

DYNAMIC ISOPERIMETRY ON GRAPHS AND WEIGHTED RIEMANNIAN MANIFOLDS

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Transport and mixing in dynamical systems are important properties for many physical, chemical, biological and engineering processes. The detection of transport barriers for dynamics with general time dependence is a difficult, but important problem, because such barriers control how rapidly different parts of phase space (which might correspond to different chemical or biological agents) interact. The key factor is the growth of interfaces that partition phase space into separate regions. In a recent paper, Froyland [1] introduced the notion of dynamic isoperimetry: the study of sets with persistently small boundary size (the interface) relative to enclosed volume, when evolved by the dynamics. Sets with this minimal boundary size to volume ratio were identified as level sets of dominant eigenfunctions of a dynamic Laplace operator. In this dissertation, we develop a data-driven approach for transport barrier detection, by extending and generalising dynamic isoperimetry to graphs and weighted Riemannian manifolds.

First, we model trajectory data as dynamics of graphs. We use minimum disconnecting cuts to search for coherent structure in dynamic graphs, where the graph dynamic arises from a general sequence of vertex permutations. We develop a dynamic spectral partitioning method via a new dynamic Laplacian matrix. We prove a dynamic Cheeger inequality for graphs, and we demonstrate the effectiveness of this dynamic spectral partitioning method on both structured and unstructured graphs. This work has been published in [2].

We then generalise the dynamic isoperimetric problem on manifolds to situations where the dynamics (i) is not necessarily volume preserving, (ii) acts on initial agent concentrations different from uniform concentrations, and (iii) occurs on a possibly

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curved phase space. Our main results include generalised versions of the dynamic isoperimetric problem, the dynamic Laplacian, the dynamic Cheeger's inequality and the Federer–Fleming theorem. We illustrate the computational approach with some simple numerical examples. This work has been published in [3].

Finally, we form a connection between the weighted graph version of our dynamic Laplacian matrix and the manifold dynamic Laplace operator. We then form a dynamic Laplacian-based manifold learning algorithm, which is designed to approximate solutions of our generalised dynamic isoperimetric problem from trajectory data. We highlight the robustness of our dynamic manifold learning method through numerical experiments.

References

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