Now the potencies a, b, c of the points A, B, C relatively to the circle O are, R being the radius of the circle,

$$a = OA^2 - R^2, \qquad b = OB^2 - R^2 \qquad c = OC^2 - R^2$$

which transform equation (1) into

$$a \, . \, BC + b \, . \, CA + c \, . \, AB + AB \, . \, BC \, . \, CA = 0 \quad - \quad - \quad (2)$$

But $CB \, . \, CA = c$, if C be the point where the tangent common to the two circles meets AB.

Thus equation (2) becomes

$$a \, . \, BC + b \, . \, CA = 0$$

or

$$\frac{CA}{CB} = \frac{a}{b} \quad - \quad - \quad - \quad - \quad - \quad (3)$$

Thus the point C is determined and the problem solved. If the problem is possible, A and B must both be inside or both outside the circle O.

Mr Muirhead suggests the following Solution.

A and B are the given points, DEF the given ⊙, and ABD the required ⊙

In virtue of the equality of angles indicated in Fig. 15, we have

$$\frac{\text{Power of A}}{\text{Power of B}} = \frac{AD \, . \, AF}{BD \, . \, BE}$$

$$= \frac{AD^2}{BD^2} = \frac{CA}{CB}$$

Discussion on Euclid's Definition of Proportion.

Papers by Prof. GIBSON and Mr W. J. MACDONALD.

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