

PROPAGATION OF WAVE FRONT SET IN GEVREY CLASS FOR AN EXAMPLE OF HYPERBOLIC SYSTEM

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Dedicated to the memory of Professor Hitoshi Kumano-go

Introduction

In this paper we show, for an example of hyperbolic system, that the wave front set in Gevrey class of a certain solution propagates precisely along "generalized broken null-bicharacteristic flow" defined by Kumano-go-Taniguchi [7] and Wakabayashi [10] [11].

Let L be a hyperbolic system of the form

$$(0.1) \quad \begin{pmatrix} p_1(t, x, D_t, D_x) & 0 \\ 0 & p_2(t, x, D_t, D_x) \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{in } R_t^1 \times R_x^4,$$

where $p_j = D_t + \lambda_j(t, x, D_x)$, $j = 1, 2$, and

$$(0.2) \quad \begin{cases} \lambda_1(t, x, \xi) = 0 \\ \lambda_2(t, x, \xi) = \xi_1 + x_1\xi_2 + x_2\xi_3 + (t^4 - 8x_1t^3 + 48x_2t^2 - 192x_3t)\xi_4. \end{cases}$$

We consider the Cauchy problem

$$(C.P) \quad LU = 0, \quad U(0, x) = G, \quad G \in \mathcal{E}'.$$

Let $\tilde{\rho}_0$ be a point $(0; (0, 0, 0, 0, 1))$ in the cotangent space $T^*(R_t \times R_x^4)$ and let \tilde{C}_0 be an integral curve in $T^*(R_t \times R_x^4)$ through $\tilde{\rho}_0$ by Hamilton vector field $H_{p_1+p_2}$. For $\tilde{\rho}_0 \in T^*(R_t \times R_x^4)$ we put $\rho_0 = (0, \eta^0) \in T^*R_x^4$, $\eta^0 = (0, 0, 0, 1) \in R_x^4 \setminus 0$. Assume that the initial value G of (C.P) satisfies

$$(0.3) \quad WF_x G = \{c\rho_0 \equiv \{(0, c\eta^0); c > 0\}, 1 < \kappa < 3\},$$

where we denote by $WF_\kappa u$ the wave front set in Gevrey class of order κ of $u \in \mathcal{D}'$ defined by Hörmander [1] (see Definition 1.1 in § 1). In the present paper we shall show that the wave front set in Gevrey class of order smaller than 3 of a certain solution U of the Cauchy problem (C.P)

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propagates precisely along the curve \tilde{C}_0 . (See Corollary 2.4 in § 2). We remark that

$$(0.4) \quad \tilde{C}_0 \subset \Sigma_1 \equiv \{\tilde{\rho} \in T^*(R_t \times R_x^n) \setminus 0; p_1(\tilde{\rho}) = p_2(\tilde{\rho}) = 0\}$$

and any Hamilton flow by H_{p_j} ($j = 1, 2$) passing \tilde{C}_0 intersects Σ_1 only once with tangency of fourth order. Namely, \tilde{C}_0 is the generalized broken null-bicharacteristic flow (but not broken null-bicharacteristic flow).

The motivation of this study is derived from the work of Kumano-go-Taniguchi [7], where they constructed the fundamental solution of the Cauchy problem for a hyperbolic system of first order with diagonal principal part by using Fourier integral operators with multi-phases. Their fundamental solution gave the following description about the propagation of C^∞ -wave front sets of solutions; if we also denote by U and G the solution and the initial value of the Cauchy problem, respectively, then we have

$$(0.5) \quad WF U(t_0) \subset \Gamma(t_0; WF G), \quad t_0 > 0, \quad (\text{see Theorem 3.4 of [7]}).$$

Here $\Gamma(t_0; \tilde{K})$ for a closed conic set \tilde{K} of $T^*R_x^n \setminus 0$ is defined as follows: For $\varepsilon > 0$ and integer $k \geq 0$ we denote by $\Gamma_\varepsilon^k(t_0; \tilde{K})$ the set composed of end points (at $t = t_0$) of all ε -admissible trajectories of step k issuing from \tilde{K} . We put

$$(0.6) \quad \Gamma(t_0; \tilde{K}) = \bigcap_{0 < \varepsilon < 1} \bigcup_{k=0}^{\infty} \Gamma_\varepsilon^k(t_0; \tilde{K}_\varepsilon),$$

where \tilde{K}_ε denotes the ε -conic neighborhood of \tilde{K} in $T^*(R_x^n) \setminus 0$. We refer the reader to [6] about the definition of ε -admissible trajectory for a hyperbolic system.

The description (0.5) seems to be rough to appearance because the set $\Gamma(t_0; \tilde{K})$ for a closed conic set \tilde{K} contains the end point (at $t = t_0$) of a limiting curve ($\varepsilon \rightarrow 0$) of the set composed of all ε -admissible trajectories issuing from \tilde{K}_ε . It should be noted, for hyperbolic system with characteristics of variable multiplicity, that the end point of the limiting curve is not always contained in the closure of the set of end points of all 0-admissible trajectories issuing from \tilde{K} , that is,

$$(0.7) \quad \left\{ \begin{array}{l} \text{there exist a } \delta_\infty \in T^*R_x^n \setminus 0 \text{ and a closed conic set } \tilde{K} \\ \text{of } T^*R_x^n \setminus 0 \text{ such that} \\ \delta_\infty \in \Gamma(t_0; \tilde{K}) \setminus \overline{\bigcup_{k=0}^{\infty} \Gamma_0^k(t_0; \tilde{K})}. \end{array} \right.$$

In fact, examples satisfying the property (0.7) were given in [4] and [5]. (See last sections of [4] and [5]).

Let's return to the present example. If π denotes the natural projection from $T^*(R_t \times R_x^4)$ to $R_t \times T^*R_x^4$ then the curve $\pi\tilde{C}_0$ is obtained as a limiting curve ($\varepsilon \rightarrow 0$) of admissible trajectories issuing from ε -conic neighborhood of ρ_0 (see Propositions 2.1 and 2.2 in § 2). If δ_∞ is the end point of $\pi\tilde{C}_0$ at $t = t_0$ and if \tilde{K} is the right hand side of (0.3) then we have the property (0.7) and furthermore for any $1 < \kappa < 3$ there exists an initial value G satisfying (0.3) such that

$$(0.8) \quad WF_\kappa U(t_0) \ni \delta_\infty ,$$

where U is the solution of (C.P) (see Theorem 2.3 in § 2). This fact shows that the description (0.5) is optimal for the present example if we consider the propagation of wave front sets in Gevrey class of order $1 < \kappa < 3$ instead of that of C^∞ wave front sets. (See also Corollary 2.5 in § 2).

Recently, Wakabayashi [11] has given a description about the propagation of C^∞ wave front sets of solutions of the Cauchy problem for symmetric hyperbolic systems, by using the work of Ivrii [3]. For hyperbolic systems of first order with diagonal principal part, the description of [11] is equivalent to (0.5). We remark that one direction of equivalence was proved in [11] (Theorem 4.4 of [11]) and another direction will be proved in the forthcoming paper [8] by K. Taniguchi and the author. The description of Wakabayashi is convenient to determine the set $\Gamma(t_0; \cdot)$ for the present example (see § 6). Finally we remark that the description (0.5) is also valid for wave front sets in Gevrey class if the symbol of hyperbolic system belongs to the corresponding Gevrey class. (See [8] and Wakabayashi [10]).

