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FINITE s-GEODESIC TRANSITIVE GRAPHS

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A *geodesic* from a vertex u to a vertex v in a graph is one of the shortest paths from u to v, and this geodesic is called an s-geodesic if the distance between u and v is s. For a graph Γ and for an integer s less than or equal to the diameter of Γ, assume that for each $i \le s$, all i-geodesics of Γ are equivalent under the group of graph automorphisms. The purpose of this thesis is to study such graphs, called *s-geodesic transitive graphs*.

In the first part of the thesis, we show that the subgraph $[\Gamma(u)]$ induced on the set of vertices of Γ adjacent to a vertex u is either: (i) a connected graph of diameter 2; or (ii) a union $m\mathrm{K}_r$ of $m \ge 2$ copies of a complete graph K_r with $r \ge 1$. This suggests a way forward for studying s-geodesic transitive graphs according to the structure of such graphs $[\Gamma(u)]$. We study further the family $\mathcal{F}(m, r)$ of connected graphs Γ such that $[\Gamma(u)] \cong m\mathrm{K}_r$ for each vertex u, and for fixed $m \ge 2, r \ge 1$. We show that each $\Gamma \in \mathcal{F}(m, r)$ is the point graph of a partial linear space $\mathcal{S}$ of order $(m, r + 1)$ which contains no triangles. Conversely, each $\mathcal{S}$ with these properties has point graph in $\mathcal{F}(m, r)$, and a natural duality on partial linear spaces induces a bijection $\mathcal{F}(m, r) \mapsto \mathcal{F}(r + 1, m - 1)$.

In the second part of the thesis, we compare 2-geodesic transitivity of graphs with another two transitivity properties, namely, 2-distance transitivity and 2-arc transitivity. It is easy to verify that if a graph is 2-arc transitive, then it is 2-geodesic transitive, which in turn implies that it is 2-distance transitive. We classify 2-geodesic transitive but not 2-arc transitive graphs of valency 4 and prime valency, and we also prove that, except for a few cases, the Paley graphs and the Peisert graphs are 2-distance transitive but not 2-geodesic transitive.

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In the third part of the thesis, we prove reduction theorems for the family of s-geodesic transitive graphs with $[\Gamma(u)]$ connected, and the family whose $[\Gamma(u)]$ is disconnected. In each case, we identify a subfamily of 'basic' s-geodesic transitive graphs such that each s-geodesic transitive graph has at least one basic s-geodesic transitive graph as a normal quotient. We study such basic graphs where a group G of graph automorphisms is quasiprimitive on the vertex set. Many of these basic graphs are Cayley graphs. This leads us to study $(G, 2)$-geodesic transitive Cayley graphs $\mathrm{Cay}(T, S)$ with G contained in the holomorph of T. This study can be further reduced to the following three problems: investigating the case where T is a minimal normal subgroup of G, studying the 2-geodesic transitive covers of these graphs, and investigating the 2-geodesic transitive covers of complete graphs.

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