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ELEVENTH MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

The annual meeting of the Association for Symbolic Logic was held (in conjunction with meetings of the American Mathematical Society and the Mathematical Association of America) at The Ohio State University, Columbus, Ohio, on December 30–31, 1948.

Three sessions for contributed papers were held—on Thursday afternoon, on Friday morning, and on Friday afternoon. Presiding officers were Alfred Tarski, J. C. C. McKinsey, and Alonzo Church. The Thursday session was a joint session with the American Mathematical Society.

There were two invited hour addresses: N. Bourbaki, *Foundations of mathematics for the working mathematician*, and F. B. Fitch, *Towards a demonstrably consistent mathematics.* The first address was read by André Weil on Friday morning, Saunders MacLane presiding; the second was given on Friday afternoon, with A. A. Bennett as presiding officer.

The Council met on Thursday afternoon. After the Council meeting the Executive Committee met briefly.

Thursday night, a dinner for the three organizations was served in Mack-Canfield Dining Hall. R. E. Langer was toastmaster; Vice President Harlan Hatcher of The Ohio State University, Saunders MacLane, and Marston Morse addressed the members. Vice President S. C. Kleene represented the Association at the speakers' table. R. P. Dilworth proposed a resolution, approved by rising vote, expressing the appreciation of the members of the three organizations to the administration of The Ohio State University, the local committee, and all who contributed to the success of the meetings.

Friday noon, the Association met for luncheon in the balcony of Mack-Canfield Dining Hall.

Abstracts of the invited addresses and of the contributed papers are appended. Of the latter, papers 14, 11, 15, 13 were presented in the first general session, papers 1, 4, 16 in the second, and papers 5, 6 in the third general session. The remaining contributed papers were presented by title. Paper 1 was read by A. R. Turquette, paper 6 by Andrzej Mostowski, paper 15 by Max A. Zorn, paper 16 by C. D. Firestone. MAX A. ZORN

N. BOURBAKI. *Foundations of mathematics for the working mathematician.*
(This appears in full in this number of this JOURNAL, pp. 1–8.)

FREDERIC B. FITCH. *Towards a demonstrably consistent mathematics.*

The aim of this paper is to provide a consistent foundation for at least as much of mathematical analysis as is required in physics. Use is made of the author's demonstrably consistent system K' and of results obtained in *An extension of basic logic* (vol. 13, pp. 95–106, this JOURNAL) and in *The Heine-Borel theorem in extended basic logic* (in this number of this JOURNAL, pp. 9–15). The terminology of the former paper will now be assumed.

The class of U-reals is redefined so as to provide for negative U-reals. The usual arithmetic operations on U-reals are defined. A "definite" class is a class completely represented in K'.

Every definite class of U-reals having an upper bound has a least upper bound, and similarly for lower bounds. Every definite convergent sequence of U-reals converges to a U-real. Every infinite definite class of U-reals in an interval has at least one U-real as a limit.

If 'a' and 'b' are U-reals then '$[a,b]$' is a "U-real pair." An expression representing in K' a class of U-real pairs is a "U-real function." Continuity of U-real functions can be defined in the metalanguage by paralleling the usual definition of continuity. A U-real function is "value-definite" if for each value of the argument the class of values of the function is definite.

If a U-real function is continuous and value-definite in an interval, then it is uniformly continuous in the interval, and it assumes a maximum finite U-real value and a minimum finite U-real value in the interval, and also all intermediate U-real values.

The theory of Lebesgue measure can be developed in the system even though all classes are denumerable.

1. J. B. Rosser and A. R. Turquette. *The Gödel completeness of m-valued functional calculi of first order.*

In a previous paper the present authors defined a set of axiom schemes for m-valued functional calculi of first order with s ($1 \leqq s < m$) designated truth-values, and proved the plausibility of the resulting formalization, i.e., it was shown that provable formulas take designated truth-values exclusively. Hence, not all formulas are provable in our formalization and the two-valued notion of "absolute consistency" is readily extended to our formalization of m-valued functional calculi of first order.

The purpose of the present paper is to show that a notion of "Gödel completeness," which is closely analogous to the corresponding two-valued notion, may be applied to our formalization of m-valued functional calculi of first order. In particular, we give conditions under which a formula of an m-valued functional calculus of first order may be said to be "analytic" or to never take an undesignated truth-value, and show that if an m-valued formula is analytic, then it is provable in our formalization of an m-valued functional calculus of first order with s designated truth-values.