§1. Wave front set in Gevrey class

Let Ω be an open set in R^n and let $\kappa \geq 1$. We denote by $\gamma^{(\kappa)}(\Omega)$ the set of all functions in Gevrey class of order κ , i.e. $u \in C^\infty(\Omega)$ and for any compact K of Ω there exists a constant C such that for any multi-index α

$$(1.1) \quad \sup_{x \in K} |D^\alpha u(x)| \leq C^{|\alpha|+1} (\alpha!)^\kappa .$$

According to the Definition 3.1 of Hörmander [1] we define the wave front set in Gevrey class of order $\kappa \geq 1$.

DEFINITION 1.1. Let $(x^0, \xi^0) \in T^*R^n \setminus 0$ and let $u \in \mathcal{D}'$. We say that (x^0, ξ^0) is in the complement of the wave front set in Gevrey class of order κ (≥ 1) of u if and only if there exist an open neighborhood U_0 of x^0 , an open conic neighborhood Γ_0 of ξ^0 and a bounded sequence $\{u_N\} \subset \mathcal{E}'$ such that

$$(1.2) \quad \begin{cases} u_N = u & \text{in } U_0 \\ |\hat{u}_N(\xi)| \leq C^{N+1}(N!)^\kappa |\xi|^{-N}, & \xi \in \Gamma_0 \\ & N = 1, 2, \dots \end{cases}$$

for a constant $C > 0$ independent of N . We denote by $WF_\kappa u$ the wave front set in Gevrey class of order κ of u .

Remark. When $\kappa = 1$ the definition above is that of analytic wave front set.

In the same way as in the proof of Corollary 1.4 of Chapter V of Treves [9] we get the following lemma in view of (2.3) of [1].

LEMMA 1.2. The point $(x^0, \xi^0) \in T^*R^n \setminus 0$ does not belong to the wave front set in Gevrey class of order κ of $u \in \mathcal{D}'$ if and only if there exist an open neighborhood U_0 of x^0 and an open conic neighborhood Γ_0 of ξ^0 such that we have the following property: For any sequence of functions $\{h_N\}$ ($N = 1, 2, \dots$) in $C_0^\infty(U_0)$ such that

$$(1.3) \quad |D^\alpha h_N| \leq (C_1 N)^{c|\alpha|}, \quad |\alpha| \leq N$$

for a constant C_1 independent of N , we get

$$(1.4) \quad |(\widehat{h_N u})(\xi)| \leq C_2^{N+1} (N!)^\kappa |\xi|^{-N}, \quad \xi \in \Gamma_0$$

for a constant C_2 independent of N .

Therefore we get the following

COROLLARY 1.3. Suppose that $\kappa > 1$. Let $(x^0, \xi^0) \in T^*R^n \setminus 0$ and let $u \in \mathcal{D}'$. Assume that there exist an increasing sequence of constants $\{C_N\}$, a sequence $\{\zeta_N; \zeta_N \in R^n \setminus 0\}$ and a sequence of functions $\{h_N\}$ in C^∞ satisfying (1.3) ($N = 1, 2, \dots$) such that

$$(1.5) \quad \begin{cases} C_N \rightarrow \infty & (N \rightarrow \infty), \\ \zeta_N / |\zeta_N| \rightarrow \xi^0 / |\xi^0|, & |\zeta_N| \rightarrow \infty \quad (N \rightarrow \infty) \end{cases}$$

$$(1.6) \quad \text{supp } h_N \rightarrow \{x^0\} \quad (N \rightarrow \infty)$$

and

$$(1.7) \quad |(\widehat{h_N u})(\zeta_N)| \geq C_N^{N+1} (N!)^k |\zeta_N|^{-N}, \quad N = 1, 2, \dots$$

Then (x^0, ξ^0) belongs to $WF_x u$.

Remark. Let $\{N_k\}$ be a subsequence of $\{1, 2, 3, \dots\}$. Clearly we have the same conclusion even if we replace $\{N\}$ by $\{N_k\}$ in the corollary.

§2. Results

Let L be the hyperbolic system in $R_t \times R_x^4$ defined by (0.1) and (0.2). Let t_0 be any (but a fixed) positive. For an even integer $\mu = 2, 4, 6, \dots$, we put

$$(2.1) \quad \rho_\mu = (0, \gamma^\mu) \in T^*R_x^4, \quad \gamma^\mu = (-(t_0/2\mu)^4, 0, 0, 1).$$

PROPOSITION 2.1. *As admissible trajectories issuing from ρ_μ one can find only the following $\mu + 2$ trajectories; two bicharacteristic curves with respect to λ_1 and λ_2 , issuing from ρ_μ , and trajectories $C(J'_r, \tilde{t}'_r, \rho_\mu)$ of step $r = 1, \dots, \mu$. Here $J'_r = (1, 2, \dots, 1)$ or $(2, 1, 2, \dots, 1)$ according to r even or odd, and $\tilde{t}'_r = (t'_1, \dots, t'_r) = (\sigma_1, \dots, \sigma_r)$, $\sigma_p = (2q - 1)\beta$ ($q = 1, \dots, \mu$, $\beta = (t_0/2\mu)$). (See Fig. 1 and 2). Furthermore, for the trajectory $C_\mu = C(J'_\mu, \tilde{t}'_\mu, \rho_\mu)$ all hypotheses of Theorem 1 of [6] are verified with $\rho_0 = \rho_\mu$.*

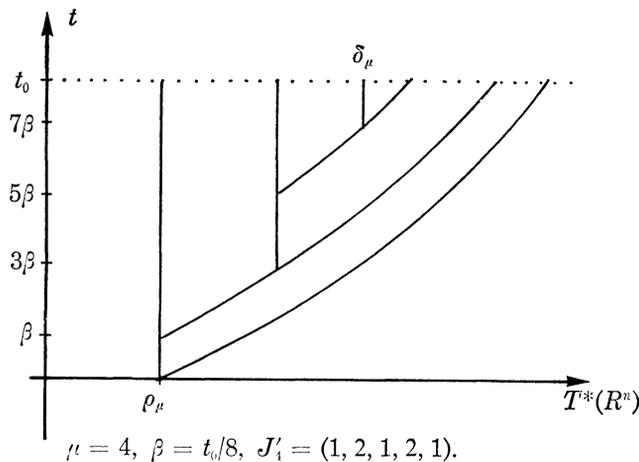


Fig. 1.

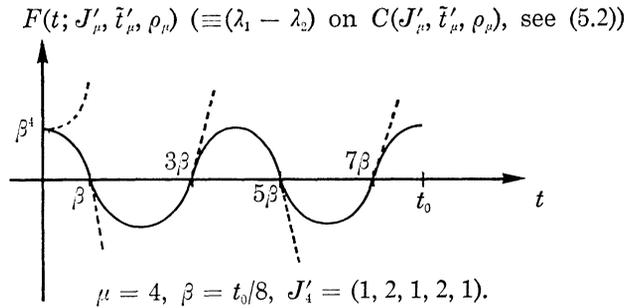


Fig. 2.

As the positive σ and the neighborhood V_0 in the hypothesis (H-3) we can take an arbitrary positive smaller than 1 and R_x^4 , respectively.

