

# New axes for the stellar mass fundamental plane

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**Abstract.** We argue that the stellar velocity dispersion observed in an elliptical galaxy is a good proxy for the halo velocity dispersion. As dark matter halos are almost completely characterized by a single scale parameter, the stellar velocity dispersion tells us the virial radius of the halo and the mass contained within. This permits non-dimensionalizing of the stellar mass and effective radius axes of the stellar mass fundamental plane by the virial radius and halo mass, respectively.

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## 1. Introduction

With 450 papers that include the words “fundamental plane” in their titles, our purpose is less to say something new than to reassemble some of what has been said before and frame it in a way that renders the fundamental plane somewhat less mysterious.

If one measures stellar velocity dispersions,  $\sigma_*$ , effective radii,  $r_e$ , and effective surface brightnesses, as measured in some filter,  $I_e$ , for elliptical galaxies, the observations lie close to a plane when plotted in the space spanned by  $\log \sigma_*$ ,  $\log r_e$  and  $\log I_e$  (Djorgovski & Davis 1987). Fitting a plane to the data yields three coefficients, as shown in equation 1.1.

The effective radius and surface brightness may be combined to produce a luminosity  $L$ . The ellipticals lie along a corresponding plane in the space spanned by  $\log \sigma_*$ ,  $\log r_e$  and  $\log L$ . If one has observations in several filters, one can calculate stellar masses,  $M_*$ , subject to considerable uncertainty in the initial mass function and its trend with velocity dispersion, and stellar surface densities,  $\Sigma_*$ . These also have associated planes (Hyde and Bernardi 2009). For the present discussion we consider the fundamental plane in the space spanned by  $\log \sigma_*$ ,  $\log r_e$  and  $\log M_*$ .

$$\underbrace{a * \log \left( \begin{array}{l} \text{surface brightness } I_e \\ \text{or luminosity } L \\ \text{or stellar mass } M_* \\ \text{or surface density } \Sigma_* \end{array} \right)}_{\text{stellar (baryonic) matter}} + b * \log (\text{half light radius } r_e) \quad (1.1)$$
$$\underbrace{+ c * \log (\text{stellar velocity dispersion } \sigma_*)}_{\text{thesis: proxy for dark matter halo dispersion } \sigma_{\text{DM}}} = 1$$

Two of these quantities, the effective radius and the stellar mass, describe a manifestly baryonic component of elliptical galaxies – the stars. We argue here that the third quantity, the stellar velocity dispersion, is a proxy for the velocity dispersion in the galaxy’s dark matter halo.

## 2. The virial theorem

Astronomers almost always use the virial theorem in the form appropriate to the global properties of equilibrium systems,

$$2T = -U. \quad (2.1)$$

By contrast, physics texts (e.g. Goldstein 1980) often present the virial theorem in a form appropriate to the orbit of a single star,

$$\langle v^2 \rangle_{\text{any object}} = \left\langle \vec{r} \cdot \vec{\nabla} \Phi \right\rangle_{\text{the orbit}}, \quad (2.2)$$

where  $\langle v^2 \rangle$  is the twice the orbit averaged kinetic energy per unit mass,  $\vec{\nabla} \Phi$  is the gradient of the gravitational potential and  $\vec{r}$  is position. The orbit averaged kinetic energies for stars and dark matter particles, are respectively,

$$\langle v^2 \rangle_{\text{any star}} \approx \left\langle \vec{r} \cdot \vec{\nabla} \Phi \right\rangle_{10^0 r_e} \quad \text{and} \quad (2.3)$$

$$\langle v^2 \rangle_{\text{dark particle}} \approx \left\langle \vec{r} \cdot \vec{\nabla} \Phi \right\rangle_{10^1 r_e}. \quad (2.4)$$

The stellar and dark matter velocity dispersions are the product of the *same* gravitational potential and differ *only* insofar as the quantity  $\vec{r} \cdot \vec{\nabla} \Phi$  varies with radius.

But there are several lines of evidence that point to little or no variation of  $\vec{r} \cdot \vec{\nabla} \Phi$  with radius.

## 3. The spheroid-halo conspiracy

It was the observation that spiral galaxies have circular velocities in excess of what was expected from their observable baryons that first lead to the conclusion that they were embedded in dark matter halos. But beyond that, H I rotation curves are remarkably flat, leading van Albada and Sancisi (1986) to conclude that there was a “disk-halo conspiracy.” The baryons dissipate to the point where they just compensate for what would otherwise be a decline in the rotation curve in the absence of baryons.