2. Abraham Robinson. *On the metamathematics of algebra.*

The principal object of the paper is the analysis and development of algebra by the methods of symbolic logic. Two lines of attack are followed up.

1. Instead of formulating and proving individual theorems as in orthodox mathematics, we consider statements about theorems in general. In particular we show that any theorem (of a certain class) which is true for one type of mathematical structure is also true for another type of mathematical structure. For example:

Any theorem, formulated within the restricted calculus of predicates in terms of addition, multiplication, equality, and order, which is true for all non-Archimedean ordered fields is true for all ordered fields.

Any theorem, formulated similarly in terms of addition, multiplication, and equality, which is true for the field of all algebraic numbers is true for any other algebraically closed field of characteristic 0.

Results such as those quoted can be used to prove actual mathematical theorems. For instance:

Let $q(x_1, \cdots, x_n)$ be a polynomial with integral coefficients which is irreducible in all extensions of the field of rational numbers. Then $q(x_1, \cdots, x_n)$, taken modulo p, is irreducible in all fields of characteristic p greater than some constant depending on q.

The only algebraic result that is used in the proof of this theorem, in addition to the metamathematical reasoning, is the fact that two polynomials which are equal for all rational arguments are identical, i.e., have the same coefficients.

2. Following the second line of attack we consider joint properties of structures of which we know only that they satisfy some specific system of axioms formulated in the restricted calculus of predicates, and that a relation of equality involving substitutivity is included in the system. We investigate various concepts which are parallel to certain standard concepts of algebra, e.g. the concepts of an algebraic number, of a polynomial ring over a given ring, and of an ideal. These general concepts are not merely analogous to their algebraic counterparts, but in the particular cases of the algebraic systems from which they are borrowed, they actually reduce to these counterparts. This shows that they can be abstracted from the specific arithmetical operation with which they are normally associated. Moreover, a number of their properties in standard algebra can be transferred to the more general case considered here.

3. R. M. Martin and J. H. Woodger. *Toward a nominalistic semantics.*

Working upon the syntactical basis provided by Quine and Goodman in *Steps toward a constructive nominalism* (this Journal, vol. 12 (1947), pp. 105–122), and using a restricted form of a name relation N, steps are taken in constructing a nominalistic semantical metalanguage for the analysis of elementary, nominalistic, or first-order systems. Several semantical concepts are built up, relativized to a particular language L, which contains a finite or denumerable number of individual constants, a finite or denumerable number of one-placed predicates, the truth functions, and quantifiers upon the individual variables. (The extension of the concepts to languages containing predicates of any finite degree presents no difficulty.) In particular, a concept *true in L* is defined and is shown to be *adequate* in the sense of Leśniewski, Kotarbiński, and Tarski. That this restricted semantics is not appreciably weaker than extensional semantics of the classical kind then follows.

4. R. M. MARTIN. *On virtual classes and real numbers.*

In a functional calculus of first order admitting at least one functional constant, restricted abstracts or schematic variables can be introduced, after the manner of D4.1 of the author's *A homogeneous system for formal logic* (this JOURNAL, vol. 8 (1943), pp. 1–23). The entities designated or quasi-designated by such abstracts may be called, following Quine, *virtual* classes relative to the system in question. The system formulated in *A homogeneous logic* was a nominalism in the sense of Tarski and Quine, and provided a foundation for elementary number theory. Working upon the basis provided in that paper, concepts leading to a definition of expressions for real numbers are presented here, and upon the axiomatic basis of that paper, restricted analogues of Huntington's postulates for real numbers (*Transactions of the American Mathematical Society*, 1905) are proved. The method rests upon a considerable expansion of the theory of virtual classes over the form presented by Quine in *On universals* (this JOURNAL, vol. 12 (1947), pp. 74–84) but without presupposing any new primitive devices beyond those of *A homogeneous logic*. The resulting theory of real numbers is comparable to that provided by intuitionistic systems.

