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A computer aided classification of certain groups of prime power order: Corrigendum

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The first four paragraphs of [1, p. 258] are a mildly erroneous over simplification of the situation. A more accurate description follows.

The analysis of two-generator 3-groups of second maximal class goes along the following lines. We first define a class of group whose structure is particularly amenable to theoretical analysis.

Let $\underline{\underline{P}}$ be a group of order p^n and class $m-1$ (for any prime p) and $s \le r$ be positive integers such that

(i) $\underline{\underline{P}}/\gamma_2(\underline{\underline{P}}) \cong C_{p^r} \times C_p$ and $[\gamma_i(\underline{\underline{P}}) : \gamma_{i+1}(\underline{\underline{P}})] = p$ for $2 \le i \le m-1$,

so that $n = m + r - 1$.

(ii) Put $\underline{\underline{M}}_2 = C_{\underline{\underline{P}}}(\gamma_2(\underline{\underline{P}})/\gamma_4(\underline{\underline{P}}))$. We require $\underline{\underline{M}}_2/\gamma_2(\underline{\underline{P}}) \cong C_{p^{r-1}} \times C_p$.

Let a_1 be a fixed element of $\underline{\underline{M}}_2$ not lying in the Frattini subgroup of $\underline{\underline{P}}$ with $a_1^p \in \gamma_2(\underline{\underline{P}})$, and let $\gamma_1(\underline{\underline{P}})$ denote $\langle \gamma_2(\underline{\underline{P}}), a_1 \rangle$.

(iii) For all $i, j \ge 1$, $[\gamma_i(\underline{\underline{P}}), \gamma_j(\underline{\underline{P}})] \subseteq \gamma_{i+j+p^{s-1}}(\underline{\underline{P}})$.

(iv) For all $i \ge 1$, $\gamma_i(\underline{\underline{P}})^p = \gamma_{i+p^{s-1}(p-1)}(\underline{\underline{P}})$.

(v) $m \ge p^{s-1}+3$.

Then $\underline{\underline{P}}$ will be said to be a *Blackburn group* of type (r, s). It

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can be shown that conditions (iii) and (iv) are independent of the choice of a_1 (see [4]).

Here we are concerned with the cases $r = 1$ or 2. If $r = 1$, so that $s = 1$, $\underline{\underline{P}}$ is just a p-group of maximal class and positive degree of commutativity as defined in [2].

Examples are easily produced. Let O denote the ring of integers in the p^sth cyclotomic number field, so that O is of rank $p^{s-1}(p-1)$ as an abelian group, and let θ be a primitive p^sth root of unity. Let A be the ideal in O generated by $\theta - 1$, so that A^i is of index p^i in O for all $i > 0$. Then the split extension of O/A^{m-1} by the cyclic group of order p^r acting via multiplication by θ is a Blackburn group of type (r, s) with O/A^{m-1} as a possible choice for $\gamma_1(\underline{\underline{P}})$ provided $m \geq p^{s-1}+3$.

The groups of second maximal class with $\underline{\underline{P}}/\gamma_2(\underline{\underline{P}}) \cong C_9 \times C_3$ and of order 3^n, where $n \leq 8$, are analysed in [1, §7]. Those in [1, Table 6] have $\left[\gamma_i(\underline{\underline{P}}) : \gamma_i(\underline{\underline{P}})^3\right] \leq 9$ for all $i \geq 2$; such a group we define to be of *maximal type*. See [1, §4] for a general explanation of the tables. All groups descended from group A contain a subgroup of maximal class and index 9. Those descended from groups G and H are Blackburn groups of type $(2, 1)$. The groups in [1, Table 7] are of *non-maximal* type; that is $\left[\gamma_i(\underline{\underline{P}}) : \gamma_i(\underline{\underline{P}})^3\right] > 9$ if $i \geq 2$ and $|\gamma_i(\underline{\underline{P}})| > 9$. This table, when continued indefinitely, will contain all Blackburn 3-groups of type $(2, 2)$. It will also contain infinitely many groups with a subgroup of index at most 27 which is of this type, and will contain only a finite number of other groups.

The groups of second maximal class with $\underline{\underline{P}}/\gamma_2(\underline{\underline{P}}) \cong C_3 \times C_3$ of order p^n, where $n \leq 10$, are analysed in [1, §6]. Those in [1, Table 2] have $\left[\gamma_i(\underline{\underline{P}}) : \gamma_i(\underline{\underline{P}})^3\right] \leq 9$ for all $i \geq 4$; such groups will also be said to be of *maximal type*. Those descended from groups B, O, and Q contain a subgroup of maximal class and index 9. Those descended from groups S

and U contain a Blackburn group of type $(2, 1)$ and index 3.

The groups in [1, Tables 4, 5] are descended from groups H and I and are of *non-maximal type*; that is, $\left[\gamma_i(\underline{P}) : \gamma_i(\underline{P})^3\right] > 9$ if $i \geq 4$ and $|\gamma_i(\underline{P})| > 9$. It can be shown (see [3]) that all descendants of H and I contain a subgroup $\underline{Q}$ of index 3 such that $\underline{Q}$ has second maximal class, $\underline{Q}/\gamma_2(\underline{Q}) \cong C_9 \times C_3$, and $\gamma_i(\underline{Q}) = \gamma_{i+1}(\underline{P})$ for all $i \geq 3$. Thus $\underline{Q}$ is also of non-maximal type, as in [1, Table 7].

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