We denote by δ_μ the end point of the trajectory $C(J'_\mu, \tilde{t}'_\mu, \rho_\mu)$ and consider a sequence of end points $\{\delta_\mu\}$ ($\mu = 2, 4, 6, \dots$). Let $\tilde{\rho}_0 \in T^*(R_t \times R_x^4)$ be $(0, (0, 0, 0, 0, 1))$ and let \tilde{C}_0 be an integral curve through $\tilde{\rho}_0$ by the Hamilton field $H_{p_1 + p_2}$.

PROPOSITION 2.2. i) *There exists a unique limiting point $\delta_\infty = \lim_{\mu \rightarrow \infty} \delta_\mu$. The limiting point δ_∞ equals the end point of the curve $\pi\tilde{C}_0 \subset R_t \times T^*R_x^4$ at $t = t_0$, where π is the natural projection from $T^*(R_t \times R_x^4)$ to $R_t \times T^*R_x^4$.*

ii) *It follows that*

$$(2.2) \quad \tilde{C}_0 \subset \Sigma_1 \equiv \{\tilde{\rho} \in T^*(R_t \times R_x^4) \setminus 0; p_1(\tilde{\rho}) = p_2(\tilde{\rho}) = 0\}$$

and any Hamilton flow of H_{p_j} ($j = 1, 2$) passing \tilde{C}_0 intersects only once with osculation of fourth order.

iii) *Let \tilde{C}_0^+ be the part of the curve \tilde{C}_0 corresponding to $t \geq 0$ and let $B_{\tilde{\rho}_0}$ be a bundle of half hamilton flows by H_{p_j} ($j = 1, 2$) emanating from \tilde{C}_0^+ , that is,*

$$(2.3) \quad B_{\tilde{\rho}_0} = \bigcup_{j=1,2} \{(\exp sH_{p_j})\tilde{\rho}; s \geq 0, \tilde{\rho} \in \tilde{C}_0^+\}.$$

If \tilde{K} is a closed conic subset in $T^*R_x^4 \setminus 0$

$$(2.4) \quad \{c\rho_0 \equiv (0, c\eta^0); c > 0, \eta^0 = (0, 0, 0, 1)\}$$

then it follows that

$$(2.5) \quad \Gamma(t_0; \tilde{K}) = \{(x, c\xi) \in T^*R_x^4 \setminus 0; c > 0, (x, \xi) \in \pi B_{\tilde{\rho}_0}|_{t=t_0}\},$$

where $\pi B_{\tilde{\rho}_0}|_{t=t_0}$ is the section of $\pi B_{\tilde{\rho}_0} \subset R_t \times T^*R_x^4$ at $t = t_0$.

Let's admit these propositions, which will be proved in Section 5 and Section 6, respectively.

THEOREM 2.3. *If \tilde{K} is the closed conic set defined by (2.4) and $\delta_\infty = \lim_{\mu \rightarrow \infty} \delta_\mu$ then we get*

$$(2.6) \quad \delta_\infty \in \Gamma(t_0; \tilde{K}) \setminus \overline{\bigcup_{k=0}^\infty \Gamma_0^k(t_0; \tilde{K})}.$$

Furthermore, for any positive $1 < \kappa_0 < 3$ there exists an initial value $G \in \mathcal{E}'$ such that

$$(2.7) \quad WFG = WF_{\kappa_0}G = \tilde{K}$$

and

$$(2.8) \quad \delta_\infty \in WF_{\kappa'}U(t_0) \quad \text{for any } \kappa_0 \leq \kappa' < 3,$$

where U is the solution of the Cauchy problem (C.P).

COROLLARY 2.4. *For any $1 < \kappa_0 < 3$ there exists an initial value $G \in \mathcal{E}'$ satisfying (2.7) such that*

$$(2.9) \quad \tilde{C}_0 \subset WF_{\kappa'}U \quad (\subset T^*(R_t \times R_x^4) \setminus 0) \quad \text{for } \kappa_0 \leq \kappa' < 3,$$

where U is the solution of (C.P).

COROLLARY 2.5. *For any $1 < \kappa_0 < 3$ there exists an initial value $G \in \mathcal{E}'$ satisfying (2.7) such that*

$$(2.10) \quad WF_{\kappa'}U(t_0) = \Gamma(t_0; \tilde{K}) \quad \text{for any } \kappa_0 \leq \kappa' < 3,$$

where U is the solution of (C.P).

§3. Plan of the proof of Theorem 2.3

Put $\mu = 2N$ for $N = 1, 2, 3, \dots$ and $\eta_N = \eta^\mu$,

$$(3.1) \quad \eta_N = (-(t_0/4N)^t, 0, 0, 1).$$

We put

$$\phi(z) = (2\pi)^{-2} \exp\left(-\sum_{j=1}^4 z_j^2/2\right), \quad z \in \mathbf{C}^4$$

and

$$(3.2) \quad g(z) = \sum_{N=1}^\infty \tau_N^{-1} \phi(\tau_N^{-1}z) \exp i\tau_N z \cdot \eta_N,$$

where γ is a positive satisfying

$$(3.3) \quad 0 < \gamma < 1/2$$

and $\{\tau_N\}_{N=1}^\infty$ is an increasing sequence of positive numbers such that τ_{N+1}/τ_N is large enough to satisfy conditions demanded later on. Let $g(x)$ be the restriction of $g(z)$ to R^4 . The function $g(x)$ is also a modification of Example 2.3 of Hörmander [2]. In the similar manner as in [6] we get

$$(3.4) \quad WF g = WF_{\kappa_0} g = \{(0, c\eta^0); c > 0, \eta^0 = (0, 0, 0, 1)\},$$

where $\kappa_0 = 1/2\gamma > 1$. Indeed, note that $\phi(z) \exp iz \cdot \eta$ for $|\eta| \leq 2$ is bounded in an open set of C^4

$$\Omega_0 = \{z \in C^4; |\operatorname{Re} z|^2 > |\operatorname{Im} z|^2 + 4|\operatorname{Im} z|\}.$$

Since we may assume $\sum \tau_N^{-1} < \infty$ the sum of the right hand side of (3.2) converges uniformly in Ω_0 . Therefore $g(z)$ is analytic in Ω_0 and $g(x)$ is real analytic except the origin. Now we have

$$\hat{g}(\xi) = \sum_N \tau_N^{4(\gamma-1)-1} \hat{\phi}((\xi - \tau_N \eta_N) \tau_N^{\gamma-1}).$$

Since we have

$$(3.5) \quad \hat{\phi}(\xi) = \exp(-|\xi|^2/2) \geq 0$$

it follows that

$$\hat{g}(\tau_N \eta_N) \geq \tau_N^{4(\gamma-1)-1},$$

which shows

$$\{(0, c\eta^0); c > 0\} \subset WF g \subset WF_{\kappa_0} g.$$

Let V be an open conic neighborhood of η^0 . By the same way as in (2.17) of [6] we have for a positive constant C_V

$$(3.6) \quad |\xi - \tau_N \eta_N| \tau_N^{\gamma-1} \geq C_V |\xi|^\gamma$$

if $\xi \notin V$ and N is large enough. It follows from (3.5) that there exists a constant $c_0 > 0$ such that

$$(3.7) \quad |\hat{g}(\xi)| \leq c_0^{-1} \exp(-c_0 |\xi|^{2\gamma}) \quad \text{if } \xi \notin V,$$

which shows another direction of inclusion of (3.4).