5. ALFRED TARSKI. *On essential undecidability.*

The theories discussed in this and the following four abstracts are assumed to be formalized within the lower predicate calculus (without variable predicates); they all have the same logical constants (connectives, quantifiers, identity symbol), logical axiom schemata, and rules of inference. Each theory $\mathfrak{T}$ is determined by its non-logical (primitive) constants—individual constants, relation and operation symbols—and non-logical axioms; in terms of these, the notions of a formula, a sentence (a formula without free variables), and a provable sentence of $\mathfrak{T}$ are defined. Theory $\mathfrak{T}$ is *finitely axiomatizable* if its set of non-logical axioms is finite or contains finitely many sentences from which all the remaining axioms can be derived by means of rules of inference. The notions of *consistency* and *completeness* are known. $\mathfrak{T}_1$ is an *extension of* $\mathfrak{T}_2$ if every sentence provable in $\mathfrak{T}_2$ is also provable in $\mathfrak{T}_1$; $\mathfrak{T}_1$ is a *finite extension of* $\mathfrak{T}_2$ if, moreover, only finitely many axioms of $\mathfrak{T}_1$ are not provable in $\mathfrak{T}_2$. $\mathfrak{T}_1$ and $\mathfrak{T}_2$ are *compatible* if they have the same non-logical constants and a consistent common extension. Consider a non-logical constant, say, the operation symbol "+", and some further non-logical constants $C_1, \cdots, C_n$ of $\mathfrak{T}$; a possible definition of "+" in terms of $C_1, \cdots, C_n$ is any sentence of the form "$\mathbf{A}_{x,y,z}(x + y = z \leftrightarrow \Phi)$" where "$\mathbf{A}$" is the universal quantifier, and "Φ" stands for any formula with the free variables "x", "y", "z" which contains no non-logical constant different from $C_1, \cdots, C_n$. $\mathfrak{T}_1$ is *consistently interpretable in* $\mathfrak{T}_2$ if $\mathfrak{T}_1$ and $\mathfrak{T}_2$ have a common consistent extension $\mathfrak{T}$ such that, for every constant C in $\mathfrak{T}_1$ which is not in $\mathfrak{T}_2$, there is a provable sentence in $\mathfrak{T}$ which is a possible definition of C in terms of constants of $\mathfrak{T}_2$ and possibly individual constants of $\mathfrak{T}$. $\mathfrak{T}$ is *decidable* if the set of its provable sentences is generally recursive, and otherwise *undecidable;* $\mathfrak{T}$ *is essentially undecidable* if it is consistent and no consistent extension of $\mathfrak{T}$ is decidable. The following general theorems can easily be established (with the help of the deduction theorem): I. *If* $\mathfrak{T}$ *is undecidable, then every theory* $\mathfrak{T}_1$ *with the same constants of which* $\mathfrak{T}$ *is a finite extension is undecidable.* II. $\mathfrak{T}$ *is essentially undecidable if, and only if, it is consistent and no consistent and complete extension of* $\mathfrak{T}$ *is decidable.* III. *If* $\mathfrak{T}$ *is essentially undecidable, finitely axiomatizable, and compatible with* $\mathfrak{T}_1$, *then* $\mathfrak{T}_1$ *is undecidable* (though not necessarily essentially undecidable). IV. *If* $\mathfrak{T}$ *is essentially undecidable, finitely axiomatizable, and consistently interpretable in* $\mathfrak{T}_1$, *then* $\mathfrak{T}_1$ *is compatible with an essentially undecidable and finitely axiomatizable theory* $\mathfrak{T}_2$, *and hence is undecidable.* In view of the last two theorems a new method of investigation into the decision problem presents itself. The applicability of this method depends, of course, on whether essentially undecidable and finitely axiomatizable theories are available which can easily be interpreted in other theories; compare the following abstracts.

