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ERRATUM

Algebraic Polymorphisms – ERRATUM

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We give three corrections to the paper [K. Schmidt and A. M. Vershik, Algebraic polymorphisms, *Ergod. Th. & Dynam. Sys.* **28** (2008), 633–642].

(1) The statement in the penultimate paragraph on [**1**, p. 634] has to be corrected as follows: if π_1 is an injection then P is (the graph of) an endomorphism, and if π_2 is an injection then P is (the graph of) an exomorphism.

In other words, the symbols π_1 and π_2 should be interchanged.

(2) [**1**, Corollary 1.7] is incorrect as stated. The correct statement should be the following.

COROLLARY. *Let $\mathsf{P} \subset G \times G$ be a correspondence and let $H \subset G$ be a closed normal subgroup. We denote by $K^{(i)}_{\mathsf{P}^n}$, $i = 1, 2$, the closed normal subgroups of G associated with the correspondence P^n, $n \geq 2$, in (1.4) by (1.9). The sequences of subgroups $(K^{(i)}_{\mathsf{P}^n}, n \geq 1)$ are non-decreasing, and we write $H^{(i)}_0 = \overline{\bigcup_{n\geq 1} K^{(i)}_{\mathsf{P}^n}}$ for the closure of $\bigcup_{n\geq 1} K^{(i)}_{\mathsf{P}^n}$. Then the following holds.*

(1) *$H^{(2)}_0$ is smallest invariant subgroup of Π_{P}.*

(2) *$H^{(1)}_0$ is the smallest co-invariant subgroup of Π_{P}.*

Proof. By definition, $\eta_{\mathsf{P}}(K^{(1)}_{\mathsf{P}} K^{(2)}_{\mathsf{P}^n}) = K^{(2)}_{\mathsf{P}^{n+1}}$ for all $n \geq 1$.

If a closed normal subgroup $H \subset G$ is invariant under Π_{P} then [**1**, Theorem 1.6(1)] shows that $K^{(2)}_{\mathsf{P}^2} = \eta_{\mathsf{P}}(K^{(1)}_{\mathsf{P}} K^{(2)}_{\mathsf{P}}) \subset \eta_{\mathsf{P}}(K^{(1)}_{\mathsf{P}} H) \subset H$. Hence

$$K^{(2)}_{\mathsf{P}^3} = \eta_{\mathsf{P}}(K^{(1)}_{\mathsf{P}} K^{(2)}_{\mathsf{P}^2}) \subset \eta_{\mathsf{P}}(K^{(1)}_{\mathsf{P}} H) \subset H$$

and, by induction, $K^{(2)}_{\mathsf{P}^n} \subset H^{(2)}_0 \subset H$ for every $n \geq 1$.

In order to verify that $H^{(2)}_0$ is invariant under Π_{P} we note that

$$K^{(2)}_{\mathsf{P}^{n+1}} = \eta_{\mathsf{P}}(K^{(1)}_{\mathsf{P}} K^{(2)}_{\mathsf{P}^n}) \subset H^{(2)}_0 \quad \text{for every } n \geq 1,$$

and by letting $n \to \infty$ we see that $\eta_{\mathsf{P}}(K_{\mathsf{P}}^{(1)} H_0^{(2)}) \subset H_0^{(2)}$. According to [**1**, Theorem 1.6(1)] this proves that $H_0^{(2)}$ is invariant.

The proof of the second assertion is analogous. □

(3) The third correction concerns the semigroup $\mathcal{P}_f(\mathbb{T}^m)$ of all finite-to-one correspondences of $\mathbb{T}^m$. Denote by $\mathcal{L}$ the semigroup of all finite index subgroups of $\mathbb{Z}^m$ with intersection as composition (and not, as stated wrongly in [**1**, p. 637], with addition). We consider the semigroup

$$\mathcal{M} = \{(Q, \Lambda) \mid Q \in \mathrm{GL}(m, \mathbb{Q}), \Lambda \in \mathcal{L}, \Lambda \subset \Lambda_Q := \mathbb{Z}^m \cap Q\mathbb{Z}^m\}, \tag{1}$$

with composition

$$(Q, \Lambda) \cdot (Q', \Lambda') = (QQ', \Lambda \cap Q\Lambda'), \tag{2}$$

where we again have replaced addition by intersection.

This correction does not affect any of the results or proofs in that section.

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REFERENCES

[1] K. Schmidt and A. M. Vershik. Algebraic polymorphisms. *Ergod. Th. & Dynam. Sys.* **28** (2008), 633–642.