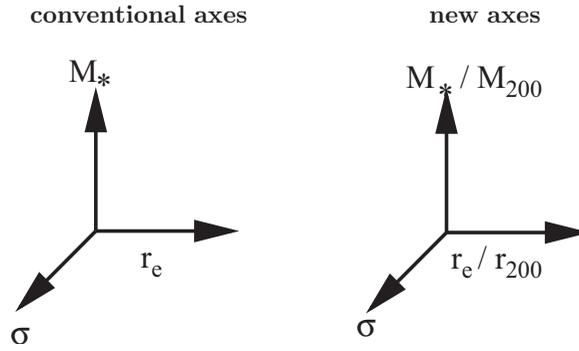
Gavazzi *et al.* (2007) subsequently used a combination of strong and weak lensing to show that there is likewise a “spheroid-halo conspiracy” for early-type systems. The gravitational potentials are isothermal giving  $\langle v^2 \rangle \sim \text{constant}$ . Humphrey and Buote (2010) use X-ray observations and likewise find a conspiracy producing isothermal potentials. Stellar velocity dispersions are therefore telling us the velocity dispersion of the dark matter.

We have marked up the fundamental plane equation 1.1 to show that while the effective radius and stellar mass are telling us about the baryons, the stellar velocity dispersion is telling us the velocity dispersion of the dark matter halo,  $\sigma_{DM}$ .

## 4. A fundamental *line* for dark matter halos

Dark matter halos are often characterized by their virial radii,  $r_{200}$  and the mass within that radius,  $M_{200}$ . It is natural to ask whether these, along with  $\sigma_{DM}$ , likewise organize themselves into a plane. They do not. Instead they lie along a tight line (Diemer *et al.* 2013). They are governed by a single parameter, an overall scale. Parameterizing the line by the maximum observed circular velocity, one has

$$M_{200} \sim V_{max}^3 \quad \text{and} \quad r_{200} \sim V_{max}. \quad (4.1)$$



**Figure 1.** Axes for the stellar mass fundamental plane, with dimensioned effective radii and stellar masses unscaled on the left and scaled by the virial radius and mass of the halo (determined from the stellar velocity dispersion) on the right.

The first of these is a consequence of defining halos in terms of densities while the second is consequence of the ongoing nature of virialization and the measurement of the virial radius at the same time for all halos.

Now if a stellar velocity dispersion tells us the velocity dispersion of the dark matter halo, and if that in turn tells us the mass and radius of the dark matter halo, then we can scale the stellar mass by the mass in dark matter (or alternatively, the total mass), and we can likewise scale the effective radius by the dark matter radius,

## 5. New axes for the stellar mass fundamental plane

This gives us new axes for the fundamental plane, as shown in Figure 1. One axis gives us the overall scale of the system, and a second axis tells us what fraction of the the baryons that have been incorporated into stars, on the assumption that the system started out with the cosmological baryon fraction.

While the above argument invokes the spheroid-halo conspiracy in its strongest form, with  $\sigma_{DM} = \sigma_*$ , the argument can still be made as long as there is a well defined relation between stellar velocity dispersion and halo dispersion.

## 6. Why not a fundamental line for ellipticals?

The stellar components of elliptical galaxies are clearly not as simple as the halos. While the baryons and dark matter were once nearly uniformly distributed, some baryons dissipated and formed stars at the centers of the halos. The  $M_*/M_{200}$  axis tells the fraction of baryons that did so. The  $r_e/r_{200}$  axis is governed both by the dissipation prior to star formation and by the subsequent remixing of stars by mergers.

If these three processes – dissipation, star formation and remixing – were strictly governed by the scale of the halo, the ellipticals would lie along a line. If there were a stochastic component to each of them, the scatter would thicken the line into a sausage. The fact that the ellipticals are confined to a plane indicates that the three processes are coupled.

## 7. Antecedents

Zaritsky *et al.* (2008) anticipated our representation of the fundamental plane. They defined an “efficiency”,  $M_*/M_{\text{baryonic}}$ , which differs from our choice of coordinate by a

factor of the cosmic baryon fraction. And they defined a “concentration”,  $r_{200}/r_e$ , which is just the inverse of our coordinate. They plot both, separately, against stellar velocity dispersion, but do not examine the three dimensional distribution and do not argue that the planarity of the elliptical distribution indicates coupling. But it is clear from their discussion that a coupling is both possible and likely.

## 8. Two postdictions

There is considerable evidence that the gravitational potential inside the effective radius, and hence the velocity dispersion, is dominated by stars rather than dark matter (e.g. Treu & Koopmans, 2004, Mediavilla *et al.* 2009). Measurement of the stellar velocity dispersion beyond the effective radius might therefore reflect the halo dispersion yet more closely than the central measurements that are typically used.

Schechter *et al.*(2014) found that if one used Einstein ring radii to calculate equivalent velocity dispersions for the SLACS (Auger *et al.*2010) sample, the resulting fundamental plane is substantially tighter than the one constructed using measured stellar velocity dispersions. It was this result that led to the present formulation, so it cannot count as confirmation.

Along the same line, Falcón-Barroso and van de Ven (2011) find that they get a tighter fundamental plane when they use velocity dispersions measured out to  $r_e$  than the one they get using dispersions interior to  $r_e/8$ . More extensive samples, and samples extending out to yet larger radii, are on the horizon.

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