6. ANDRZEJ MOSTOWSKI and ALFRED TARSKI. *Undecidability in the arithmetic of integers and in the theory of rings.*

Using the notations of the preceding abstract, we consider a formalized theory of integers, $\mathfrak{T}_1$, with the following non-logical constants: "I", "$+$", "$\cdot$", and "$<$"; "I" denotes the set of all integers. ["x is an integer" is symbolized by "$I(x)$"]. The set of non-logical axioms of $\mathfrak{T}_1$ consists of (i) finitely many sentences which characterize I as a ring with unit under $+$ and $\cdot$ that is ordered, but not densely ordered, by $<$; (ii) all the sentences which are particular cases of the induction principle, i.e., which express the idea that, if some integer satisfies a formula Φ and if $x + 1$ and $x - 1$ satisfy Φ whenever x satisfies Φ, then every integer satisfies Φ. Theory $\mathfrak{T}_1$ is essentially undecidable—a result due essentially to Rosser (II 52); but presumably $\mathfrak{T}_1$ is not finitely axiomatizable. By using, however, methods of Gödel (418 *3*) and Rosser (II 52), one shows that theory $\mathfrak{T}_2$ obtained from $\mathfrak{T}_1$ by removing all the axioms (ii) is still essentially undecidable; of course, $\mathfrak{T}_2$ is finitely axiomatizable. Hence, by Theorem III of the preceding abstract, *every theory $\mathfrak{T}$ with the same constants as $\mathfrak{T}_1$ and whose axioms are true sentences of the arithmetic of integers—and, more generally, every theory compatible with $\mathfrak{T}_2$—is undecidable.* Only particular cases of this result are known from the literature: the case $\mathfrak{T} = \mathfrak{T}_1$; the case when $\mathfrak{T}$ has no non-logical axioms (the undecidability of the lower predicate calculus—a result of Church); the case when all true sentences of the arithmetic of integers involving the constants of $\mathfrak{T}_1$ are axioms of $\mathfrak{T}$. The result extends to theories which contain only "I", "$+$", and "$\cdot$" as non-logical constants (since "$<$" is definable in terms of "$+$" and "$\cdot$" in the arithmetic of integers); also to theories in which I is replaced by the set P of positive integers. The first of these extensions implies that *the theory of rings and that of commutative rings* (formalized within the lower predicate calculus) *are undecidable.* These theories, however, are not essentially undecidable since, e.g., the theory of Boolean rings and that of the ring of real numbers are decidable (results of Tarski).

7. ALFRED TARSKI. *Undecidability of group theory.*

The results of the preceding abstract extend to the theories of integers with "I", "$+$", and "$\mid$" (the symbol of the divisibility relation) as the only non-logical constants. This follows from the fact that multiplication can be defined in the arithmetic of integers in terms of addition and divisibility. [For x^2 can be so defined in view of the fact that $x^2 + x$ is a least common multiple of x and $x + 1$; hence a definition of $x \cdot y$ can be obtained by considering the familiar formula for $(x + y)^2$.] Therefore there is an essentially undefinable and finitely axiomatizable theory $\mathfrak{T}_1$ whose axioms are true sentences of the arithmetic of integers with no non-logical constant different from "I", "$+$", and "$\mid$". Let now $\mathfrak{T}_2$ be the theory of groups with two non-logical constants: "G" denoting the set of elements of a group, and "$\bigcirc$" denoting the group operation; as non-logical axioms of $\mathfrak{T}_2$ we take the usual postulates characterizing the notion of a group. Consider the system $\Gamma = \langle G, \bigcirc, c, I, +, \mid \rangle$ where (i) G is a free group generated by an element c and infinitely many other elements $d_0, d_1, \cdots$, with the group operation $\bigcirc$ and "defining equations" $c^{n+2} \bigcirc d_n = d_n \bigcirc c^{n+2}$ for $n = 0, 1, \cdots$ (the power of an element being henceforth undersgood in group theoretical sense); (ii) I is the set of all powers of c, $+$ is an operation defined over couples of elements of I and which coincides on these couples with the group operation $\bigcirc$, and $\mid$ is the relation which holds between two elements x and y of I in case y is a power of x. Let $\mathfrak{T}$ be the theory whose non-logical constants are "G", "$\bigcirc$", "c", "I", "$+$", and "$\mid$", and whose set of non-logical axioms consists of all sentences which hold in Γ. $\mathfrak{T}$ is clearly a consistent extension of $\mathfrak{T}_1$ and $\mathfrak{T}_2$. Moreover, among axioms of $\mathfrak{T}$ we find possible definitions of "I", "$+$", and "$\mid$" in terms of "G", "$\bigcirc$", and "c", e.g.:

$$\mathbf{A}_x[I(x) \leftrightarrow G(x) \wedge x \bigcirc c = c \bigcirc x],$$

$$\mathbf{A}_{x,y,z}[x + y = z \leftrightarrow G(x) \wedge G(y) \wedge G(z) \wedge x \bigcirc c = c \bigcirc x \wedge y \bigcirc c = c \bigcirc y \wedge x \bigcirc y = z],$$

$$\mathbf{A}_{x,y}[x \mid y \leftrightarrow G(x) \wedge G(y) \wedge x \bigcirc c = c \bigcirc x \wedge \mathbf{A}_z(x \bigcirc z = z \bigcirc x \rightarrow y \bigcirc z = z \bigcirc y)].$$

