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Constructing Representations of Finite Simple Groups and Covers

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Abstract. Let G be a finite group and χ be an irreducible character of G. An efficient and simple method to construct representations of finite groups is applicable whenever G has a subgroup H such that χ_H has a linear constituent with multiplicity 1. In this paper we show (with a few exceptions) that if G is a simple group or a covering group of a simple group and χ is an irreducible character of G of degree less than 32, then there exists a subgroup H (often a Sylow subgroup) of G such that χ_H has a linear constituent with multiplicity 1.

1 Introduction

Let G be a finite group and χ be an irreducible character of G. An efficient and simple method to construct representations of finite groups has been presented in [5]. This is applicable whenever G has a subgroup H such that χ_H has a linear constituent with multiplicity 1. We call such a subgroup H, a χ-subgroup. The problem in using this method to construct representations of G is finding a χ-subgroup for each irreducible character χ of G. We may need to examine the full lattice of subgroups of G to find a χ-subgroup. Indeed there is no guarantee that for a given character χ any χ-subgroup exists. Examples of solvable groups where no such subgroups exist are given by G. Glauberman [9]. Also one can find non-solvable examples. For instance, the covering group $6.A_7$ of the group A_7 has three characters of degree 36 and for two of them there is no such subgroup.

Suppose G is a simple group or a covering group of a simple group which is listed in the Atlas [1] (see also [2]). Using a combination of theory and computation we find, with a few exceptions, a χ-subgroup for each nontrivial irreducible character χ of G of degree < 32. In the exceptional cases we show that the restriction of χ to some maximal subgroup of G is irreducible. The bound 32 on the degrees of irreducible characters has been chosen with an eye to applications. The main theorems described in [3, Chapter 5] only hold for characters of degrees less than 32. Also as the degree of χ becomes larger there seem to be increasingly many examples of groups which contain no χ-subgroup.

The results of this paper form an important part of the theoretical basis for a general program which the author has developed to compute representations of finite groups (see [4]) and for the computational reason we have tried to find easily described χ-subgroups.

We now turn to examine specific classes of simple groups and their covers.

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2 Alternating Groups

We recall some facts about characters of the symmetric group S_n (see [8]). Since the number of irreducible characters of a group is equal to the number of conjugacy classes, which in the case of S_n is the number of partitions of n, the irreducible characters of S_n are labelled by partitions of n. If $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_l)$ is a partition of n then $[\lambda] = [\lambda_1, \lambda_2, \dots, \lambda_l]$ denotes the irreducible character labelled by λ. In the present paper we say the partition λ has *level* k if $k = \lambda_2 + \cdots + \lambda_l (= n - \lambda_1)$. Similarly we say that the corresponding irreducible character $[\lambda]$ of S_n has level k. This is a nonstandard terminology.

Theorem 2.1 *Let $k \geqslant 0$ be fixed. Suppose $[\lambda] = [n-k, \lambda_2, \dots, \lambda_l]$ is an irreducible character of S_n of level k. Then $[\lambda](1)$ is a polynomial in n of degree k.*

Proof Let H_{ij} be the hook of the diagram of $[\lambda]$ corresponding to the node (i, j). Then $|H_{ij}| = h_{ij} \leqslant k$ for $i \geqslant 2$. Also there exist $n - k$ hooks, H_{1j}, such that $|H_{1j}| = h_{1j}$ has a value of the form $(n - m_j)$ with $m_1 < m_2 < \cdots < m_{n-k}$ for $1 \leqslant j \leqslant n - k$. Simplifying the hook formula [8, Theorem 2.3.21],

$$[\lambda](1) = \frac{n!}{\prod_{(i,j)\in\lambda} h_{i,j}} = \frac{n!}{(\prod_{(1,j)\in\lambda} h_{1,j})(\prod_{(i\geqslant 2,j)\in\lambda} h_{i,j})}$$

we find that only k factors remain in the numerator, and so $[\lambda](1)$ is a polynomial in n of degree k. ∎

This theorem shows that the degrees of irreducible characters of S_n increase when n increases. Therefore using [1], if $[\lambda]$ is an irreducible characters of S_n of degree < 32 such that $[\lambda]_{A_n}$ is irreducible, then $[\lambda]$ has level $\leqslant 2$ for $n \geqslant 9$ and has level $\leqslant 1$ for $n \geqslant 10$.

If λ is a partition of n, then λ', the *conjugate partition* of λ, is the partition of n whose Young diagram is obtained by reflecting the Young diagram of λ in the main diagonal. If $\lambda \neq \lambda'$ then $[\lambda]_{A_n}$ is irreducible [8, Theorem 2.5.7]. In particular $\lambda \neq \lambda'$ when $\lambda_1 \neq l$.

If we consider the characters of levels 1, 2 and 3, then the following characters are irreducible: $[n-1,1]_{A_n}$ for $n \geqslant 4$, $[n-2,2]_{A_n}$ for $n \geqslant 5$, $[n-2,1^2]_{A_n}$ and $[n-3,3]_{A_n}$ for $n \geqslant 6$, $[n-3,1^3]_{A_n}$ for $n \geqslant 8$ and $[n-3,2,1]_{A_n}$ for $n \geqslant 7$.

The following theorem describes a χ-subgroup for each of these irreducible characters.

