several bounds and consistency results, e. g. the consistency of $\mathfrak{s} < \mathfrak{s}_{1/2}$ and $\mathfrak{s}_{1/2} < \mathrm{non}(\mathcal{N})$, as well as several results about possible values of $\mathfrak{i}_{1/2}$. Most proofs are of a combinatorial nature; one of the more sophisticated proofs utilises a creature forcing poset already introduced in Chapter B.

[1] T. Yorioka, *The cofinality of the strong measure zero ideal*. ***Journal of Symbolic Logic***, vol. 67 (2002), no. 4, pp. 1373–1384.

[2] A. Fischer, M. Goldstern, J. Kellner, and S. Shelah, *Creature forcing and five cardinal characteristics in Cichón's diagram*. ***Archive for Mathematical Logic***, vol. 56 (2017), no. 7–8, pp. 1045–1103.

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Ari Meir Brodsky, ***A Theory of Stationary Trees and the Balanced Baumgartner–Hajnal–Todorcevic Theorem for Trees***, University of Toronto, Canada, 2014. Supervised by Stevo Todorcevic. MSC: Primary 03E02, Secondary 03C62, 05C05, 05D10, 06A07. Keywords: combinatorial set theory, nonspecial trees, stationary trees, stationary subtrees, partial orders, diagonal union, regressive function, normal ideal, Pressing-Down Lemma, balanced partition relation, partition calculus, Erdős–Rado Theorem, Baumgartner–Hajnal–Todorcevic Theorem, elementary submodels, nonreflecting ideals, very nice collections.

Abstract

Building on early work by Stevo Todorcevic, we develop a theory of stationary subtrees of trees of successor-cardinal height. We define the diagonal union of subsets of a tree, as well as normal ideals on a tree, and we characterize arbitrary subsets of a nonspecial tree as being either stationary or nonstationary.

We then use this theory to prove the following partition relation for trees:

Main Theorem. *Let κ be any infinite regular cardinal, let ξ be any ordinal such that $2^{|\xi|} < \kappa$, and let k be any natural number. Then*

$$\textit{non-}(2^{<\kappa})\textit{-special tree} \rightarrow (\kappa + \xi)^2_k .$$

This is a generalization to trees of the Balanced Baumgartner–Hajnal–Todorcevic Theorem, which we recover by applying the above to the cardinal $(2^{<\kappa})^+$, the simplest example of a non-$(2^{<\kappa})$-special tree.

An additional tool that we develop in the course of proving the Main Theorem is a generalization to trees of the technique of nonreflecting ideals determined by collections of elementary submodels.

As a corollary of the Main Theorem, we obtain a general result for partially ordered sets:

Theorem. *Let κ be any infinite regular cardinal, let ξ be any ordinal such that $2^{|\xi|} < \kappa$, and let k be any natural number. Let P be a partially ordered set such that $P \rightarrow (2^{<\kappa})^1_{2^{<\kappa}}$. Then*

$$P \rightarrow (\kappa + \xi)^2_k .$$

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Aaron Thomas-Bolduc, ***New Directions for Neo-logicism***, University of Calgary, Canada, 2018. Supervised by Richard Zach. MSC: 00A30, 03A99. Keywords: neo-logicism, philosophy of mathematics, arithmetic, analysis.

Abstract

In this dissertation, I focus on a program in the philosophy of mathematics known as neo-logicism that is a direct descendant of Frege's logicist project. That program seeks to reduce mathematical theories to logic and definitions in order to put those theories on stable epistemic and logical footing. The definitions that are of greatest importance are abstraction principles, biconditionals associating identity statements for abstract objects on one side, with equivalence classes on the other. Abstraction principles are important because they provide connections between logic on the one hand, and mathematics and its ontology on the other.

Throughout this work, I advocate that the epistemic goals of neo-logicism be taken into account when we're looking to solve problems that are of central importance to its success. Additionally, each chapter either discusses or advocates for a methodological shift, or sets up and implements a novel methodological position I believe to be broadly beneficial to the neo-logicist project.

Chapter 2 traces thinking about the status of higher-order logic through the mid-twentieth century, setting the stage for issues dealt with in later chapters. Chapter 3 asks neo-logicists to look beyond set theory and consider other foundational theories, or something entirely new, when looking for reductions of foundational mathematical theories. Chapter 4 is an extended argument involving nonstandard analysis showing that Hume's Principle ought not be considered analytic in Frege's sense of the term.[1]

Chapters 5 and 6 move away form the (somewhat) historical work in the first three chapters, and set up new strategies for solving central neo-logicist problems by integrating formal and epistemic considerations. Chapter 5 introduces the notion of a canonical equivalence relation via a discussion of content carving, the latter notion being a particular way of understanding the relationship between equivalence relations and abstracts. Finally, Chapter 6 makes use of canonical equivalence relations to introduce a new direction in the search for solutions to the Bad Company objection.

As whole, the project can be seen as providing, as the title suggests, new directions that ought to be considered by those wishing to vindicate neo-logicism.

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William D. Simmons, ***Completeness of Finite-Rank Differential Varieties***, University of Illinois at Chicago, USA, 2013. Supervised by David Marker. MSC: 03C60, 12H05, 12H20. Keywords: model theory of fields, differentially closed fields, complete variety, quantifier elimination, differential algebra.

Abstract

The fundamental theorem of elimination theory states that projective varieties over an algebraically closed field K are *complete*: If V is such a variety and W is an arbitrary variety over K, then the projection map $\pi : V \times W \to W$ takes Zariski-closed sets to Zariski-closed sets. This property is tightly linked to projectiveness, as shown by the example of the affine hyperbola $xy - 1 = 0$. The images of the projections to either axis lack 0 but contain every other point of the affine line. We must "close up" the variety with a point at infinity to ensure a closed projection.

The situation is much more complicated in differential algebraic geometry. A *differential ring* is a commutative ring R with 1 and a finite set of maps Δ such that for each $\delta \in \Delta$ (*derivations* on R) and $x, y \in R$, $\delta(x + y) = \delta(x) + \delta(y)$ and $\delta(xy) = \delta(x)y + x\delta(y)$. We consider differential fields of characteristic zero with a single derivation δ. Over such a field,

[1]A version of Chapter 4 has been published as E. Darnell and A. Thomas-Bolduc, "Is Hume's Principle Analytic?" *Synthese*, 2018. DOI: 10.1007/s11229-018-01988-8.