Hence, by the preceding abstract of the author, *On essential undecidability*, $\mathfrak{T}_1$ is consistently interpretable in $\mathfrak{T}_2$ and, consequently, the *theory of groups (formalized within the lower predicate calculus) is undecidable*. This theory, however, is not essentially undecidable since, as was shown by Mrs. Szmielew, the theory of Abelian groups is decidable.

8. Julia Robinson. *Undecidability in the arithmetic of integers and rationals and in the theory of fields.*

In the arithmetic of positive integers, addition is definable in terms of multiplication and the successor operation S $(Sx = x + 1)$; for the following formula clearly holds for arbitrary positive integers x, y, and z:

$$x + y = z \leftrightarrow S(x \cdot z) \cdot S(y \cdot z) = S(S(x \cdot y) \cdot z^2).$$

An analogous, although slightly more involved, formula holds in the arithmetic of arbitrary integers (and, more generally, in the theory of integral domains with unit elements). Hence the results stated in the preceding abstract of Mostowski and Tarski, *Undecidability in the arithmetic of integers and in the theory of rings*, extend to the theories of integers which contain "P", "S", and "$\cdot$" (or "I", "S", and "$\cdot$") as the only non-logical constants.

Furthermore, multiplication is definable in the arithmetic of positive integers in terms of the successor operation S and the divisibility relation | (though the definition in this case is rather involved). Hence the results referred to above extend also to the theories of positive integers which contain "P", "S", and "|" as the only non-logical constants.

Finally, *in the arithmetic of rationals the notion of an integer is definable in terms of addition and multiplication*. For, let "$R(x)$" express the fact that x is rational; then the following equivalence holds:

$$I(z) \leftrightarrow R(z) \wedge \mathsf{A}_{x,y}\{R(x) \wedge R(y) \wedge \Phi(x, y, 0) \wedge \mathsf{A}_u[R(u) \wedge \Phi(x, y, u) \rightarrow \Phi(x, y, u + 1)] \rightarrow \Phi(x, y, z)\}$$

where expressions of the form "$\Phi(x, y, z)$" are used as abbreviations for:

$$\sim \mathsf{A}_{r,s,t}[R(r) \wedge R(s) \wedge R(t) \rightarrow 2 + x \cdot y \cdot z^2 + y \cdot r^2 \neq s^2 + x \cdot t^2].$$

The proof of this equivalence is based upon the results of Hasse concerning quadratic forms in ***Journal für die reine und angewandte Mathematik***, vol. 152 (1923), pp. 129–148. In consequence, the results of Mostowski and Tarski extend to the arithmetic of rationals. Thus, *every theory whose non-logical constants are "R", "$+$", and "$\cdot$" and whose non-logical axioms are true sentences in the arithmetic of rationals is undecidable*. In particular, *the theory of fields (formalized within the lower predicate calculus) is undecidable*—an improvement of the result of Mostowski and Tarski regarding the theory of rings. However, the theory of fields is not essentially undecidable since (as was shown by Tarski) the theories of algebraically closed fields and real closed fields are decidable.

