation (8), have been discussed elsewhere (Rees, 1974a, b) and I shall just list them here. They include the following possibilities.

(1) The effective cross-section per electron may be $\ll \sigma_T$ e.g. if the radiation is in the gamma-ray band, or at frequencies $\ll \nu_{\rm L}$, where $h\nu_{\rm L} \simeq 1.2\,(B/10^{11}\ {\rm G})$ keV.

(ii) In a non-spherical situation (e.g. the accretion column on a magnetised star) the radiation may escape sideways so that it is ineffective in opposing the inflow.

(iii) In spherical symmetry, the radiation pressure may be unable to stem the accretion even if it can decelerate the inflow.

(iv) $L_{\rm edd}$ is irrelevant in explosive or rapidly varying conditions.

Even though $L_{\rm edd}$ may well be exceeded in accreting magnetised neutron stars for one or other of these reasons, it is harder to see how this could happen in a disc geometry. As discussed by Shakura and Sunyaev (1973) a disc stops being thin (i.e. h becomes of order r) when $L \simeq L_{\rm edd}$. However when this happens, radiation pressure becomes dynamically important in the radial direction as well as in the vertical direction (Mészáros and Rees, 1975; Maraschi *et al.*, 1976). The radial pressure gradient alters the orbital velocities from their ordinary Keplerian values, and tends to move the innermost stable orbit outward. This reduces the power radiated per unit mass swallowed by the hole.

It is possible to conjecture that, even though the luminosity may never exceed $L_{\rm edd}$, the efficiency always decreases as $\dot{M}$ increases in such a way that the black hole can swallow at an arbitrary rate. This is more readily shown for spherical accretion, when one finds that, as $\dot{M}$ rises above the critical accretion rate, the opacity increases in such a way that the radiation is progressively less able to maintain a net outward diffusion rate through the matter which is itself falling inward (cf. Tamazawa *et al.*, 1975). Thus when $\dot{M}$ is supercritical most of the radiation is itself swallowed by the hole, and so does not contribute to L. If this conjecture were true in the disc case, it would imply that black holes in a sufficiently dense environment could grow on a timescale much less than the $\sim 10^8$ yr commonly assumed (see, e.g. Salpeter, 1964). The question of what fraction of the material is swallowed and what fraction expelled as a wind also seems quite open.

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DISCUSSION

Hutchings: Do you predict non-isotropic X-radiation for the accretion mechanism involving a stellar wind? 1700–37 has an extreme wind and shows lower X-ray flux at phase 0.5.

Rees: If absorption occurs in the wind, this would cause the escaping radiation to be anisotropic. Moreover, in sources which involve accretion from a wind on to a magnetised neutron star there may not be a disc at all. The accretion on to the polar caps could then be modulated during each spin period, as discussed by Feigelson (1975), and it would then be misleading to interpret the pulse shape as due to a rotating beam with a constant pattern.

Ostriker: Have you worried about the assumptions of instantaneous conversion of bulk turbulent motions to thermal atomic motion? It seems to me that – as in the equivalent solar problem – much of the energy will be transformed to waves whose energy will be deposited in a hot corona above the disc. This will radiate hard bremsstrahlung and may produce a wind violating the $\dot{m}$ = constant assumption. Much important physics is not in the standard models.

Rees: Yes, I have worried about these effects. In fact Icke will discuss some of them in his paper.

Shaviv: Your note (i) on the Eddington Limit says that you should substitute another σ value for σ_T. But before you get to L_{edd} matter becomes convectively unstable. As $L \rightarrow L_{edd}$ convection becomes more and more inefficient and v_{conv} increases. The assumption of hydrostatic equilibrium is not valid and you go over slowly into a steady flow. Under such conditions clearly the Eddington limit formula does not apply.

Wilson: If the angular momentum can only partly be accepted by the black hole, and therefore angular momentum accumulates in the disc, wouldn't one wind up with an outer disc radius which is larger than that corresponding to the angular momentum of the infalling matter?

Rees: Yes. This is in fact found in the computer simulations carried out by Lin and Pringle

Meszaros: I would like to remark that spherical accretion on to black holes need not be inefficient in converting gravitational energy into radiation. Spherical flows should be subject to the same viscous dissipation mechanisms at the discs (turbulence, magnetic viscosity), and this converts kinetic into thermal energy at all radii. If the turbulent or magnetic energy reaches equipartition, as flux freezing would imply, turbulent dissipation and field line reconnection lead to an efficiency comparable to that of discs. This is important for binary systems with a black hole fed by a stellar wind, since then the specific angular momentum is small and a spherical configuration is more likely. This appears to be the case in Cygnus X-1, and some calculations I did recently indicate that a spherical model is able to reproduce the spectrum adequately.

Rees: Yes, I think these spherical calculations you mention provide a more natural explanation of the hard component of the spectrum of Cyg X-1 than do the disc models.