TWO-DIMENSIONAL NUMERICAL MODELS OF THE BOUNDARY LAYER OF ACCRETION DISKS IN CATACLYSMIC VARIABLES

W. KLEY and G. HENSLER
Institut für Astronomie und Astrophysik der Universität
München, München, F.R.G.

ABSTRACT. Nearly half of the total available accretion energy can be released in the boundary layer (BL) if the accreting object is slowly rotating. The spectral distribution of the emitted radiation depends crucially on the internal structure of the BL. Up to now no detailed models concerning the BL exist. We have developed an explicit two-dimensional numerical method written for axisymmetric accretion flows including viscosity effects and angular momentum. We display our first models concerning the BL structure incorporating variation of the stellar rotation and of the fraction of the released energy. The first results show a strong dependence of the BL structure on the local rate of cooling and on the rotation of the primary.

1. INTRODUCTION

Theoretical models of accretion disks in Cataclysmic Variables (CV) are usually one-dimensional and based on the so-called α-prescription (Shakura & Sunyaev, 1973). In this "standard model" the disk rotates almost Keplerian and is thin (H<<R). If the accreting (primary) star is non-magnetic the matter grazes its surface at the inner edge of the accretion disk and the disk matter has to be decelerated to stellar rotational velocity. In case of slow rotation as it is usually assumed up to half of the total available accretion energy is dissipated in a very small region close to the primary the so-called boundary layer.

The spectral distribution of this energy depends crucially on the BL structure and the energy transfer processes in turn may determine the global BL structure (King and Shaviv, 1984). In addition this high energy release in the vicinity of the white dwarf may cause the observed winds in Dwarf Novae at outburst (Klare et al., 1982; Cordova and Mason, 1982).

In theoretical considerations the BL is expected to be optically thin for low $\dot{M}$ (Pringle and Savonije, 1979) and will then radiate primarily thermal bremsstrahlung in the hard X-rays (Tylanda, 1981) and should be optically thick at high M values with a stronger soft X-ray radiation.

The observations are in general compatible with this ideas but the

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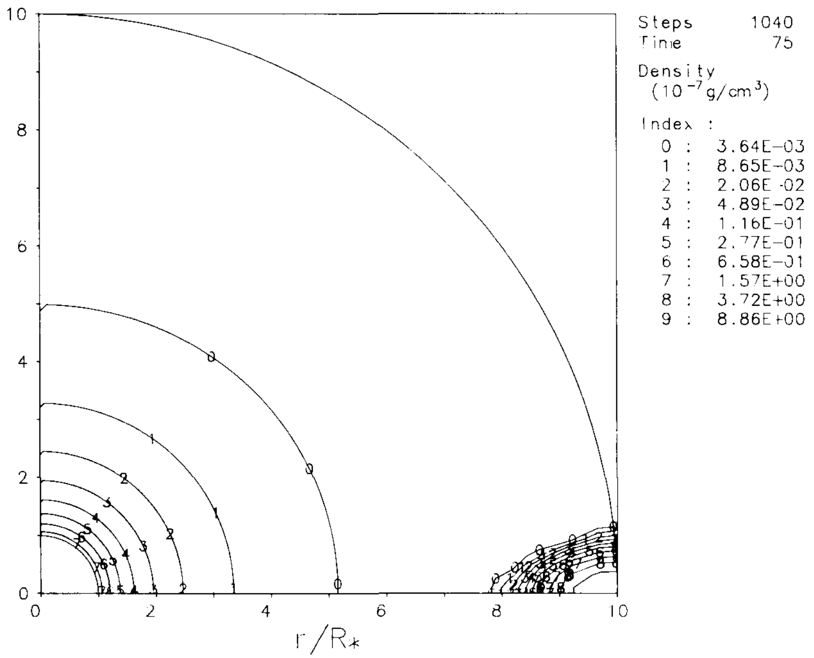

Fig. 1. Quasi-initial density configuration of the BL models.