9. Alfred Tarski. *Undecidability of the theories of lattices and projective geometries.*

By the preceding abstract of Mrs. Robinson, there is an essentially undecidable and finitely axiomatizable theory $\mathfrak{T}_1$, with "R", "$+$", and "$\cdot$" as non-logical constants, the axioms of which are true sentences in the arithmetic of rationals. Let $\mathfrak{T}_2$ be the theory of modular lattices; its non-logical constants are the symbols "L", "$\cup$," and "$\cap$" denoting the set of elements of a lattice and the operations of join and meet; the non-logical axioms of $\mathfrak{T}_2$ are the usual postulates characterizing the notion of a modular lattice. Consider the system $\mathrm{A} = \langle L, \cup, \cap, a, b, c, d, R, +, \cdot \rangle$ where (i) L is the set of all linear subspaces of the two dimensional projective geometry whose points have homogeneous rational coordinates; (ii) $\cup$ and $\cap$ are the usual join and meet operations on elements of L; (iii) a, b, c, and d are points (0-dimensional elements of L) with the coordinates (0, 0, 1), (0, 1, 0), (1, 0, 0), and (1, 1, 1,), respectively; (iv) R is the set of all points representable as $(x, 0, 1)$, and $+$ and $\cdot$ are operations on these points corresponding to ordinary arithmetical operations on their first coordinates in the above representation, e.g., $(x, 0, 1) + (y, 0, 1) = (x + y, 0, 1)$. Let $\mathfrak{T}$ be the theory whose non-logical constants are "L", "$\cup$", etc. and whose non-logical axioms are all the sentences which hold in A. $\mathfrak{T}$ is clearly a consistent extension of $\mathfrak{T}_1$ and $\mathfrak{T}_2$. Among axioms of $\mathfrak{T}$ we find possible definitions of "R", "+", and "·" in terms of "L", "∪", "∩", "a", "b", "c", and "d"; a method of constructing such definitions is known from projective geometry. Hence, by the preceding abstract *On essential undecidability*, $\mathfrak{T}_1$ is consistently interpretable in $\mathfrak{T}_2$ and, consequently, *the theory of modular lattices (formalized within the lower predicate calculus) is undecidable.* The same applies to *the theory of arbitrary lattices* (an older result of the author originally obtained by an analogous method, but without the help of Mrs. Robinson's results), *the theory of complemented modular lattices*, and *that of abstract projective geometries*. None of these theories is essentially undecidable since, as was shown by the author, the theories of Boolean algebras and of real projective geometry are decidable. For the notions involved above see Birkhoff, ***Lattice theory.***

10. Haskell B. Curry. *A theory of formal deducibility.*

This is a revision of a paper read in 1937 (***Bulletin of the American Mathematical Society***, vol. 43, p. 615, abstract no. **325**). The fundamental idea is to apply the inferential-rule methods of Gentzen to propositions relating to a formal system, so as to make a semantic analysis of the "compound" propositions formed from the elementary propositions of the system by the connectives of propositional algebra and predicate calculus. The new results relate chiefly to extending these methods to include the classical as well as the intuitionistic approaches. When negation is introduced four types of system are considered, viz.: M, the minimal system (Johansson); J, the intuitionistic system (Heyting); K, the classical system; and D, a minimal system with excluded middle (Johansson), applicable when the underlying formal system is decidable. For each type of system there are three types of formulation called respectively L, T, H; the L formulation is like that so called by Gentzen, T is Gentzen's "natural" formulation, and H is a more orthodox calculus. Relations between these types of system and formulation are considered, including a generalization of the Glivenko theorem. The paper will be published in booklet form by the University of Notre Dame.

11. Haskell B. Curry. *The permutability of rules in the classical inferential calculus.*

Suppose we have a rule-theoretic system à la Gentzen with elementary statements of the form $X_1, X_2, \cdots, X_m \rightarrow Y_1, Y_2, \cdots, Y_n$. With reference to the rules of such a system we distinguish as *parameters* those X_i, Y_j which go over unchanged from premises to conclusion, as *components* those which appear in the premises but not in the conclusion, and as *principal constituents* those which occur in the conclusion only. Suppose the rules are such that the same parameters appear in all the premises, and that parameters can be added to and deleted from all premises and conclusion simultaneously without destroying the validity of the inference. Then the following theorem is true: if a rule R_1 is followed by a rule R_2 in such a way that the principal constituents for R_1 are parameters for R_2, then the rules can be interchanged. This simple observation includes the strong form of the elimination theorem ("Hauptsatz") which is valid in Gentzen's system LK. In the system LJ it is not possible to add parameters on the right, and consequently a proof of the elimination theorem by this method breaks down.

12. Haskell B. Curry. *The elimination theorem when necessity is present.*

In the Notre Dame lectures on formal deducibility (see the preceding abstract) a proof of the fundamental theorem, called the elimination theorem (Gentzen's "Hauptsatz"), was lacking for the case of systems involving necessity. This hiatus is filled in the present paper. This enables the treatment of modal systems to be completed; and we now have relations between L, T, and H, formulations of propositional algebra just like those for non-modal systems. In particular the procedure gives a decision process for the Lewis system S4; however the relation of this procedure to those previously known for this system has not been investigated. The generalized form of the elimination theorem so obtained leads to some simplifications in the previous theory, notably the connections between the LC (classical positive) and LA (intuitionistic positive) systems.