We choose, as the initial value G of (C.P),

$$(3.8) \quad G(x) = {}^t(\psi_0(x)g(x), \psi_0(x)g(x)) ,$$

where $\psi_0(x)$ is a function in $\gamma^{(\kappa_0)} \cap C_0^\infty$ ($\kappa_0 = 1/2\gamma$) such that $0 \leq \psi_0 \leq 1$, $\psi_0(x) = 1$ for $|x| \leq R_0$. Here R_0 is a positive large number. Clearly we get $G \in \mathcal{E}'$ and

$$(3.9) \quad WF_{\kappa_0} G = WF G = \{(0, c\eta^0); c > 0, \eta^0 = (0, 0, 0, 1)\} .$$

We put $\delta_\mu = \delta_{2N} = (x_N, \xi_N)$ and $\delta_\infty = (x^0, \xi^0)$. Note that $\delta_\infty = \lim_{N \rightarrow \infty} \delta_{2N}$ by means of Proposition 2.2. Let $\{f_N(x)\}$ ($N = 1, 2, \dots$) be a sequence of functions in C_0^∞ such that $0 \leq f_N \leq 1$, $f_N = 1$ on $|x - x^0| \leq 1$, $f_N = 0$ except $|x - x^0| \leq 2$ and for any α

$$(3.10) \quad |D^\alpha f_N| \leq (CN)^{|\alpha|} , \quad |\alpha| \leq N$$

for a constant $C > 0$ independent of N . It is easy to construct $\{f_N\}$ (see Lemma 1.1 of Chapter V of [9]). Let $\{\beta_N\}_{N=1}^\infty$ be an increasing sequence of positive numbers such that $\beta_N \leq \log N$. If we put

$$h_N(x) = f_N(x^0 + \beta_N(x - x^0)) ,$$

the sequence $\{h_N\}$ satisfies conditions (1.3) and (1.6). We may assume that β_N is slowly increasing such that

$$(3.11) \quad h_N = 1 \quad \text{near } x_N .$$

Let $U = {}^t(u_1, u_2)$ be the solution of (C.P) for the G defined by (3.8) and (3.2). We shall show that for any N there exists a constant $c_1 > 0$ independent of N such that

$$(3.12) \quad |(\widehat{h_N u_1})(t_0, \zeta_N)| \geq c_1^{N+1} N^{3N} |\zeta_N|^{-N+4(\gamma-1)-1} ,$$

where $\zeta_N = \tau_N \xi_N$. Admitting this, by means of Corollary 1.3 we can obtain the last part of Theorem 2.3. The first part of Theorem 2.3 will be proved in Section 5 with Proposition 2.1.

§4. Proof of (3.12)

When we consider $h_N u_1(t_0, x)$, in view of (2.4) of [6] we may assume $G = {}^t(g, g)$ instead of (3.8). Indeed, we have

$$h_N(x)(1 - \psi_0(\pi_{J_\nu}(\tilde{t}_\nu)x)) = 0$$

if R_0 is large enough. By the same way as in getting (2.20) of [6] we have

$$(4.1) \quad \widehat{(h_N u_1)}(t_0; \tau_N \xi_N) = \sum_{N'} \tau_{N'}^{4(\gamma-1)-1} (w_{(1)}(\tau_N, \tau_{N'})) \\ + \sum_{\nu=1}^{\infty} \sum_{\substack{J_\nu \in \Pi_\nu^0 \\ j_1=1}} \int_{\mathcal{A}_\nu} w_{J_\nu}(\tau_N, \tau_{N'}; \tilde{t}_\nu) d\tilde{t}_\nu,$$

where $w_{J_\nu}(\tau_N, \tau_{N'}; \tilde{t}_\nu) = G_{J_\nu}(\tau_N, \tau_{N'}; \tilde{t}_\nu) \exp i\tau_N F_{J_\nu}(\tilde{t}_\nu)$ ($\nu \geq 0, J_0 = (1)$),

$$(4.2) \quad F_{J_\nu}(\tilde{t}_\nu) = -\xi_N \cdot d_{J_\nu}(\tilde{t}_\nu) \quad (\text{note } y^0 = 0)$$

and

$$(4.3) \quad G_{J_\nu}(\tau_N, \tau_{N'}; \tilde{t}_\nu) = \int \exp(iy \cdot (\tau_{N'} \eta_{N'} - \tau_N {}^t M_{J_\nu}(\tilde{t}_\nu) \xi_N) \tau_{N'}^{\gamma-1}) \\ \times h_N(M_{J_\nu}(\tilde{t}_\nu)(\tau_{N'}^{\gamma-1} y) + d_{J_\nu}(\tilde{t}_\nu)) \phi(y) (-i)^\nu dy.$$

Here we used $\beta_{J_\nu} = (-i)^\nu$ because $\det M_{J_\nu} = 1$. By the similar way as in (2.24) of [6] we get for a γ_0 satisfying $0 < \gamma_0 < \gamma$

$$(4.4) \quad |\tau_{N'} \eta_{N'} - \tau_N {}^t M_{J_\nu}(\tilde{t}_\nu) \xi_N| \tau_{N'}^{\gamma-1} \geq \tau_N^{\gamma_0}$$

if $|\tau_N - \tau_{N'}|$ is large enough. We remark that directions of $\eta_{N'}$ and ξ_N do not concern the derivation of (4.4). Integrating by parts with respect to y $[(N + 3 + 4(\gamma - 1))/\gamma_0]$ times, we obtain

$$(4.5) \quad |G_{J_\nu}(\tau_N, \tau_{N'}; \tilde{t}_\nu)| \leq C_N^\nu \tau_N^{-(N+2)+4(\gamma-1)} \\ \text{if } |\tau_N - \tau_{N'}| \text{ large enough,}$$

where C_N is a constant independent of ν and N' . Note that $\sum \tau_{N'}^{4(\gamma-1)-1} < +\infty$ and that the volume of \mathcal{A}_ν is $t_\nu^\nu/\nu!$. If we denote by $R(\tau_N)$ the sum with respect to $N' \neq N$ in the right hand side of (4.1), there exists a constant C'_N such that

$$(4.6) \quad |R(\tau_N)| \leq C'_N \tau_N^{-(N+2)+4(\gamma-1)}$$

if $|\tau_N - \tau_{N'}|$ for $N' \neq N$ is so large that the inequality (4.4) holds.