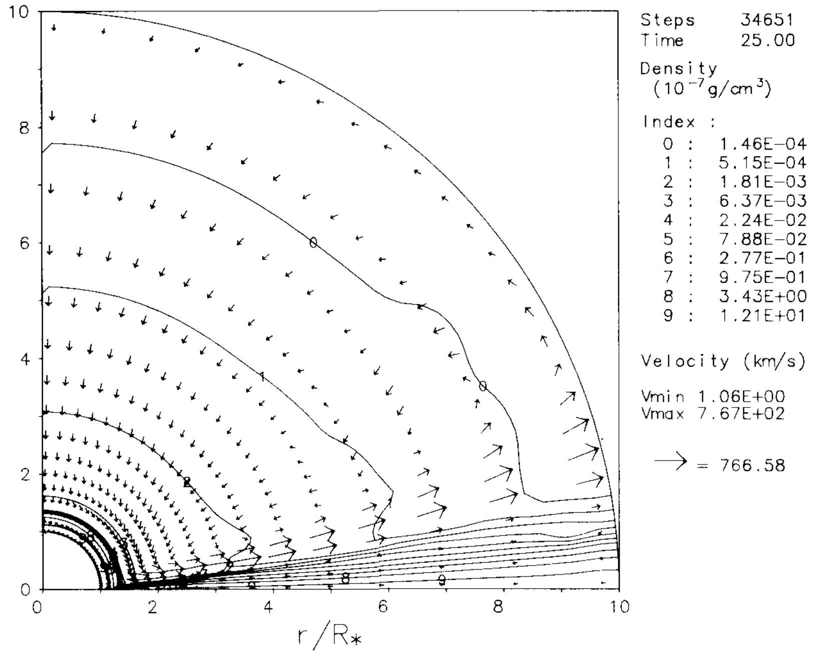

Fig. 2. Model at t=25. Linear scaled velocity vectors, logarithmically spaced contours.

observed soft X-ray luminosity is according to Ferland et al. (1982) 10^2 to 10^4 times weaker than that inferred from the simple standard model of the BL as cited above.

Despite the importance of the Bl structure on several features (spectrum, winds, accretion mechanism, spin-up, etc) up to now nearly no numerical effort has been spent on the investigation of the BL structure. Only recently, a numerical method based on the flux-corrected transport algorithm FCT (Boris and Book,1973) has been developed by Robertson and Frank (1985). They follow the evolution of equilibrium torii under the influence of friction.

We have developed a general explicit Eulerian two-dimensional hydrodynamic code devised for axisymmetric flows including viscosity effects and angular momentum. The method is based on the second-order method of van Leer (1977). Although radiative transfer is not included yet the first results show a strong dependence of the BL structure on the local rate of cooling and on the primary´s rotation.

2. ASSUMPTIONS

Since friction plays an important role we have to use the full set of hydrodynamic equations including the viscosity terms. With no knowledge of the origin of the rather high viscosity in accretion disks we make here the assumption that the kinematic viscosity coefficient $\nu = \eta/\rho$ is constant. As equation of state we use the ideal gas law.

In the energy equation we have omitted the energy transport terms and any radiative losses. In order to investigate roughly the influence of the energy balance, the model calculations are performed either taking the total dissipated energy into account (trapped frictional energy) or by setting the dissipation function equal to zero which allows for an instantaneous emission of frictional energy (adiabatic gas). In this paper we don´t go into the details of the numerical method of solution which is described with full particulars in our paper (Kley and Hensler, 1986) together with the testruns performed with the code.

3. RESULTS

The parameter set which is altered for different models consists of the fraction $f_\star$ of the stellar rotation to Keplerian velocity (at the surface), and the fraction f_Φ of the dissipated energy that is trapped in the system. The star is treated as a rigid body, i.e. no material inflow is allowed. For the mass and radius of the central White Dwarf we adopt $M_\star = 1\ M_\odot$, $R_\star = 10^9$ cm. The boundary and initial conditions are also described in detail in Kley & Hensler (1986). In the following we present some of our models with various parameters. The figures show a cut through the meridional plane where the unit of time is 200 sec.

Model 1 (f_Φ=0, $f_\star$=0)
From the inital set-up (Fig. 1) the inflowing disk matter moves inwards while losing angular momentum, reaches the stellar surface at t≈10 and evolves after t=25 to the configuration shown in Fig. 2. A thin disk has been formed as a consequence of the instantaneously emitted radiation and the star is enveloped by a shell of rather dense material. Above the disk the matter is spun up and moves out rapidly; elsewhere, it is falling radially towards the White Dwarf because most of the pressure