Theorem 2.2

(1) *If $n \geqslant 4$ and $\chi = [n-1,1]_{A_n}$, then $\mathrm{Syl}_{A_4}(3)$ is a χ-subgroup.*
(2) *If $n \geqslant 6$ and $\chi = [n-2,2]_{A_n}$ or $[n-2,1^2]_{A_n}$, then $\mathrm{Syl}_{A_6}(3)$ is a χ-subgroup.*
(3) *If $n \geqslant 8$ and $\chi = [n-3,3]_{A_n}$, $[n-3,2,1]_{A_n}$ or $[n-3,1^3]_{A_n}$, then $\mathrm{Syl}_{A_8}(2)$ is a χ-subgroup.*

Proof Suppose $k \in \{1,2,3\}$. If we denote $\chi = [n-k, \lambda_2, \dots, \lambda_l]_{A_n}$ for $k = \lambda_2 + \cdots + \lambda_l$, then for $n - r > k + l$ all constituents of $\chi_{A_{n-r}}$ are irreducible. Now using [3, Theorem 4.1.12], for $n - r > k + l$ we can write $\chi_{A_{n-r}} = \rho + \sum m_i \rho_i$ such that $\rho = [n-k-r, \lambda_2, \dots, \lambda_l]_{A_{n-r}}$ and ρ_i are the other constituents. Since ρ is with multiplicity one, if H is a subgroup of A_{n-r} and $\phi \in \mathrm{Irr}(H)$ a linear character such that $\langle \rho_H, \varphi \rangle = 1$ and $\langle (\rho_i)_H, \varphi \rangle = 0$ for all i, then $\langle \chi_H, \varphi \rangle = 1$. Simple computations show that for $k = 1, 2, 3$ and n greater than or equal to $5, 8, 11$, respectively, the Sylow subgroups $\mathrm{Syl}_{A_4}(3)$, $\mathrm{Syl}_{A_6}(3)$ and $\mathrm{Syl}_{A_8}(2)$ have this property and are χ-subgroups, respectively. ■

With the exception of the characters covered in the theorem above, there are only a few cases where an alternating group has a nontrivial irreducible character of degree < 32. In these cases a χ-subgroup was computed directly using GAP [7]. These exceptions are listed in Table 6 at the end of this paper. Table 6 also contains χ-subgroups for the covering groups of alternating groups and other simple groups and covers listed in [1] for which there is no general theorem about their χ-subgroups when $\chi(1) < 32$. These were also found by a direct computation. In most cases we have found a p-subgroup which is a χ-subgroup. Exceptions occur for $6.A_6$, $2.A_7$, $3.A_7$, $6.A_7$ and $2.A_8$ which for some χ do not have χ-subgroups which are p-groups. However in the exceptional cases, computation in GAP enabled us to find the following solvable χ-subgroups of G containing the centre of G.

If $G = 6.A_6$ and $\chi(1) = 12$, then G has a χ-subgroup of order 60. If $G = 2.A_7$ and $\chi(1) = 20$, then G has a χ-subgroup of order 40. If $G = 3.A_7$ and $\chi(1) = 21$ or 24, then G has an abelian χ-subgroup of order 36 and a χ-subgroup of order 60, respectively. If $G = 6.A_7$ and $\chi(1) = 20, 21$ or 24, then G has χ-subgroups of order 120, 72 and 120, respectively. And finally, if $G = 2.A_8$ and $\chi(1) = 24$, then G has a χ-subgroup of order 30.

3 $\mathrm{PSL}(2,q)$ and Its Cover

The group $\mathrm{SL}(2, q)$ is the unique covering group of the simple group $\mathrm{PSL}(2, q)$, except for $q = 9$. In the latter case $\mathrm{PSL}(2, 9) \cong A_6$ and this has been dealt with in the previous section. Also $\mathrm{PSL}(2,q)$ is the factor group of $\mathrm{SL}(2, q)$ by its centre so its characters correspond to the characters of $\mathrm{SL}(2, q)$ whose kernels contain the centre. Thus it is enough to find χ-subgroups for the irreducible characters χ of $\mathrm{SL}(2,q)$.

Let $G = \mathrm{SL}(2,q)$ where $q = p^n$ for some prime p and let

$$H = \left\{ \begin{pmatrix} 1 & 0 \\ \beta & 1 \end{pmatrix} \;\middle|\; \beta \in \mathbb{F}_q \right\}.$$

Then H is an abelian Sylow p-subgroup of G of order q. The following tables are the tables of values of characters of G on elements 1 and $1 \neq h \in H$, when q is odd and when q is even (see [6, pp. 228, 235]).

Now we show H is a χ-subgroup for all irreducible characters of G. We shall need the following lemma.

Table 1: Values of characters of $\mathrm{SL}(2, q)$ on elements of H when q is even: $1 \leqslant i \leqslant q/2$ and $1 \leqslant j \leqslant (q-2)/2$.

	$\mathbf{1}$	ρ	ψ_i	θ_j
1	1	q	$q-1$	$q+1$
h	1	0	-1	1

Table 2: Values of characters of $\mathrm{SL}(2,\, q)$ on elements of H when q is odd: $\epsilon = (-1)^{(q-1)/2}$, $1 \leqslant i \leqslant (q-1)/2$ and $1 \leqslant j \leqslant (q-3)/2$. Note that $\eta_1(h)+\eta_2(h) = -1$ and $\xi_1(h)+\xi_2(h) = 1$ for all $1 \neq h \in H$.

	$\mathbf{1}$	η_1	η_2	ξ_1	ξ_2	ρ	ψ_i	θ_j
1	1	$\frac{(q-1)}{2}$	$\frac{(q-1)}{2}$	$\frac{(q+1)}{2}$	$\frac{(q+1)}{2}$	q	$q-1$	$q+1$
h	1	$\frac{(-1\mp\sqrt{\epsilon q})}{2}$	$\frac{(-1\mp\sqrt{\epsilon q})}{2}$	$\frac{(1\mp\sqrt{\epsilon q})}{2}$	$\frac{(1\mp\sqrt{\epsilon q})}{2}$	0	-1	1

Lemma 3.1 *Let χ be an irreducible character of group G and suppose $p \nmid (|G|/\chi(1))$ for some prime p. Then $\chi(g) = 0$ whenever $p \mid o(g)$. In particular if G has a Sylow subgroup H and an irreducible character χ such that $|H| = \chi(1)$, then χ_H is the regular character of H and so $\langle \chi_H, \varphi \rangle = 1$ for each linear character φ of H.*