From now on we shall consider the term with $N' = N$ in the right hand side of (4.1), that is,

$$\tau_N^{4(\gamma-1)-1} \sum_{\nu=0}^{\infty} I_{J_\nu}(\tau_N), \quad (J_0 = (1), J_\nu \in \Pi_\nu^0 \text{ such that } j_1 = 1),$$

where $I_{J_\nu}(\tau)$ is defined by (2.26) of [6] with G_{J_ν} as follows:

$$(4.7) \quad G_{J_\nu}(\tau; \tilde{t}_\nu) = (-i)^\nu \int \exp i\tau^\gamma (\eta_{N'} - {}^t M_{J_\nu}(\tilde{t}_\nu) \xi_N) \cdot y \\ \times h_N(M_{J_\nu}(\tilde{t}_\nu)(\tau^{\gamma-1} y) + d_{J_\nu}(\tilde{t}_\nu)) \phi(y) dy.$$

Since it follows from Proposition 2.1 that, for the trajectory $C(J'_\mu, \tilde{t}'_\mu, \rho_\mu)$ ($\mu = 2N$), all hypotheses of Theorem 1 of [6] are satisfied, we can use the same method as in Section 3 of [6] and obtain the result corresponding to Lemma 2.1 of [6], that is; there exist two constant $c_\mu > 0$ and $C_\mu > 0$ independent of ν such that

$$(4.8) \quad |I_{J'_\mu}(\tau)| \geq c_\mu \tau^{-\mu/2} \quad \text{if } \tau \geq \tau(\mu)$$

and for any $J_\nu \neq J'_\mu$

$$(4.9) \quad |I_{J_\nu}(\tau)| \leq C_\mu \nu \tau^{-\mu/2 - \sigma\tau/4} / \nu! \quad \text{if } \tau \geq \tau(\mu),$$

where $\tau(\mu)$ is a large number depending on $\mu = 2N$. Here we used (3.11) to show $c_\mu > 0$ (see Remark 1 in § 3 of [6]).

Since $\mu = 2N$, the conjunction of (4.6), (4.8) and (4.9) shows that there exists a constant $C''_N > 0$ such that

$$(4.10) \quad |\widehat{(h_N u_1)}(t_0; \tau_N \xi_N)| \geq (c_{2N} - C''_N \tau_N^{-\sigma\tau/4}) \tau_N^{-N+4(r-1)-1}$$

if $\{\tau_N\}$ satisfies

$$(4.11) \quad \begin{cases} \tau_N \geq \tau(2N) \\ |\tau_N - \tau_{N'}| \text{ (} N \neq N' \text{)} \end{cases} \quad \text{large enough to get (4.4).}$$

If we choose $\{\tau_N\}$ furthermore such that

$$(4.12) \quad c_{2N} - C''_N \tau_N^{-\sigma\tau/4} \geq c_{2N}/2,$$

for the proof of (3.12) it remains to show that for any N there exists a constant $c_1 > 0$ independent of N such that

$$(4.13) \quad c_{2N} = c_1^N N^{3N} \quad (\text{or } c_\mu = c_1^{\mu/2} (\mu/2)^{3\mu/2}).$$

In consideration of the formula (3.35) of [6] and Remark 1 of Section 1 of [6], it suffices to show that for any $\mu = 2N$ there exists a constant $c_2 > 0$ independent of μ such that

$$(4.14) \quad c_2^{-\mu} \mu^{-3\mu} \leq |\det(\partial_{t_p} \partial_{t_q} \xi_N \cdot d_{J'_\mu}(\tilde{t}'_\mu))| \leq c_2^\mu \mu^{-3\mu}.$$

This inequality will be proved in the next section.

§ 5. Proof of Proposition 2.1

If H_j denotes the Hamilton vector field of p_j , that is, $H_j = H_{p_j}$ ($j = 1, 2$) then it follows that

$$(N.5) \quad H_{j_1} H_{j_2} \cdots H_{j_s} p_{j_s} = 0, \quad j_k \in \{1, 2\}.$$

The condition (N.m) with integer $m \geq 1$ was already defined in Introduction of [4], to which we refer the reader about motivation of the definition and results under the condition with $m \leq 3$. By the same way as in the proof of Lemma 2.1 of [4] we get

LEMMA 5.1. *If the condition (N.5) is verified then $H_{j_1} H_{j_2} \cdots H_{j_s} p_{j_s}$ ($j_k \in \{1, 2\}$) are constants along any trajectory.*

By this lemma we see that along any trajectory issuing from ρ_μ

$$(5.1) \quad \begin{cases} H_1^4 p_2 = H_2^4 p_1 = H_1^2 H_2^2 p_1 = H_2^2 H_1^2 p_2 = 24 \\ H_2 H_1^3 p_2 = H_1 H_2^3 p_1 = H_1 H_2 H_1^2 p_2 = H_2 H_1 H_2^2 p_1 = -24. \end{cases}$$

Let $\{(t, x(t), \xi(t)); t \in [0, t_0]\} \subset R_t \times T^*R_x^4$ be a trajectory $C(J_\nu, \tilde{t}_\nu, \rho_\mu)$. Then $H_1^3 p_2(t, x(t), \xi(t))$, $-H_2^3 p_1(t, x(t), \xi(t))$ and $H_1 H_2^2 p_1(t, x(t), \xi(t))$ ($\equiv -H_2 H_1^2 p_2(t, x(t), \xi(t))$),

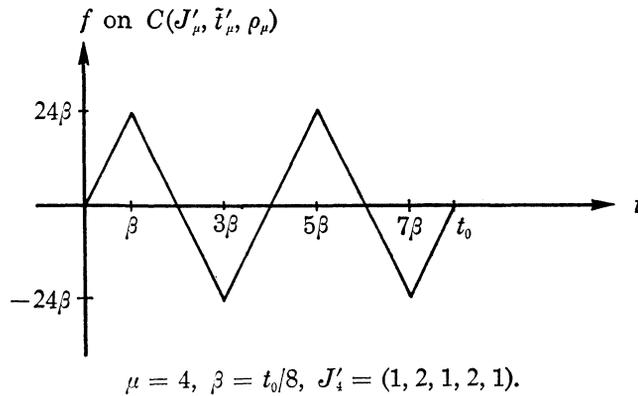


Fig. 3.

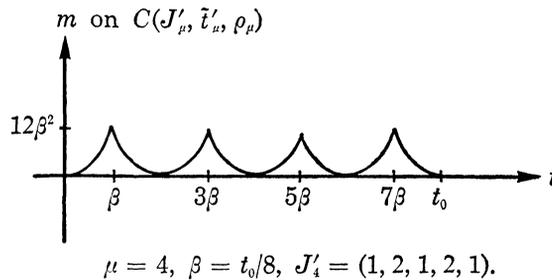


Fig. 4.

$\xi(t)$, which follows from the Jacobi identity) are equal to $f(t)$, a continuous piecewise linear function such that $f'(t)$ on (t_q, t_{q-1}) is 24 or -24 according to $j_q = 1$ or 2 and $f(0) = 0$. Indeed, by means of (5.1) we can see, for example, that $d(H_1^3 p_2)/dt$ on (t_q, t_{q-1}) is equal to $H_1^4 p_2 = 24$ or $H_2 H_1^3 p_2 = -24$ according to $j_q = 1$ or 2. Clearly $H_1^3 p_2(0, \rho_\mu) = 0$ and it follows from (N.5) that $d^2(H_1^3 p_2)/dt^2 = 0$. (See Fig. 3).

On the other hand, it follows from (N.5) and (5.1) that $H_1^2 p_2(t, x(t), \xi(t))$ and $H_2^2 p_1(t, x(t), \xi(t))$ are equal to $m(t)$ a continuous piecewise quadratic function such that $m(0) = 0$, $m'' = 24$ and m' on (t_q, t_{q-1}) is equal to f or $-f$ according to $j_q = 1$ or 2. (See Fig. 4)

If we put

$$(5.2) \quad F(t; J, \tilde{t}, \rho_\mu) = (\lambda_1 - \lambda_2)(t, x(t), \xi(t))$$

for the trajectory $C(J, \tilde{t}, \rho_\mu) = \{(t, x(t), \xi(t)); t \in [0, t_0]\}$ then F is a continuously differentiable function of degree 1 and piecewise polynomials of order 4 such that

$$(5.3) \quad \begin{cases} F'''' = 24 \text{ or } -24, & F''' = -f \text{ or } f \\ F'' = -m \text{ or } m \text{ on } (t_q, t_{q-1}) & \text{according to } j_q = 1 \text{ or } 2, \end{cases}$$

because $d^j F/dt^j$ ($j = 1, 2, \dots$) are equal to $H_1^j(p_1 - p_2) = -H_1^j p_2$ or $H_2^j(p_1 - p_2) = H_2^j p_1$ on (t_q, t_{q-1}) according to $j_q = 1$ or 2 and because we have $H_1 p_2 = -H_2 p_1$ trivially.