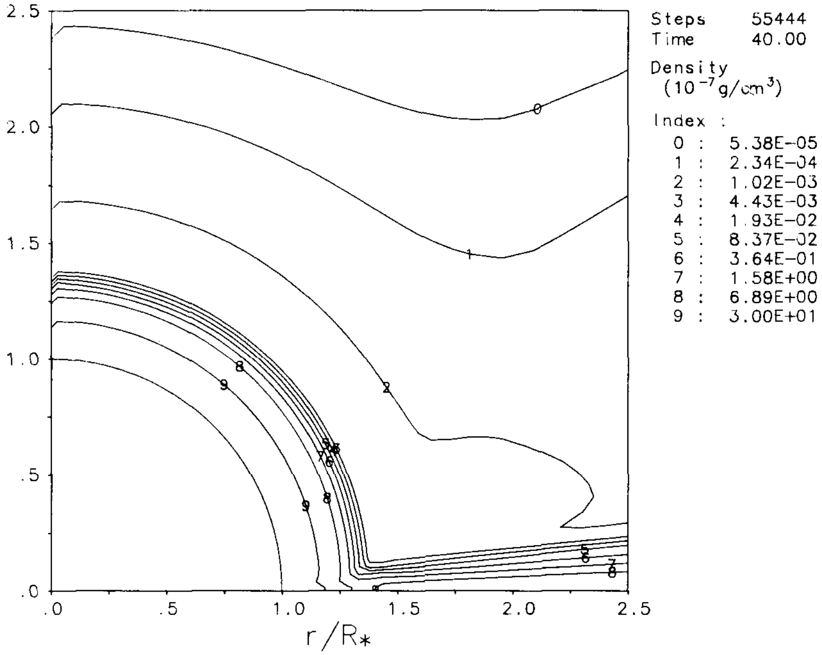

Fig. 3. Model 1 at t=40; density contours close to the WD.

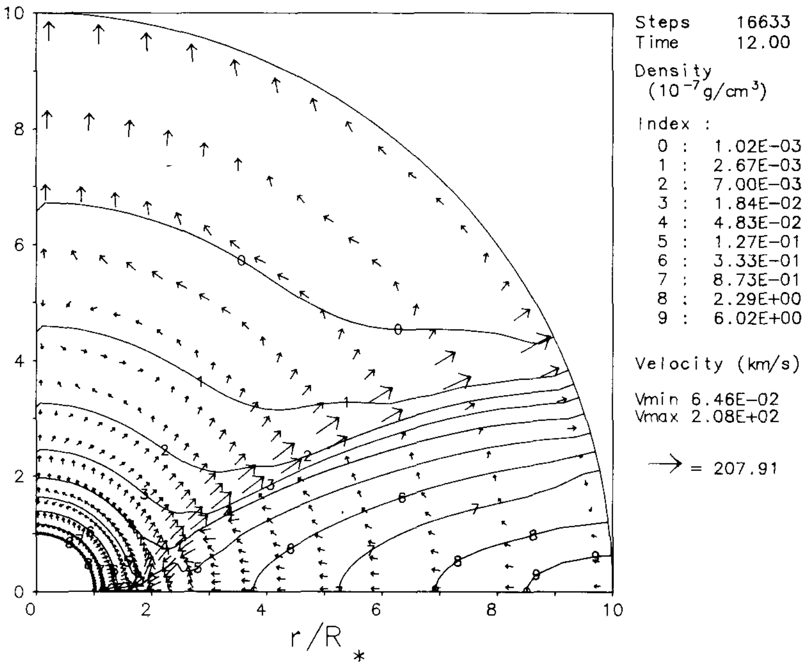

Fig. 4. Model 2 at t=12; scaling as in Figure 2.

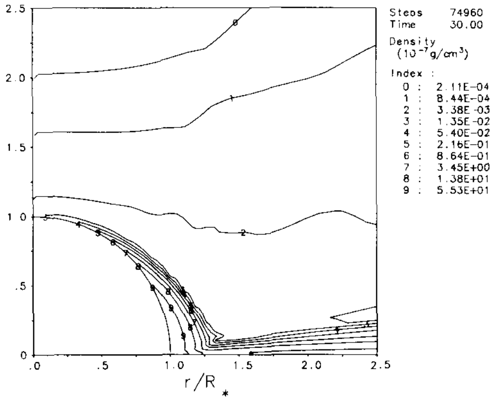

Fig. 5 Model 3 (f_r=.6); density contours close to the WD

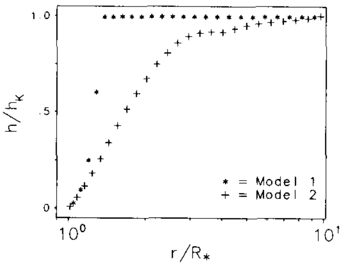

Fig. 6 The rotational velocity in the disk in units of the Keplerian value

support is lost by cooling of the inner parts. During the time sequence of the calculations the shell grows and the density gradient at the edge gradually steepens (Fig.3).

