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DEGREE BOUNDED GEOMETRIC SPANNING TREES WITH A BOTTLENECK OBJECTIVE FUNCTION

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In this thesis, we introduce and investigate the *δ-minimum bottleneck spanning tree problem* (*δ-MBST*). Given a weighted graph $G = (V, E)$, where the weights of E represent the lengths of the edges of E, the δ-MBST problem involves finding a spanning tree of G such that no vertex in the tree has a degree that exceeds δ and the length of the longest edge in the tree is minimum. We specifically focus on geometric versions of the problem, where vertices of V represent points embedded in a geometric space (for example, the Euclidean plane) and the weights of the edges of E are the distances between their endpoints in the space.

We begin by producing several initial results for the δ-MBST problem. We establish that the δ-MBST problem is NP-complete for any $\delta \geq 2$, and we show that when edge lengths of the graph are Euclidean distances between points in the plane, the problem is NP-hard for $\delta = 2$ and 3 and tractable for $\delta \geq 5$. We also give a dual approximation method for the general graph version of the problem and we describe several constant factor approximation algorithms for the geometric and Euclidean versions of the problem. Next, we perform a series of computational experiments to compare and contrast the effectiveness of a variety of heuristic and approximation algorithms for the δ-MBST problem and its min-sum variant. These experiments involve existing algorithms as well as several new heuristics. In order to obtain suitable test instances for the algorithms, we also formulate a method for reliably producing sets of points such that the minimum spanning trees for the point sets contain a certain number of vertices of specific degrees.

Finally, we explore the δ-MBST problem in three-dimensional Euclidean space and three-dimensional rectilinear space. We show that these problems are NP-hard

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for $\delta \in \{2, 3, 4, 5\}$ and we provide inapproximability results for these cases. We also describe new approximation algorithms for solving these three-dimensional variants, and then analyse their worst-case performance by considering the sets of points that yield worst performances for the algorithms.

Some of the research from this thesis has appeared in [1–3].

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