Noting that $H_1 p_2(0, \rho_\mu) = 0$ and $(\lambda_1 - \lambda_2)(0, \rho_\mu) = \beta^4$ ($\beta = t_0/2\mu$) we get

$$(5.4) \quad \begin{cases} F_-(t) \equiv F(t; (1), \emptyset, \rho_\mu) = \beta^4 - t^4 \\ F_+(t) \equiv F(t; (2), \emptyset, \rho_\mu) = \beta^4 + t^4 \end{cases} \quad (\text{see Fig. 2}),$$

where we denote by $C((1), \emptyset, \rho_\mu)$ and $C((2), \emptyset, \rho_\mu)$ two bicharacteristic curves with respect to λ_1 and λ_2 , respectively. Furthermore, for $r = 1, \dots, \mu$, we have

$$(5.5) \quad \begin{aligned} F_r(t) \equiv F(t; J_r', \tilde{t}_r', \rho_\mu) &= (-1)^{r-q}(\beta^4 - (t - 2(r - q)\beta)^4) \\ &\text{on } t \in [t_q^r, t_{q-1}^r] \quad (q = 1, \dots, r + 1, t_{r+1}^r = 0, t_0^r = t_0). \end{aligned}$$

(See Fig. 2).

By means of (5.3)-(5.5) we get the first part of Proposition 2.1. The hypothesis (H-1) of Theorem 1 of [6] follows from (5.5). By the similar way as in getting Proposition 5.2 of [4], in view of (5.3)-(5.5) we get (5.3) of [4], which shows that the hypothesis (H-3) of Theorem 1 of [6] is satisfied.

As the σ of (H-3) we can take an arbitrary positive smaller than 1 because constant times ε is inferior to ε^σ for small $\varepsilon < 1$. As the V_0 of (H-3) we can take R_x^t because the ξ variables of end points of admissible trajectories issuing from ρ_μ are different each other, which is easily checked by the direct calculation if we take notice of the ξ_s component.

Before the check of the hypothesis (H-2) of Theorem 1 we shall show the first part of Theorem 2.3. By the same way as in the last paragraph of Section 7 of [4], noting that $(\lambda_1 - \lambda_2)(0, \rho_0) = 0$ ($\rho_0 = (0, \eta^0)$) we have by (5.3) and (5.4)

$$(5.6) \quad \overline{\bigcup_k \Gamma_0^k(t_0; \tilde{K})} = \overline{\Gamma_0^0(t_0; \tilde{K})} = \Gamma_0^0(t_0; \tilde{K})$$

for the set \tilde{K} defined by (2.4). We get

$$(5.7) \quad \delta_\infty \notin \Gamma_0^0(t_0; \tilde{K}),$$

because $\lambda_1 - \lambda_2 = 0$ at (t_0, δ_∞) and it is not zero on $\Gamma_0^0(t_0; \tilde{K})$.

Now we shall check the hypothesis (H-2), that is,

$$(4.14)' \quad c_2^{-\mu} \mu^{-3\mu} \leq |\det(\partial_{t_p} \partial_{t_q} \xi^\mu \cdot d_{J'_\mu}(\tilde{t}'_\mu))| \leq c_2^\mu \mu^{-3\mu}$$

for a constant $c_2 > 0$ independent of μ . Here we putted $\delta_\mu = (x^\mu, \xi^\mu)$ ($= \delta_{2N} = (x_N, \xi_N)$). We recall that $F_{J'_\mu}(\tilde{t}'_\mu) = -\xi^\mu \cdot d_{J'_\mu}(\tilde{t}'_\mu)$. By means of (3.13) and (3.14) of [6] with $y^0 = 0, \eta^0 = \eta^\mu$ and $\xi^0 = \xi^\mu$ we get

$$(5.8) \quad \partial_{t_q} F_{J'_\mu} = (\lambda_{j_q} - \lambda_{j_{q+1}})(t_q, x^q, \xi^q) + ({}^t M_{J'_\mu}^{-1} \eta^\mu - \xi^\mu) \cdot \partial_{t_q} d_{J'_\mu}.$$

Differentiating with respect to t_p and putting $\tilde{t}_\mu = \tilde{t}'_\mu$ we have

$$(5.9) \quad \begin{aligned} a_{p,q} &\equiv \partial_{t_p} \partial_{t_q} F_{J'_\mu}(\tilde{t}'_\mu) \\ &= (t_0^3/2\mu^3) \delta_{p,q} + (\partial_{t_p} ({}^t M_{J'_\mu}^{-1} \eta^\mu) \cdot \partial_{t_q} d_{J'_\mu})(\tilde{t}'_\mu) \quad \text{if } p \leq q, \end{aligned}$$

where we used $(dF(t; J'_\mu, t'_\mu, \rho_\mu)/dt)|_{t=(2\mu-2q+1)\beta} = (-1)^{q-1} 4\beta^3$ to obtain the derivative of the first term of the right hand side of (5.8). Here $\delta_{p,q}$ is the δ of Kronecker. Since the direct calculation shows that the second term of the right hand side of (5.9) is equal to $(-1)^{q-1}$ multiplied by a constant independent of q , we get by (5.9)

$$(5.10) \quad a_{p,q} + a_{p,q+1} = (t_0/2\mu^3) \delta_{p,q} \quad \text{if } p \leq q.$$

Furthermore, the direct calculation gives

$$(5.11) \quad a_{1,1} = (4\mu - 1)t_0^3/2\mu^3, \quad a_{p-1,p} + a_{p,p} = -3t_0^3/2\mu^3, \quad 2 \leq p \leq \mu.$$

If we denote by T_μ a $\mu \times \mu$ matrix

$$\begin{pmatrix} 1 & 0 & & 0 \\ 1 & 1 & \cdot & \\ & 1 & \ddots & \\ 0 & & \cdot & 1 & 0 \\ & & & 1 & 1 \end{pmatrix}$$

then it follows from (5.10), (5.11) and $a_{p,q} = a_{q,p}$ that

$$T_\mu(a_{p,q})^t T_\mu = (t_0/2\mu^3)^\mu S_\mu.$$

Here we denoted by S_q for $1 \leq q \leq \mu$, $q \times q$ matrix of Jacobi

$$\begin{pmatrix} b_1 & d_2 & \cdot & 0 \\ d_2 & b_2 & \cdot & \\ & \cdot & \cdot & d_q \\ 0 & & d_q & b_q \end{pmatrix} \quad \begin{matrix} b_1 = 4\mu - 1, \\ b_q = -2, \quad d_q = 1, \\ q \geq 2. \end{matrix}$$

If we put $\square_q = \det S_q$ then we have

$$\begin{aligned} (5.12) \quad \square_q &= b_q \square_{q-1} - d_q^2 \square_{q-2} \\ &= -2 \square_{q-1} - \square_{q-2} \quad \text{if } q \geq 3, \end{aligned}$$

which shows

$$(5.13) \quad \det(a_{p,q}) = (t_0/2\mu^3)^\mu (1 - 4\mu^2).$$

Thus we obtain (4.14)′.