Model 2 (f_Φ=1, f_*=0)
In contrast to the former model the dissipated energy is now trapped in the system, i.e. f_Φ=1. As expected the frictional energy generation leads to an additional pressure that thickens the disk (Fig. 4). The matter above the disk is flowing out again and the entire atmosphere is now set into an outward motion. The further evolution is much slower than in the previous case and a pronounced shell is not formed at all.

Model 3 (f_Φ=0., f_*=.6)
If the star rotates fast, the shell becomes strongly ellipsoidic (Fig. 5). Apart from this expected change of the shape of the shell structure with the rotation of the White Dwarf the model is very similar to the non-rotating case (model 1). The main difference seems to be the stronger mass loss from the atmosphere.

Mass flux and rotational velocity
The standard disk models are usually calculated applying a constant mass flux $\dot{M}$. The outer boundary condition in our models allows for varying mass in- and outflow rates. The averaged values of the final quasistationary accretion rates show an expected relation: at lower kinematic viscosity the inflow rate is also lower (from models not shown here).

The radial dependence of the angular velocity of the disk is shown in Fig. 6. In the thin disk models with instantaneously emitted frictional energy, the disk rotation is approximately Keplerian down to $r \approx 1.4$ R$_*$. In the conventional approach the boundary layer extends up to that point where the angular velocity begins to lie significantly below the Keplerian velocity. Using this definition our BL has a vertical extent of ≈.4 stellar radii which is identical to the dimensions of the dense shell of material enveloping the White Dwarfs.

On the other hand for model 2(f_Φ =1) h/h_K decreases gradually to zero at the stellar surface indicating the absence of any well-defined boundary layer as was already shown by the density contours. But down to $r \approx 3.5\ R_*$ the fraction is well above the 90% level, implying a BL of this thickness.

4. DISCUSSION

The f_Φ=0 models develop a quasi-stationary disk structure that is in good agreement with the results of the standard theory for thin accretion disks (i.e. $u_r/u_\varphi \approx .01 - .02$, $u_\varphi \approx u_K$, concave shape). Concerning the BL structure the main result is a strong dependence on the cooling efficiency. If the cooling time is negligibly short (f_Φ=0) a dense shell is formed. On the other hand, if not, the shell expands and is ill-defined. These first results agree qualitatively with the calculations of Robertson and Frank (1985). The latter behavior is predicted by estimates of King and Shaviv (1984) who showed that in the case of inefficient cooling the BL expands to form a hot corona.

If we define the BL to be identical to the dense shell our results indicate a layer that, firstly, envelopes the star completely and, secondly, is much thicker than predicted what might contribute to the solution of the ´missing X-ray flux´ problem. Besides the induced outflow of the atmosphere, our calculations so far give no hint of a strong wind emanating near the White Dwarf, but radiative transfer has not yet been taken into account.

To mimic physical reality in more detail, calculations including radiative transport and different treatments of the viscosity are necessary. Additionally, the outer and inner boundary conditions might need modification allowing for example accretion through the inner boundary.

References

Boris, J.P., Book, D.L.: 1973, J. Comp. Phys. 18, 248

Cordova, F.A., Mason, K.O.: 1982, Astrophys. J. 260, 879

Ferland, G.J., Langer, S.H., Mac Donald, J., Pepper, G.H., Shaviv, G., Truran, J.W.: 1982, Astrophys. J. 262, L53

King, A.R., Shaviv, G.: 1984, Nature 308, 519

Klare, G., Krautter, J., Wolf, B., Stahl, O., Vogt, N., Wargau, W., Rahe, J.: 1982, Astron. Astrophys. 113, 76

Kley, W., Hensler, G.: 1986 submitted to Astron. Astrophys.

Pringle, J.E., Savonije, G.J.: 1979, Monthly Notices Roy. Astron. Soc. 197, 777

Robertson, J.A., Frank, J.: 1985, MPI for Astrophysics, Munich, preprint MPA 207, submitted to Monthly Notices Roy. Astron. Soc.

Shakura, N.I., Sunyaev, R.A.: 1973, Astron. Astrophys. 24, 337

Tylenda, R.: 1981, Acta Astron. 31, 267

van Leer, B.: 1977, J. Comp. Phys. 23, 276