Proof See [12, Theorem 8.17] ■

Theorem 3.2 *Let $G = \mathrm{SL}(2, q)$ for $q = p^n \geqslant 4$ and H be a Sylow p-subgroup of G. Then for all irreducible characters χ of G, H is a χ-subgroup.*

Proof By Lemma 3.1 the character ρ_H of degree q is the regular character of H. Since H is abelian, all irreducible characters $\varphi_1 := \mathbf{1}, \varphi_2, \dots, \varphi_q$ of H are linear. On the other hand, $\psi_j(h) = -1$ and $\theta_i(h) = 1$ for all $1 \neq h \in H$ so

$$(\psi_j)_H = \rho_H - \mathbf{1}$$

and

$$(\theta_i)_H = \rho_H + \mathbf{1}.$$

Also when q is odd we have $\eta_1(h) + \eta_2(h) = -1$ and $\xi_1(h) + \xi_2(h) = 1$ for all $1 \neq h \in H$ so

$$(\eta_1)_H + (\eta_2)_H = \rho_H - \mathbf{1}$$

and

$$(\xi_1)_H + (\xi_2)_H = \rho_H + \mathbf{1}.$$

Now since $\rho_H = \sum_{i=1}^{q} \varphi_i$ and $q \geqslant 4$, therefore the restriction of each irreducible character of G to H has at least one linear constituent with multiplicity 1. ■

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4 PSL$(3, q)$, PSU$(3, q)$ and Covers

By [13, Theorem 7.1.1] the group SL$(3, q)$ where $q = p^n > 2$ and p is a prime, is the unique covering group of the simple group PSL$(3, q)$ except when $q = 4$ (the group PSL$(3, 4)$ has 7 different covering groups, see Table 6). Also PSU$(3, q)$ is a simple group of twisted Lie type ${}^2A_2(q)$ and the group SU$(3, q)$ is the unique covering group of the simple group PSU$(3, q)$ (see [10, Corollary 5.1.3]).

As we mentioned for the groups PSL$(2, q)$ in Section 3, the irreducible characters of PSL$(3, q)$ and PSU$(3, q)$ are obtained from characters of SL$(3, q)$ and SU$(3, q)$, respectively. Thus it is enough to find a χ-subgroup for each irreducible character χ of SL$(3, q)$ and SU$(3, q)$.

Suppose H is a Sylow p-subgroup of $G = \mathrm{SL}(3, q)$ where q is a power of a prime p. Using the character table of G in [14], Guzel [11] constructs the primitive idempotents of the complex group algebra of G. Let χ be an irreducible character of G and ψ a linear character of H. If e_χ and e_ψ are the orthogonal central idempotents afforded by χ and ψ, respectively, then $e_\chi e_\psi$ is a primitive idempotent of $\mathbb{C}G$ corresponding to χ. Using this fact he determines the pairs χ, ψ such that $\langle \chi, \psi^G \rangle = 1$. This implies that the Sylow p-subgroup H is a χ-subgroup for all $\chi \in \mathrm{Irr}(G)$. For a different proof of this result see [3, Theorems 4.3.3, 4.3.9]. In what follows we denote

$$\mathrm{LT}(a, b, c) = \begin{pmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{pmatrix}.$$

Now suppose $G = \mathrm{SU}(3, q)$. Define $H := \{\mathrm{LT}(a, b, c) \mid a, b, c \in \mathbb{F}_q\}$. Then the order of H is q^3 and H is a Sylow p-subgroup for G. We use the character values of G restricted to H to show that H or an abelian subgroup of order q^2 of H is a χ-subgroup for $\chi \in \mathrm{Irr}(G)$.

The character table of G is known by the work of J. S. Frame and W. A. Simpson [14]. We shall use that table to get the values of characters on the different conjugacy classes of G which contain the elements of H.

Table 3 is a part of Table 1a of [14] that shows the structure of conjugacy classes of G which contain some elements of the Sylow p-subgroup H. In this section $d = \gcd(3, q + 1)$, $\epsilon \in \mathrm{GF}(q^2)$ and $\epsilon^3 \neq 1$. In Table 3, ω is a complex primitive cube root of unity. Each element of H is contained in one of the conjugacy classes $\mathcal{C}_1^{(0)}$, $\mathcal{C}_2^{(0)}$ and $\mathcal{C}_3^{(0,l)}$ of G. The centre $Z(H) = \{\mathrm{LT}(0, z, 0) \mid z \in \mathbb{F}_q\}$ is an elementary abelian p-group of order q. By using the canonical representative elements of conjugacy classes $\mathcal{C}_1^{(0)}$, $\mathcal{C}_2^{(0)}$ and $\mathcal{C}_3^{(0,l)}$ we see that the minimal polynomials of elements of these conjugacy classes have degrees 1, 2 and 3, respectively and the minimal polynomials of nontrivial elements of $Z(H)$ have degree 2 so nontrivial elements of $Z(H)$ are contained in the conjugacy class $\mathcal{C}_2^{(0)}$.

The following lemma gives us some properties of H.

Lemma 4.1 *Suppose $G = \mathrm{SU}(3,\ q)$ where q is a power of a prime p. If H is a Sylow p-subgroup of G then we have:*

(1) *H has $q^2 + q - 1$ conjugacy classes.*

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Table 3: Conjugacy classes of $\mathrm{SU}(3, q)$ which contain elements of the Sylow p-subgroup H for $d = 1, 3$.

