A computer aided classification of certain groups of prime power order: Corrigendum

Judith A. Ascione, George Havas, and C.R. Leedham-Green

The first four paragraphs of [1, p. 258] are a mildly erroneous over simplification of the situation. A more accurate description follows.

The analysis of two-generator 3-groups of second maximal class goes along the following lines. We first define a class of group whose structure is particularly amenable to theoretical analysis.

Let \( P \) be a group of order \( p^n \) and class \( m - 1 \) (for any prime \( p \)) and \( s \leq r \) be positive integers such that

(i) \( P/Y_2(P) \cong C_{p^r} \times C_p \) and \( [Y_i(P), Y_{i+1}(P)] = P \) for \( 2 \leq i \leq m-1 \),

so that \( n = m + r - 1 \).

(ii) Put \( M_2 = C_{\Sigma}(Y_2(P)/Y_1(P)) \). We require \( M_2/Y_2(P) \cong C_{p^{r-1}} \times C_p \).

Let \( a_1 \) be a fixed element of \( M_2 \) not lying in the Frattini subgroup of \( P \) with \( a_1^{p^r} \in Y_2(P) \), and let \( Y_1(P) \) denote \( \langle Y_2(P), a_1 \rangle \).

(iii) For all \( i, j \geq 1 \), \( [Y_i(P), Y_j(P)] \subseteq Y_{i+j+p^{s-1}}(P) \).

(iv) For all \( i \geq 1 \), \( Y_i(P)^{P} = Y_{i+p^{s-1}(p-1)}(P) \).

(v) \( m \geq p^{s-1}+3 \).

Then \( P \) will be said to be a Blackburn group of type \( (r, s) \). It

Received 25 August 1977.

317
can be shown that conditions (iii) and (iv) are independent of the choice of \( a_1 \) (see [4]).

Here we are concerned with the cases \( r = 1 \) or \( 2 \). If \( r = 1 \), so that \( s = 1 \), \( P \) is just a \( p \)-group of maximal class and positive degree of commutativity as defined in [2].

Examples are easily produced. Let \( O \) denote the ring of integers in the \( p^s \)th cyclotomic number field, so that \( O \) is of rank \( p^{s-1}(p-1) \) as an abelian group, and let \( \theta \) be a primitive \( p^s \)th root of unity. Let \( A \) be the ideal in \( O \) generated by \( \theta - 1 \), so that \( A^i \) is of index \( p^i \) in \( O \) for all \( i > 0 \). Then the split extension of \( O/A^{m-1} \) by the cyclic group of order \( p^r \) acting via multiplication by \( \theta \) is a Blackburn group of type \((r, s)\) with \( O/A^{m-1} \) as a possible choice for \( y(F) \) provided \( m > p^{s-1}+3 \).

The groups of second maximal class with \( G/Y_2(G) \cong C_9 \times C_3 \) and of order \( 3^n \), where \( n \leq 8 \), are analysed in [1, §7]. Those in [1, Table 6] have \( Y_i(G) : Y_i(G)^3 \leq 9 \) for all \( i \geq 2 \); such a group we define to be of maximal type. See [1, §4] for a general explanation of the tables. All groups descended from group \( A \) contain a subgroup of maximal class and index 9. Those descended from groups \( G \) and \( H \) are Blackburn groups of type \((2, 1)\). The groups in [1, Table 7] are of non-maximal type; that is \( Y_i(G) : Y_i(G)^3 > 9 \) if \( i \geq 2 \) and \( |Y_i(G)| > 9 \). This table, when continued indefinitely, will contain all Blackburn 3-groups of type \((2, 2)\). It will also contain infinitely many groups with a subgroup of index at most 27 which is of this type, and will contain only a finite number of other groups.

The groups of second maximal class with \( G/Y_2(G) \cong C_9 \times C_3 \) of order \( p^n \), where \( n \leq 10 \), are analysed in [1, §6]. Those in [1, Table 2] have \( Y_i(G) : Y_i(G)^3 \leq 9 \) for all \( i \geq 4 \); such groups will also be said to be of maximal type. Those descended from groups \( B, O \), and \( Q \) contain a subgroup of maximal class and index 9. Those descended from groups \( S \)
and $U$ contain a Blackburn group of type $(2, 1)$ and index 3.

The groups in [1, Tables 4, 5] are descended from groups $H$ and $I$ and are of non-maximal type; that is, $\left[ \gamma_i^*(P) : \gamma_{i+1}^*(P) \right] > 9$ if $i \geq 4$ and $|\gamma_i^*(P)| > 9$. It can be shown (see [3]) that all descendants of $H$ and $I$ contain a subgroup $Q$ of index 3 such that $Q$ has second maximal class, $Q/\gamma_2(Q) \cong C_9 \times C_3$, and $\gamma_i(Q) = \gamma_{i+1}(P)$ for all $i \geq 3$. Thus $Q$ is also of non-maximal type, as in [1, Table 7].

References


Department of Mathematics, Institute of Advanced Studies, Australian National University, Canberra, ACT;