§6. Proof of Proposition 2.2

It is easy to see that when $\mu \rightarrow \infty$ the trajectory C_μ given in Proposition 2.1 converges the part of $\pi\tilde{C}_0$ corresponding to $t \in [0, t_0]$. Then the assumption i) of Proposition 2.2 is obvious. Since it follows from (5.5) that the difference between λ_1 and λ_2 is estimated by $\beta^t = (t_0/2\mu)^t$ on the trajectory C_μ we get (2.2). By means of the argument in the previous section it is not difficult to see the rest of the assertion ii) of Proposition 2.2 because $H_k^j(\lambda_1 - \lambda_2) = 0$ ($k = 1, 2, j = 0, 1, 2, 3$) on $\pi\tilde{C}_0$.

As stated in Introduction, for the proof of the assertion iii) of Proposition 2.2 we need to review the result of Wakabayashi [11] and [10], where the propagation of C^∞ wave front sets and that of wave front sets in Gevrey classes, respectively, were studied for the hyperbolic Cauchy problem. If we apply the result of [11] (resp. [10]) to the Cauchy problem

for any hyperbolic system of the form (0.1), we get the following description about C^∞ wave front set (resp. the one in Gevrey class) of solutions: If U and G denote the solution and the initial value of the Cauchy problem, respectively, then we get

$$(6.1) \quad WF U \subset \{K_\rho^+; \tilde{\rho} \in p^{-1}(0) \cap \pi^{-1}(\{0\} \times WF G)\},$$

$$(6.1)' \quad WF_{\kappa_1} U \subset \{K_\rho^+; \tilde{\rho} \in p^{-1}(0) \cap \pi^{-1}(\{0\} \times WF_{\kappa_1} G)\} \\ (1 < \kappa_1 < 2),$$

where

$$p^{-1}(0) = \bigcup_{j=1,2} p_j^{-1}(0), \quad p_j^{-1}(0) = \{\tilde{\rho} \in T^*(R_t \times R_x^n) \setminus 0, p_j(\tilde{\rho}) = 0\}$$

and K_ρ^+ is defined as follows:

$$(6.2) \quad K_\rho^+ = \{\tilde{\rho}(s) \in T^*(R_t \times R_x^n); s \geq 0 \text{ and } \tilde{\rho}(s) \text{ is a Lipschitz} \\ \text{continuous curve satisfying } d\tilde{\rho}(s)/ds \in \Gamma_\rho^+(s) \text{ for} \\ \text{a.e. } s \text{ and } \tilde{\rho}(0) = \tilde{\rho}\}. \quad (\text{See (1.2) of [11]}).$$

Here Γ_ρ^+ is the set of vector fields in $T^*(R_t \times R_x^n)$ defined by

$$(6.3) \quad \Gamma_\rho^+ = \{(\sum_{j \in \mathfrak{S}} \alpha_j H_{p_j})(\tilde{\rho}); \alpha_j \geq 0\} \quad \text{if } \tilde{\rho} \in \bigcap_{j \in \mathfrak{S}} p_j^{-1}(0),$$

where \mathfrak{S} is a subset of $\{1, 2\}$.

Noting that $WF U \subset p^{-1}(0) \subset T^*(R_t \times R_x^n) \setminus 0$ we can see that the description (6.1) is equivalent to (0.5) in Introduction because we have the following:

PROPOSITION 6.1. *For any closed conic set $\tilde{K} \subset T^*R_x^n \setminus 0$ we get*

$$(6.4) \quad \Gamma(t_0; \tilde{K}) = \{\pi K_\rho^+|_{t=t_0}; \tilde{\rho} \in \pi^{-1}(\{0\} \times \tilde{K}) \cap p^{-1}(0)\},$$

where $\pi K_\rho^+|_{t=t_0}$ is the section of $\pi K_\rho^+ \subset R_t \times T^*R_x^n$ at $t = t_0$.

The inclusion relation that the left hand side contains the right hand side was proved in [11] (see Theorem 4.4 of [11]) and another direction of inclusion will be proved in the forthcoming paper [8] by K. Taniguchi and the author.

On account of this proposition, for the proof of the assertion iii) of Proposition 2.2 it suffices to show

$$(6.5) \quad B_{\rho_0} = K_{\rho_0}^+.$$

In the similar manner as in the observation of [11] for a simple example of hyperbolic system given by [4] (see Example 4.9 of [11]) we get the following: Since $\Sigma_1 \equiv p_1^{-1}(0) \cap p_2^{-1}(0) = \{\tilde{\rho} \in T^*(R_t R_x^4) \setminus 0; \lambda_2(\pi\tilde{\rho}) = 0\}$ we get

$$(6.6) \quad T_{\tilde{\rho}}\Sigma_1 \cap \Gamma_{\tilde{\rho}}^\sigma = \begin{cases} \Gamma_{\tilde{\rho}}^\sigma & \text{if } \partial_t \lambda_2(\pi\tilde{\rho}) = 0 \\ \{0\} & \text{otherwise,} \end{cases}$$

because it follows from $(\alpha_1 H_{p_1} + \alpha_2 H_{p_2})(\tilde{\rho}) \in T_{\tilde{\rho}}\Sigma_1$ that $(\alpha_1 + \alpha_2)\partial_t \lambda_2(\pi\tilde{\rho}) = 0$. Put $\Sigma_2 = \{\tilde{\rho} \in \Sigma_1; \partial_t \lambda_2 = 0\}$. Then we get

$$(6.7) \quad T_{\tilde{\rho}}\Sigma_1 \cap \Gamma_{\tilde{\rho}}^\sigma = \begin{cases} \Gamma_{\tilde{\rho}}^\sigma & \text{if } t^2 - 4x_1 t + 8x_2 = 0 \\ \{\alpha(H_{p_1} + H_{p_2})(\tilde{\rho}); \alpha \geq 0\} & \text{otherwise,} \end{cases}$$

because it follows from $(\alpha_1 H_{p_1} + \alpha_2 H_{p_2})(\tilde{\rho}) \in T_{\tilde{\rho}}\Sigma_2$ that

$$(\alpha_1 - \alpha_2)(t^2 - 4x_1 t + 8x_2) = 0.$$

Putting $\Sigma_3 = \{\tilde{\rho} \in \Sigma_2; t^2 - 4x_1 t + 8x_2 = 0\}$ we have

$$(6.8) \quad T_{\tilde{\rho}}\Sigma_3 \cap \Gamma_{\tilde{\rho}}^\sigma = \begin{cases} \Gamma_{\tilde{\rho}}^\sigma & \text{if } t = 2x_1 \\ \{\alpha H_{p_1+p_2}(\tilde{\rho}); \alpha \geq 0\} & \text{otherwise,} \end{cases}$$

because we have $(\alpha_1 - \alpha_2)(t - 2x_1) = 0$ by the same argument. If we put $\Sigma_4 = \{\tilde{\rho} \in \Sigma_3; t = 2x_1\}$ then we have

$$(6.9) \quad T_{\tilde{\rho}}\Sigma_4 \cap \Gamma_{\tilde{\rho}}^\sigma = \{\alpha H_{p_1+p_2}(\tilde{\rho}); \alpha \geq 0\}.$$

In view of (6.6)-(6.9) we obtain (6.5), noting the assertion ii) of Proposition 2.2.