Conjugacy class	Canonical representative	Parameters
$\mathcal{C}_1^{(k)}$	$\begin{pmatrix} \omega^k & 0 & 0 \\ 0 & \omega^k & 0 \\ 0 & 0 & \omega^k \end{pmatrix}$	$0 \leqslant k \leqslant (d-1)$
$\mathcal{C}_2^{(k)}$	$\begin{pmatrix} \omega^k & 0 & 0 \\ 1 & \omega^k & 0 \\ 0 & 0 & \omega^k \end{pmatrix}$	$0 \leqslant k \leqslant (d-1)$
$\mathcal{C}_3^{(k,l)}$	$\begin{pmatrix} \omega^k & 0 & 0 \\ \epsilon^l & \omega^k & 0 \\ 0 & \epsilon^l & \omega^k \end{pmatrix}$	$0 \leqslant k, l \leqslant (d-1)$

(2) *H has q^2 linear characters and $q - 1$ non-linear characters of degree q such that their values on nontrivial elements of $Z(H)$ are 1 and $q\omega^i$ for some $1 \leqslant i \leqslant p$ respectively, where ω is a primitive p-th root of unity.*

(3) *If τ is an irreducible character of degree q of H, then $\tau(x) = 0$ for $x \notin Z(H)$, and $\sum_{1 \neq z \in Z(H)} \tau(z) = -q$.*

Proof First of all we show $H/Z(H)$ is abelian. Let $x, y \in H$ so it is enough to show $x^{-1}y^{-1}xy \in Z(H)$. Let $x = \mathrm{LT}(a, b, c)$ and $y = \mathrm{LT}(d, e, f)$ then $x^{-1}y^{-1}xy = \mathrm{LT}(0, af - dc, 0)$. Hence $H/Z(H)$ is abelian and $H' \subseteq Z(H)$. Conversely if $z = \mathrm{LT}(0, t, 0) \in Z(H)$ then $z = x^{-1}y^{-1}xy \in H'$ where $x = \mathrm{LT}(t, b, c)$ and $y = \mathrm{LT}(0, 1, e)$ for $b, c, e \in \mathbb{F}_q$. Therefore $H' = Z(H)$.

Now suppose $h = \mathrm{LT}(h_1, h_2, h_3) \in H \backslash Z(H)$ so at least one of h_1, h_3 is not 0. Then $x^{-1}hx = h^x = \mathrm{LT}(h_1, h_1c - ah_3 - h_2, h_3)$.

As x runs over H, $h_1c - ah_3 - h_2$ runs over $\mathbb{F}_q$. Thus the conjugacy class $\{h^x \mid x \in H\}$ has order q. Therefore each conjugacy class of H has order 1 or q and H has q single element conjugacy classes, since $|Z(H)| = q$. If n is the number of conjugacy class of order q then $|H| = (q \times 1) + (n \times q)$ and so $n = q^2 - 1$. Thus H has $q^2 + q - 1$ conjugacy classes.

Since $|H : H'| = q^2$ therefore H has q^2 linear characters and since the number of conjugacy classes of H is $q^2 + q - 1$ so H has $q - 1$ non-linear characters. Let τ be a non-linear irreducible character of H. Since $Z(H) \subseteq Z(\tau)$ and by [12, Corollary 2.30]

$$\tau^2(1) \leqslant |H : Z(\tau)| \leqslant |H : Z(H)| = q^2, \tag{4.1}$$

so $\tau(1) \leqslant q$. On the other hand, the number of conjugacy classes of H is $q^2 + q - 1$

and the order of H is q^3 so.

$$q^3 = |H| = \sum_{i=1}^{q^2} \varphi_i(1)^2 + \sum_{j=1}^{q-1} \tau_j(1)^2,$$

where φ_i and τ_j are linear and non-linear irreducible characters of H, respectively. Since $\tau_j(1) \leqslant q$, therefore $\tau_j(1) = q$ and (4.1) implies $Z(H) = Z(\tau)$. Since $H' = Z(H)$, the value of all linear characters of H on $Z(H)$ is 1. Also for an irreducible character τ of degree q, if ρ is a representation which affords τ, then $\rho(z)$ is a scalar for all $1 \neq z \in Z(H)$. Thus $\tau(z) = q\omega^j$ for some $1 \leqslant j \leqslant p$, where ω is a primitive p-th root of unity.

Since $\tau^2(1) = q^2 = |H : Z(H)|$, [12, Corollary 2.30] shows that $\tau(x) = 0$ for all $x \notin Z(H)$. Using the first orthogonality relation we get

$$\frac{1}{|H|} \sum_{x \in H} \tau(x) 1(x^{-1}) = \frac{1}{|H|} \sum_{x \in H} \tau(x) = \frac{1}{|H|} \sum_{z \in Z(H)} \tau(z) = 0.$$

Therefore $\tau(1) = q$ implies

$$\sum_{1 \neq z \in Z(H)} \tau(z) = -q \tag{4.2}$$

and this completes the proof. ∎

The following lemmas are simple consequences of Clifford's theorem and the Frattini argument.

Lemma 4.2 *Let H be a subgroup of any group G, $x \in N_G(H)$ and ϑ and ψ be characters of H. Then $\langle \vartheta^x, \psi^x \rangle = \langle \vartheta, \psi \rangle$. In particular taking $\psi = \vartheta$, ϑ^x is irreducible if and only if ϑ is irreducible.*

Lemma 4.3 *Let G be a normal subgroup of a group L and H be a Sylow subgroup of G. Let χ and ϑ be irreducible characters of G and H, respectively. Let $l \in L$. Then*

$$\langle \chi_H, \vartheta \rangle = \langle \chi_H^l, \vartheta^x \rangle \text{ for some } x \in N_L(H).$$

In particular $\langle \chi_H, \mathbf{1} \rangle = \langle \chi_H^l, \mathbf{1} \rangle$.

Table 4 and Table 5 taken from [14] show the values of the restriction of the irreducible characters of the groups $\mathrm{SU}(3, q)$ on elements of the Sylow subgroup H when $d = 1$ and $d = 3$, respectively.

