THE SUNYAEV-ZEL'DOVICH EFFECT AND $H_0$

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ABSTRACT. Observational data on the Sunyaev-Zel'dovich effect for three clusters are used to obtain limits to the value of the Hubble constant. Only a wide bound on the value of $H_0$, $10 \text{ km s}^{-1} \text{ Mpc}^{-1} < H_0 < 160 \text{ km s}^{-1} \text{ Mpc}^{-1}$, can be derived, mostly because of the large uncertainty in the measured temperature of the intracluster medium in each cluster. Better information on this temperature and on the distribution of the intracluster medium are needed for this method to yield a good measurement of $H_0$.

1. THE USE OF THE SUNYAEV-ZEL'DOVICH EFFECT TO MEASURE $H_0$

The Sunyaev-Zel'dovich effect is a change of brightness of the microwave background radiation produced by its inverse-Compton scattering by electrons in a hot intracluster medium (Sunyaev & Zel'dovich 1972). The X-ray surface brightness of the intracluster medium is usually approximated by

$$l_X \approx l_0 \left(1 + \frac{\theta^2}{\theta_c^2}\right)^{\frac{1}{2}-3\beta}$$

with $\beta \approx 0.5$. If the intracluster medium is isothermal, the predicted microwave background decrement is

$$\Delta T_{RJ} = \Delta T_0 \left(1 + \frac{\theta^2}{\theta_c^2}\right)^{\frac{1}{2}-3\beta}$$

where $\Delta T_0$, the central decrement, and $l_0$, the central X-ray surface brightness, are related by

$$\frac{\Delta T_0^2}{l_0} = 55 \frac{T_r^2}{\Lambda(T_g)} \left(\frac{k T_g}{m_e c^2}\right)^2 \sigma_T^2 D_A \theta_c$$

for $\beta = 0.5$. $\Delta T_0$, $l_0$, $\theta_c$ and $\beta$ can be fitted from the observed Sunyaev-Zel'dovich data and X-ray images, the gas temperature $T_g$ can be derived from X-ray spectra, the X-ray emissivity $\Lambda(T_g)$ is a known function of $T_g$, and $T_r$ (the radiation temperature of the microwave background radiation), $k$, $m_e$, $c$, and $\sigma_T$ are known constants. This relation can, therefore, be used to determine the angular diameter distance $D_A$ of a cluster of galaxies, and hence the Hubble constant. This method has been described by a number of authors (Gunn 1978; Silk & White 1978; Cavaliere et al. 1979; Birkinshaw 1979).
2. THE DATA

The Sunyaev-Zel'dovich data are taken from the results of OVRO observations made over the period 1982-6, which add the data taken during the 1985-6 season to those summarized in Birkinshaw & Moffet (1986). These data have been corrected for the presence of weak non-thermal radio sources lying near the clusters, and they have been adjusted to make an allowance for the maximum likely residual systematic errors. The central microwave background decrements, $\Delta T_0$, (deconvolved for the telescope beam) are given in Table 1.

<table>
<thead>
<tr>
<th>cluster</th>
<th>$\Delta T_0$/mK</th>
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<tbody>
<tr>
<td>0016+16</td>
<td>$-0.92 \pm 0.19$</td>
</tr>
<tr>
<td>A 665</td>
<td>$-1.01 \pm 0.16$</td>
</tr>
<tr>
<td>A 2218</td>
<td>$-0.63 \pm 0.13$</td>
</tr>
</tbody>
</table>

The values of $\theta_c$ needed in the deconvolution were taken from the X-ray data of White et al. (1981), Forman & Jones (private communication) and Boynton et al. (1981). The central X-ray surface brightnesses, $I_0$, were derived from the fluxes given in these papers, and approximate gas temperatures were derived from the Einstein X-ray data for 0016+16 ($T_g > 6$ keV; White et al. 1981) and A 2218 ($T_g = 11^{+8}_{-4}$ keV; Perrenod & Henry 1981). For A 665 and A 2218, estimates of $T_g$ may also be derived from the cluster velocity dispersions, $\sigma_v$ (subject to the assumed value of $\beta$), since $\beta = \mu m_p\sigma_v^2/kT_g$ (Cavaliere & Fusco-Femiano 1976). The results for an assumed $\beta = \frac{1}{2}$ are $29 \pm 16$ keV and $24 \pm 7$ keV, respectively, and are probably overestimates (Mushotzsky 1984). The uncertainties in the temperatures derived from the velocity dispersions are too large for these estimates to be useful at present, but $T_g$ may be derived in this way in the future if more accurate values of $\sigma_v$ can be measured and if $\beta$ can be determined independently.