§7. Proofs of Corollaries 2.4 and 2.5

If U is the solution of (C.P) it follows from Theorem 5.1 of [1] that

$$WF_x U \subset p^{-1}(0) \equiv \bigcup_{j=1,2} p_j^{-1}(0) \quad \text{for } \kappa \geq 1.$$

If $\delta(t') \in T^*R_x^4 \setminus 0$ denotes the end point of $\pi\tilde{C}_0 \subset R_t \times T^*(R_x^4)$ at $t = t'$, for the proof of Corollary 2.4 it suffices to show for any t'

$$(7.1) \quad \delta(t') \in WF_x U(t') \quad \text{for } \kappa_0 \leq \kappa' < 3$$

because $\tilde{C}_0 = \pi^{-1}(\pi\tilde{C}_0) \cap p^{-1}(0)$.

Let t_0 be a fixed positive and let G be the initial value defined by (3.8) and (3.2). Unfortunately, G depends on t_0 . However, if we choose the sequence $\{\tau_N\}$ suitably we have (7.1) for any t' of a countable set

$$(7.2) \quad \{t'_j; j = 1, 2, \dots, t'_j = (r'_j/r_j)t_0, r_j, r'_j > 0 \text{ integers}\}$$

Indeed: For g of (3.2) we put

$$(7.3) \quad \begin{cases} g(x) = \Sigma\phi_N(x, \tau_N) \\ \phi_N(x, \tau) = \tau^{-1}\phi(\tau^{-1}x) \exp i\tau x \cdot \eta_N \\ \eta_N \equiv \eta_N(t_0) = (-t_0/4N)^t, 0, 0, 1. \end{cases}$$

Note that for a $t' = (r'/r)t_0$ ($r, r' > 0$ integers) we have $\eta_N(t_0) = \eta_{(Nr'/r)}(t')$. Put $\tilde{N} = Nr'/r$ for $N = rk$ ($k = 1, 2, \dots$). Let $U^{\tilde{N}} = {}^t(u_1^{\tilde{N}}, u_2^{\tilde{N}})$ denote the solution of (C.P) for the initial value

$${}^t(\psi_0(x)\phi_N(x, \tau_N), \psi_0(x)\phi_N(x, \tau_N)) \quad \text{with } N = \tilde{N}r/r',$$

where $\psi_0(x)$ is the cut function given in (3.8). If $(x_{\tilde{N}}, \xi_{\tilde{N}})$ is the end point of admissible trajectories issuing from $(0, \eta_{\tilde{N}})$ ($= (0, \eta_N)$) of step $2\tilde{N}$, then by the same way as in the proof of (3.11) we see that there exists a constant $c_3 > 0$ independent of \tilde{N} such that

$$(7.4) \quad |(\widehat{h_N u_1^{\tilde{N}}})(t'; \tau \xi_{\tilde{N}})| \geq c_3^{\tilde{N}+1} \tilde{N}^{3\tilde{N}} \tau^{-\tilde{N}+4(\gamma-1)-1} \quad \text{if } \tau \geq \tau(\tilde{N}, t'),$$

where $\tau(\tilde{N}, t')$ is a large number determined in the same manner as for $\tau(2N)$ of (4.8) and (4.9). Recall that we have (4.6) only if $|\tau_N - \tau_{N'}|$ for $N \neq N'$ is large enough to hold (4.4). If the sequence $\{\tau_N\}$ satisfies

$$(7.5) \quad \tau_N \geq \tau(\tilde{N}, t') \quad \text{for } N = rk \text{ (} k = 1, 2, 3, \dots \text{)}$$

then there exists a constant $C_{\tilde{N}} > 0$ such that

$$(7.6) \quad |(\widehat{h_N u_1})(t'; \tau_N \xi_{\tilde{N}})| \geq (c_3^{\tilde{N}+1} \tilde{N}^{3\tilde{N}} - C_{\tilde{N}} \tau_N^{-1}) \tau_N^{-\tilde{N}+4(\gamma-1)-1},$$

where $U = {}^t(u_1, u_2)$ is the solution of (C.P) for the G defined by (3.8) and (3.2). If we assume that $\tau(\tilde{N}, t')$ is large enough to hold for τ_N satisfying (7.5)

$$c_3^{\tilde{N}+1} \tilde{N}^{3\tilde{N}} - C_{\tilde{N}} \tau_N^{-1} \geq c_3^{\tilde{N}+1} \tilde{N}^{3\tilde{N}}/2$$

it follows from Corollary 1.3 and its remark that we have (7.1) for $t' = (r'/r)t_0$. If we choose the sequence $\{\tau_N\}$ such that

$$\tau_N \geq \max_{1 \leq j \leq N} \tau(\tilde{N}_j, t'_j)$$

we get (7.1) for any $t' \in \{t'_j\}_{j=1}^{\infty}$. Here we putted $\tau(\tilde{N}_j, t'_j) = 0$ if $\tilde{N}_j = Nr'_j/r_j$ is not integer.

Since $\{t'_j\}$ is dense in R^+ we have (7.1) for any $t' > 0$ because $WF_x U$ is closed set in $T^*(R_t \times R_x^4) \setminus 0$. Since we can obtain the same result as Theorem 2.3 for negative t_0 , we get (7.1) for any t' .

In the rest of this section we shall prove Corollary 2.5. We first remark that if U and G are the solution and the initial value, respectively, of the Cauchy problem for any hyperbolic system of the form (0.1) then it follows that for any $\kappa > 1$

$$WF_x U(t_0) \subset \Gamma(t_0; WF_x G), \quad t_0 > 0.$$

In view of Proposition 6.1 the inclusion with $1 < \kappa < 2$ follows from (6.1)'. The case for $\kappa \geq 2$ will be proved in [8].

For a fixed $t_0 > 0$ let t' be a positive such that $0 < t' < t_0$. Let \tilde{C}_1 be the continuous curve in $T^*(R_t \times R_x^4)$ such that

$$\tilde{C}_1 = \begin{cases} \tilde{C}_0 & \text{for } 0 \leq t \leq t' \\ \text{Hamilton flow by } H_{p_j} & (j = 1, 2) \text{ for } t' \leq t \leq t_0. \end{cases}$$

If $\tilde{\delta} \in T^*R_x^4 \setminus 0$ denotes the end point at $t = t_0$ of the curve $\pi\tilde{C}_1$ then it follows that

$$(7.7) \quad \tilde{\delta} \in WF_{\kappa'} U(t_0), \quad \kappa_0 \leq \kappa' < 3.$$

For the proof of (7.7) it suffices to consider the sequence of admissible trajectories issuing from ρ_μ which converges the curve $\pi\tilde{C}_1$ and to find an initial value G corresponding to the sequence of admissible trajectories, as in the proof of Theorem 2.3. Though the initial value G depends on $\tilde{\delta} \in \Gamma(t_0; \tilde{K})$ we can complete the proof of Corollary 2.5 by the same argument as in the proof of Corollary 2.4.

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