By the values of characters ω_m and γ_n on the conjugacy classes $\mathcal{C}_1^{(0)}$, $\mathcal{C}_2^{(0)}$ and $\mathcal{C}_3^{(0,l)}$ in Table 1b of [14], we have

$$\{(\omega_1)_H, (\omega_2)_H, (\omega_3)_H\} = \{(\gamma_1)_H, (\gamma_2)_H, (\gamma_3)_H\}. \tag{4.3}$$

Table 4: Values of characters of $\mathrm{SU}(3, q)$ on elements of H when $d = 1$: $1 \leqslant i, j \leqslant q$, $1 \leqslant r \leqslant (q^2 - q)/6$, $1 \leqslant s \leqslant (q^2 - q - 2)/2$ and $1 \leqslant t \leqslant (q^2 - q)/3$.

	$\mathcal{C}_1^{(0)}$	$\mathcal{C}_2^{(0)}$	$\mathcal{C}_3^{(0,0)}$
1	1	1	1
ψ	$q^2 - q$	$-q$	0
ρ	q^3	0	0
ζ_i	$q^2 - q + 1$	$-q + 1$	1
η_j	$q^3 - q^2 + q$	q	0
ε_r	$q^3 - 2q^2 + 2q - 1$	$2q - 1$	-1
μ_s	$q^3 + 1$	1	1
ν_t	$q^3 + q^2 - q - 1$	$-q - 1$	-1

Table 5: Values of characters of $SU(3, q)$ on elements of H when $d = 3$: $1 \leqslant i, j \leqslant q$, $1 \leqslant r \leqslant (q^2 - q - 2)/6$, $1 \leqslant s \leqslant (q^2 - q - 2)/2$, $1 \leqslant t \leqslant (q^2 - q - 2)/3$ and $1 \leqslant k, m, n \leqslant 3$.

	$\mathcal{C}_1^{(0)}$	$\mathcal{C}_2^{(0)}$	$\mathcal{C}_3^{(0,l)}$
1	1	1	1
ψ	$q^2 - q$	$-q$	0
ρ	q^3	0	0
ζ_i	$q^2 - q + 1$	$-q + 1$	1
η_j	$q^3 - q^2 + q$	q	0
θ_k	$(q^3 - 2q^2 + 2q - 1)/3$	$(2q - 1)/3$ or $(-q - 1)/3$	$(2q - 1)/3$ or $(-q - 1)/3$
ε_r	$q^3 - 2q^2 + 2q - 1$	$2q - 1$	-1
μ_s	$q^3 + 1$	1	1
ν_t	$q^3 + q^2 - q - 1$	$-q - 1$	-1
ω_m	$(q^3 + q^2 - q - 1)/3$	$(-q - 1)/3$ or $(2q - 1)/3$	$(-q - 1)/3$ or $(2q - 1)/3$
γ_n	$(q^3 + q^2 - q - 1)/3$	$(-q - 1)/3$ or $(2q - 1)/3$	$(-q - 1)/3$ or $(2q - 1)/3$

Theorem 4.4 *Let $G = \mathrm{SU}(3, q)$ where $q > 2$ is a power of the prime p. Let H be a Sylow p-subgroup of G. Then H is a χ-subgroup for all irreducible characters χ of G such that $\chi(1) \neq q^2 - q$. If $\chi(1) = q^2 - q$, then there is an abelian subgroup of order q^2 in H which is a χ-subgroup.*

Proof Let ψ be the irreducible character of degree $q^2 - q$ of G and τ an irreducible character of degree q of H. Then using Table 4 and Table 5 for the value of ψ on the conjugacy class $\mathcal{C}_2^{(0)}$ containing the nontrivial elements of $Z(H)$, together with

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Lemma 4.1, we have

$$\begin{aligned}\langle\psi_H,\tau\rangle &= \frac{1}{|H|}\sum_{x\in H}\psi_H(x)\overline{\tau(x)}\\ &= \frac{1}{q^3}\Big(\psi_H(1)\tau(1)+\sum_{1\neq z\in Z(H)}\psi_H(z)\overline{\tau(z)}+\sum_{z\notin Z(H)}\psi_H(z)\overline{\tau(z)}\Big)\\ &= \frac{1}{q^3}\big((q^2-q)q+(-q)(-q)+0\big) = 1.\end{aligned}$$

Since H has $q-1$ irreducible characters of degree q, we have

$$\psi_H=\sum_{i=1}^{q-1}\tau_i. \tag{4.4}$$

Let ρ be the irreducible character of degree q^3 of G. Since $\rho(1) = |H|$, Lemma 3.1 shows that ρ_H is the regular character of H. But H has q^2 linear characters so for each linear character φ of H we have $\langle\rho_H,\varphi\rangle = 1$ and by (4.4) we have $\langle\psi_H,\varphi\rangle = 0$. On the other hand, Tables 4 and 5 show that:

$$\begin{aligned}(\zeta_i)_H &= \psi_H+\mathbf{1},\\ (\eta_j)_H &= \rho_H-\psi_H,\\ (\varepsilon_r)_H &= \rho_H-2\psi_H-\mathbf{1},\\ (\mu_s)_H &= \rho_H+\mathbf{1},\\ (\nu_t)_H &= \rho_H+\psi_H-\mathbf{1}.\end{aligned}$$