3. RESULTS

The gas temperature, the central Sunyaev-Zel'dovich decrement, the central X-ray surface brightness, and the structural data can now be combined to produce an independent estimate of the distance (and hence the Hubble constant) for each cluster. The values obtained for $H_0$ are $> 11$ km s$^{-1}$ Mpc$^{-1}$ from the data for 0016+16 and $70 \pm 90$ km s$^{-1}$ Mpc$^{-1}$ from the data for A 2218 (using the X-ray inferred gas temperatures). The large errors in these estimates reflect mostly the uncertainties in the values of $T_g$, and indicate that accurate estimates of the Hubble constant can be derived from this method only with better (error less than 1 keV) measurements of $T_g$. At present it is more useful to invert the argument and predict the gas temperatures that would be inferred from good X-ray spectra for these clusters for different assumed values of the Hubble constant. These temperatures are given in Table 2.

<table>
<thead>
<tr>
<th>cluster</th>
<th>$T_g$/keV for $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$</th>
<th>$T_g$/keV for $H_0 = 100$ km s$^{-1}$ Mpc$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0016+16</td>
<td>$8 \pm 3$</td>
<td>$12 \pm 4$</td>
</tr>
<tr>
<td>A 665</td>
<td>$10 \pm 2$</td>
<td>$14 \pm 2$</td>
</tr>
<tr>
<td>A 2218</td>
<td>$10 \pm 2$</td>
<td>$13 \pm 3$</td>
</tr>
</tbody>
</table>

It can be seen that overall the predicted gas temperatures are significantly higher for $H_0 = 100$ km s$^{-1}$ Mpc$^{-1}$. The difference between the gas temperatures for the two cases suggests that a test for the value of $H_0$ between the bounds of 50 and 100 km s$^{-1}$ Mpc$^{-1}$ may be possible with X-ray spectra with high signal/noise near 10 keV. The values of $T_g$ predicted in Table 2 were...
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derived for an isothermal model with $\beta = \frac{1}{2}$, but are found to depend only weakly on $\beta$.

4. THE FUTURE
This method of limiting the value of the Hubble constant may become important if good X-ray data on these clusters of galaxies, or good Sunyaev-Zel'dovich data on nearer clusters of galaxies (for which extensive X-ray data are already available) can be obtained. High quality data should allow the systematic uncertainties in the $H_0$ estimate caused by the unknown equation of state of the gas (assumed here to be isothermal) and the uncertain value of $\beta$ (taken to be $\frac{1}{2}$) to be resolved.

5. ACKNOWLEDGEMENTS
The Sunyaev-Zel'dovich data on which these calculations are based were obtained in collaboration with A.T. Moffet and S.F. Gull on the OVRO 40-m telescope.

6. REFERENCES

DISCUSSION

ELLIS: Why in the case of 0016+16 were you only able to determine an upper limit to the angular diameter distance? Is the X-ray data inadequate? I thought it was one of the strongest distant X-ray clusters.

BIRKINSHAW: The limit to $D_\circ$ arises from the lower limit to the gas temperature ($T < 6$ keV) for this cluster. Although 0016+16 is very X-ray luminous, its flux is small, and it is difficult to extract any temperature information from the Einstein data.

CANIZARES: How sensitive is your determination of $H_0$ to the assumption that the X-ray gas is isothermal?
BIRKINSHAW: Quite sensitive - but in principle this assumption, like the \( \beta \)-parameter assumption, can be tested by comparing X-ray and Sunyaev-Zel'dovich effect maps of the clusters, by using a spatially-resolved X-ray spectrum, or by doing a better job of modelling the X-ray emitting gas.

MANDOLISI: Have you taken into account the emission from weak, unresolved radio sources?

BIRKINSHAW: Yes. The emission from these sources was measured in a multi-frequency VLA survey of the fields of each of these three clusters.