13. Henry Blumberg. *Conception of set; elimination of the paradoxes of set theory.*

This paper sets forth a conception of set—termed "genetic"—which, as the author submits, constitutes an appropriate solution of the problem of the set-theoretic paradoxes. This conception is close to mathematical experience, requires no special devices or new constructions, and permits a logically unimpeachable development of the salient points of Cantor's discoveries—in particular, of Zermelo's theorem on the normal order. The principal idea is to validate the concepts and modes of conceptual derivation which mathematicians had no hesitation in accrediting before the phenomenon of the paradoxes, and developing the implications of such validation. This idea is supported by the fact that the set-theoretic paradoxes may be eliminated frontally; in other words, as the author shows, the paradox maker, in every case, commits an error in the argument allegedly establishing the paradox. Such direct refutation is facilitated by the proposed genetic conception of set. This conception does not fix once for all what a set is; a satisfactory conception of set cannot be expected to do this. Legitimate sets are derived from accredited sets by accredited associations; but the phrases, "all legitimate sets," and "there exists a legitimate set" have no meaning. There is clarification, too, of the deficiencies of other proposed conceptions of set.

14. I. L. Novak. *The relative consistency of von Neumann's and Zermelo's axioms for set theory.*

The system of axioms for class and set theory which was adapted from von Neumann's by Bernays (this Journal, vol. 2 (1937), pp. 65–77, and vol. 6 (1941), pp. 1–17) and Gödel (*Consistency of the continuum hypothesis*, Princeton 1940) differs from Zermelo's (including the Aussonderungs- and Ersetzungsaxiome) by admitting classes (non-elements) as well as sets. In the present paper a model of the von Neumann-Bernays-Gödel system is constructed in the syntax of Zermelo's. The syntax employed has axioms stating (i) certain basic signs and their combinations exist, (ii) quantification theory, (iii) induction, and (iv) identity theory may be used to derive metatheorems, (v) Zermelo's system is consistent. The 'ϵ' of Zermelo's system is reinterpreted as a syntactic relation between names of sets with help of a syntactically defined predicate which proves true of all Zermelo's theorems and true of any two formulas if and only if true of their conjunction, and true of any formula if and only if not true of its denial. The construction turns of the fact that the well-ordering hypothesis is consistent with Zermelo's system. The existence of this model shows von Neumann's system consistent relative to this syntax of Zermelo's system.

15. Ernst Snapper and Max A. Zorn. *On transfinite induction.*

A new mathematical proof for a variant of transfinite induction up to the first epsilon-number is given. The proof is (i) capable of generalization, (ii) of metamathematical interest.

16. C. D. Firestone and J. B. Rosser. *The consistency of the hypothesis of accessibility.*

This paper is concerned with a formalization of the system of axiomatic set theory described by Gödel in *The consistency of the continuum hypothesis*, and with the relation to this system of a statement which asserts that all cardinal numbers are accessible. It is shown that if this system of set theory is consistent, then the system obtained by adding an axiom which states that all cardinals are accessible is also consistent. This is equivalent to proving that if the system of set theory is consistent, then no statement which implies the existence of an inaccessible cardinal is provable in the system.

The proof is obtained by means of a modeling process based on that used by Gödel in the work cited above.

After the presentation of this paper it was pointed out by Prof. Andrzej Mostowski that a slightly weaker result for a modification of the Zermelo system of 1908 was announced, without proof, by C. Kuratowski, *Sur l'état actuel de l'axiomatique de la théorie des ensembles*, *Annales de la Société Polonaise de Mathématique*, vol. 3, p. 146 f.

THE ASSOCIATION FOR SYMBOLIC LOGIC announces the following elections, each for a term of three years from January 1, 1949:

As members of the Executive Committee, Professor George D. W. Berry of Princeton University, and Professor Emil L. Post of the College of the City of New York.

As member of the Council, Professor Andrzej Mostowski of the University of Warsaw.

The Council has appointed Professor Max Black as editor of the JOURNAL for a further term of three years from January 1, 1949.

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