Therefore if φ is a non-principal linear character of H then, since $\langle\rho_H,\varphi\rangle = 1$ and $\langle\psi_H,\varphi\rangle = 0$, we get

$$\langle(\eta_j)_H,\varphi\rangle = \langle(\varepsilon_r)_H,\varphi\rangle = \langle(\mu_s)_H,\varphi\rangle = \langle(\nu_t)_H,\varphi\rangle = 1$$

and

$$\langle(\zeta_i)_H,\mathbf{1}\rangle = 1.$$

Now for the case $\psi(1) = q^2-q$ we proved as follows. Define

$$K := \{\mathrm{LT}(a,b,a) \mid \text{ for } a,b\in\mathbb{F}\}.$$

Then K is an abelian subgroup of H of order q^2 and $Z(H)\subset K$. Let $k = \mathrm{LT}(a,b,a)\in K\backslash\{1\}$. Then

$$(k-1)^2=\begin{pmatrix}0&0&0\\0&0&0\\a^2&0&0\end{pmatrix}.$$

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Thus, if $(k-1)^2 = 0$, then $a = 0$ and $k \in Z(H)$. Otherwise the minimal polynomial for k has degree 3. Since the minimal polynomials of elements in the conjugacy classes $\mathcal{C}_1^{(0)}$, $\mathcal{C}_2^{(0)}$ and $\mathcal{C}_3^{(0,0)}$ have degrees 1, 2 and 3, respectively, we have $k \in \mathcal{C}_2^{(0)}$ when $1 \neq k \in Z(H)$ and $k \in \mathcal{C}_3^{(0,0)}$ when $k \notin Z(K)$. Let ϕ be a non-principal linear character of K. Then using the values of ψ on $\mathcal{C}_2^{(0)}$ and $\mathcal{C}_3^{(0,0)}$ we have

$$\begin{aligned}\langle \psi_K, \phi \rangle &= \frac{1}{|K|} \sum_{k \in K} \psi_K(k)\overline{\phi(k)} \\ &= \frac{1}{|K|}\Big(\psi_K(1)\phi(1) + \sum_{1 \neq k \in Z(H)} \psi_K(k)\overline{\phi(k)} + \sum_{k \notin Z(H)} \psi_K(k)\overline{\phi(k)}\Big) \\ &= \frac{1}{q^2}\Big((q^2-q) + (-q) \sum_{1 \neq k \in Z(H)} \overline{\phi(k)} + 0\Big).\end{aligned}$$

Put $Z := Z(H)$ then $Z \subset K$ and ϕ_Z is a linear character of Z. Using the first orthogonality relation we have $\sum_{k \in Z} \overline{\phi(k)} = \sum_{k \in Z} \overline{\phi_Z(k)} = 0$. Therefore

$$\sum_{1 \neq k \in Z(H)} \overline{\phi(k)} = -1.$$

This shows $\langle \psi_K, \phi \rangle = 1$ as required.

For the case that $d = 3$ the only remaining characters to consider are θ_k, ω_m and γ_n for $1 \leqslant k, m, n \leqslant 3$.

Suppose φ is a non-principal linear character of H. Then

$$\langle \psi_H, \varphi \rangle = 0 \quad \text{and} \quad \langle \rho_H, \varphi \rangle = 1,$$

so

$$\langle (\eta_j)_H, \varphi \rangle = \langle (\varepsilon_r)_H, \varphi \rangle = \langle (\mu_s)_H, \varphi \rangle = \langle (\nu_t)_H, \varphi \rangle = 1 \text{ and } \langle (\zeta_i)_H, \varphi \rangle = 0.$$

Using Frobenius reciprocity we have,

$$\langle \eta_j, \varphi^G \rangle = \langle \varepsilon_r, \varphi^G \rangle = \langle \mu_s, \varphi^G \rangle = \langle \nu_t, \varphi^G \rangle = 1 \text{ and } \langle \zeta_i, \varphi^G \rangle = 0.$$

Also if we define

$$K_k = \langle (\theta_k)_H, \varphi \rangle, \quad M_m = \langle (\omega_m)_H, \varphi \rangle \quad \text{and} \quad N_n = \langle (\gamma_n)_H, \varphi \rangle,$$

then

$$\langle \theta_k, \varphi^G \rangle = K_k, \quad \langle \omega_m, \varphi^G \rangle = M_m \quad \text{and} \quad \langle \gamma_n, \varphi^G \rangle = N_n$$

for $1 \leqslant k, m, n \leqslant 3$.

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Now if we induce φ to G, we get

$$\varphi^G = \rho + q\eta_j + \big((q^2 - q - 2)/6\big)\,\varepsilon_r + \big((q^2 - q - 2)/2\big)\,\mu_s + \big((q^2 - q - 2)/3\big)\,\nu_t + \sum_{k=1}^{3} K_k\theta_k + \sum_{m=1}^{3} M_m\omega_m + \sum_{n=1}^{3} N_n\gamma_n.$$

But $\varphi^G(1) = |G{:}H|\varphi(1)$, so if we calculate the value at 1 and simplify the above equation we have

$$|G{:}H| = q^2 - 2q^3 + q^5 + \sum_{k=1}^{3} K_k\theta_k(1) + \sum_{m=1}^{3} M_m\omega_m(1) + \sum_{n=1}^{3} N_n\gamma_n(1).$$

Since $|G{:}H| = q^5 - q^3 + q^2 - 1$, we get

$$\sum_{k=1}^{3} K_k\theta_k(1) + \sum_{m=1}^{3} M_m\omega_m(1) + \sum_{n=1}^{3} N_n\gamma_n(1) = q^3 - 1.$$

Since $\theta_k(1) = (q^3 - 2q^2 + 2q - 1)/3$ and $\omega_m(1) = \gamma_n(1) = (q^3 + q^2 - q - 1)/3$, we have

$$\Big(\sum_{k=1}^{3} K_k\Big)\big((q^3-2q^2+2q-1)/3\big) + \Big(\sum_{m=1}^{3} M_m + \sum_{n=1}^{3} N_n\Big)\big((q^3+q^2-q-1)/3\big) = q^3-1.$$

Hence by considering $K = \sum_{k=1}^{3} K_k$, $M = \sum_{m=1}^{3} M_m$ and $N = \sum_{n=1}^{3} N_n$ we get

$$K\big((q^3 - 2q^2 + 2q - 1)/3\big) + (M + N)\big((q^3 + q^2 - q - 1)/3\big) = q^3 - 1,$$

so

$$(K+M+N)q^3 - \big(2K - (M+N)\big)\,q^2 + \big((2K - (M+N)\big)\,q - (K+M+N) = 3(q^3 - 1).$$

Thus

$$(A - 3)(q^3 - 1) = B(q^2 - q) \tag{4.5}$$

where $A = K + M + N$ and $B = 2K - (M + N)$. Since $q \mid B(q^2 - q)$, we have $q \mid A - 3$ and this means that $A - 3 = tq$ for some integer t. Hence simplifying (4.5) implies $B = t(q^2 + q + 1)$. Therefore

$$0 \leqslant 3K = A + B = 3 + t(q + 1)^2$$

and

$$0 \leqslant 3(M + N) = 2A - B = 6 - t(q^2 - q + 1).$$

If $q > 2$ then the first inequality shows that $t \geqslant 0$ and the second shows that $t \leqslant 0$. So $t = 0$, $A = 3$ and $B = 0$, which gives $K = 1$ and $M + N = 2$. Hence $\sum_{k=1}^{3} K_k = 1$ and $\sum_{m=1}^{3} M_m + \sum_{n=1}^{3} N_n = 2$. Therefore, for some k, $K_k = 1$ and $\langle (\theta_k)_H, \varphi \rangle = 1$.

Let $\langle (\theta_1)_H, \varphi \rangle = 1$. Then the characters θ_1, θ_2 and θ_3 are conjugate in $L = \mathrm{GU}(3, q)$, (see [14, §4]). Hence by Lemma 4.3 we have

$$\langle (\theta_1)_H, \varphi \rangle = \langle (\theta_2)_H, \varphi^x \rangle = \langle (\theta_3)_H, \varphi^y \rangle = 1$$

for some $x, y \in N_L(H)$. On the other hand, by Lemma 4.2, φ^x and φ^y are linear characters of H so the restriction of characters θ_1, θ_2 and θ_3 to H have at least a constituent of degree one with multiplicity one.

Also equation (4.3) shows $\sum_{m=1}^{3} M_m = \sum_{n=1}^{3} N_n$ and so both sums equal 1. Therefore for some m and n we have $N_n = 1$ and $M_m = 1$, which means $\langle (\omega_m)_H, \varphi \rangle = \langle (\gamma_n)_H, \varphi \rangle = 1$. Without loss in generality we can suppose $\langle (\omega_1)_H, \varphi \rangle = \langle (\gamma_1)_H, \varphi \rangle = 1$. Since the elements of each set of characters $\{\omega_1, \omega_2, \omega_3\}$ and $\{\gamma_1, \gamma_2, \gamma_3\}$ are conjugate in $L = \mathrm{GU}(3, q)$ (see [14, §4]), therefore by Lemma 4.3 and Lemma 4.2 there exist $r, s, t, u \in N_L(G)$ such that $\varphi^r, \varphi^s, \varphi^t$ and φ^u are linear characters of H and

$$\langle (\omega_2)_H, \varphi^r \rangle = \langle (\omega_3)_H, \varphi^s \rangle = \langle (\gamma_2)_H, \varphi^t \rangle = \langle (\gamma_3)_H, \varphi^u \rangle = 1.$$

Hence for $1 \leqslant m, n \leqslant 3$ the characters $(\omega_m)_H$ and $(\gamma_n)_H$ have a linear constituent with multiplicity 1. This completes the proof. ∎

5 Other Simple Groups and Covers

We have shown above that for each irreducible character χ of degree less than 32 of the alternating groups and their covers there exists a χ-subgroup (often a Sylow subgroup). Also without any restriction on the degree of characters, if G is one of the groups $\mathrm{PSL}(2, q)$, $\mathrm{PSL}(3, q)$, $\mathrm{PSU}(3, q)$ or their covers and χ is an irreducible character of G, then there exists a Sylow subgroup or a p-subgroup of G which is a χ-subgroup.

Lemma 3.1 shows that if a group G has a Sylow subgroup P and an irreducible character χ such that $|P| = \chi(1)$, then χ_P is the regular character of P. In this case $\langle \chi_P, \varphi \rangle = 1$ for each linear character φ of P (*i.e.*, P is a χ-subgroup). Using these results and some computations in GAP, we found all the other cases listed in [1], where G is a simple group or a cover of a simple group and χ an irreducible character of G with degree less than 32, for which there exists a Sylow subgroup which is a χ-subgroup. We have summarized our results in the Table 6.

For the groups $3.O_7(3)$, $3.U_6(2)$ and the covering groups of $U_4(3)$ we have not been able to determine whether their characters of degree less than 32 have χ-subgroups. However, in [4] we have used an alternative approach to construct the representations based on the fact that we can show that the character remains irreducible on some proper subgroup. Suppose $\tilde{G}$ is one of these groups and χ is an irreducible character of $\tilde{G}$ of degree less than 32. We shall use [1] to find a maximal subgroup $\tilde{M}$ of $\tilde{G}$ such that $\chi_{\tilde{M}}$ is irreducible. It is enough to find a maximal subgroup $\tilde{M}$ such that

$$\langle \chi_{\tilde{M}}, \chi_{\tilde{M}} \rangle = \langle \chi_{\tilde{M}} \bar{\chi}_{\tilde{M}}, \mathbf{1} \rangle = 1.$$

Since χ is irreducible, $\langle \chi, \chi \rangle = \langle \chi\bar{\chi}, \mathbf{1} \rangle = 1$. Note that the kernel of $\chi\bar{\chi}$ contains the centre of $\tilde{G}$ and so we can consider $\chi\bar{\chi}$ as a character of $G = \tilde{G}/Z(\tilde{G})$. So

$$\chi_M\bar{\chi}_M = \mathbf{1} + \sum_{\mathbf{1}\neq\psi_i\in\mathrm{Irr}(G)} m_i(\psi_i)_M, \tag{5.1}$$

where $M = \tilde{M}/Z(\tilde{G})$. Now if we find a maximal subgroup M of G such that for each constituent $(\psi_i)_M$ of equation (5.1), $\langle (\psi_i)_M, \mathbf{1} \rangle = 0$ then $\langle \chi_M\bar{\chi}_M, \mathbf{1} \rangle = 1$. This means that the restriction $\chi_{\tilde{M}}$ of χ to the inverse image $\tilde{M}$ of M in the centre of $\tilde{G}$ is irreducible.

Suppose $G = U_4(3)$. The covering groups $2.G$ and $4.G$ have one and two characters of degree 21, respectively (see [1]). The covering group $3_1.G$ has two characters of degree 15 and two characters of degree 21. Finally the covering group $6_1.G$ has two characters of degree 6. If χ is an irreducible character of degree less than 32 of one of these covers such that $\chi(1) \neq 21$, then for the maximal subgroup $M \cong 3^4{:}A_6$ of index 112 of G we have $\langle \chi_M\bar{\chi}_M, \mathbf{1} \rangle = 1$ which means $\chi_{\tilde{M}}$ is irreducible. If $\chi(1) = 21$, then G has a maximal subgroup M isomorphic to $\mathrm{PSL}(3,4)$ of index 162 such that $\langle \chi_M\bar{\chi}_M, \mathbf{1} \rangle = 1$ and so $\chi_{\tilde{M}}$ is irreducible.

If $G = O_7(3)$, then the covering group $3.G$ has two characters of degree 27, and G has a maximal subgroup M of index 364 such that $M \cong 3^5{:}U_4(2){:}2$. For each character χ of degree 27, $\chi_{\tilde{M}}$ is irreducible.

Finally for $G = U_6(2)$ the covering group $3.G$ has two irreducible characters of degree 21, and G has a maximal subgroup M of index 891 such that $M \cong 2^9{:}\,L_3(4)$. We find that $\chi_{\tilde{M}}$ is irreducible when χ is character of degree 21. For more details about these maximal subgroups see [3].

Table 6 describes χ-subgroups for the characters of degree less than 32 for simple groups and covers which have not been already described in the theorems.

Table 6:

G	Degree	χ-subgroup
A_5	3	Syl(3)
$2.A_5$	2, 3, 4, 5, 6	Syl(5)
A_6	8	Syl(2)
$2.A_6$	4, 5, 8, 9, 10	Syl(3)
$3.A_6$	3, 5, 6, 8, 9, 10, 15	Syl(2)
$6.A_6$	3, 4, 5, 6, 8, 9	Syl(5)
	10	Syl(3)
	15	Syl(2)
A_7	10, 21	Syl(3)
$2.A_7$	4, 6, 10, 14, 15, 21	Syl(3)
$3.A_7$	4, 6, 10, 14, 15, 20	Syl(2)
$6.A_7$	4, 6, 10	Syl(7)
	14	Syl(3)
	15	Syl(2)

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G	Degree	χ-subgroup
M_{11}	10, 11, 16	Syl(11)
A_8	14, 21	Syl(3)
$2.A_8$	7, 8	Syl(7)
	14, 20, 21	Syl(3)
	28	Syl(2)
$2.L_3(4)$	10, 20, 28	Syl(2)
$3.L_3(4)$	15, 20, 21	Syl(2)
$4_1.L_3(4)$	8, 10, 20, 28	Syl(2)
$4_2.L_3(4)$	10, 20, 28	Syl(2)
$6.L_3(4)$	6, 10, 15, 20, 21, 28	Syl(2)
$12_1.L_3(4)$	6, 10, 15, 20, 21, 24, 28	Syl(2)
$12_2.L_3(4)$	6, 10, 15, 20, 21, 28	Syl(2)
$U_4(2)$	6	Syl(5)
	5, 10, 15, 20, 24, 30	Syl(3)
$Sp_4(3)$	6	Syl(5)
	4, 5,10, 15, 20, 24, 30	Syl(3)
$Sz(8)$	14	Syl(13)
M_{12}	11, 16	Syl(2)
$2.M_{12}$	10, 11, 12, 16	Syl(2)
A_9	21	Syl(2)
$2.A_9$	8	Syl(7)
	21, 27, 28	Syl(3)
M_{22}	21	Syl(2)
$2.M_{22}$	10	Syl(3)
	21	Syl(2)
$3.M_{22}$	21	Syl(2)
J_2	14, 21	Syl(5)
$2.J_2$	6, 14, 21	Syl(5)
$S_4(4)$	18	Syl(5)
$S_6(2)$	7	Syl(7)
	15, 21, 27	Syl(3)
$2.S_6(2)$	7, 8	Syl(7)
	15, 21, 27	Syl(3)
$2.A_{10}$	9, 16	Syl(5)
$U_4(3)$	21	Syl(2)
$G_2(3)$	14	Syl(13)
$3.G_2(3)$	14, 27	Syl(2)
$S_4(5)$	13	Syl(13)
$Sp_4(5)$	12, 13	Syl(3)
$L_4(3)$	26	Syl(3)
$L_5(2)$	30	Syl(2)
M_{23}	22	Syl(23)

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G	Degree	χ-subgroup
$U_5(2)$	10, 11	Syl(11)
$^2F_4(2)'$	26, 27	Syl(3)
$2.A_{11}$	10, 16	Syl(11)
HS	22	Syl(2)
$3.J_3$	18	Syl(17)
$O_8^+(2)$	28	Syl(5)
$2.O_8^+(2)$	8, 28	Syl(5)
$^3D_4(2)$	26	Syl(7)
M_{24}	23	Syl(23)
$2.G_2(4)$	12	Syl(13)
M^cL	22	Syl(2)
$S_6(3)$	13	Syl(13)
$2.S_6(3)$	13, 14	Syl(13)
$U_6(2)$	22	Syl(3)
$2.Ru$	28	Syl(29)
$6.Suz$	12	Syl(13)
Co_3	23	Syl(23)
Co_2	23	Syl(23)
$2.Co_1$	24	Syl(23)

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