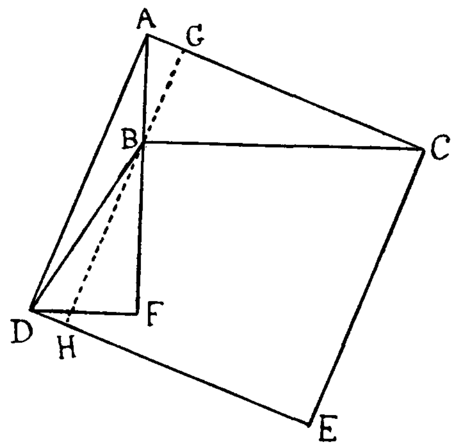
Join DB: then the triangle ADB is equal to half the rectangle with AB as base and FD as altitude; that is, to half the square on AB.



Now through B draw GBH parallel to AD to meet AC in G and DE in H. Then the rectangle $ADHG$, being equal to twice the triangle ADB, is equal to the square on AB.

Similarly the rectangle $CGHE$ is equal to the square on BC. Thus, on adding, we find that the square on AC is equal to the sum of the squares on AB and BC.

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Note on a Vanishing Determinant.

1. In one of Sir Thomas Muir's more recent historical papers on the theory of determinants (*Proc. R. S. Edin.*, XLIII, 1922, p. 129), is included a result due to V. Jung, commented on as being "verified in an unsuggestive way for the first three cases." The theorem is that the determinant

$$\begin{vmatrix} 1 & 1^2 & \dots & 1^n & 1^{n+1} \\ 2 & 2^2 & \dots & 2^n & 2^{n+1} \\ \dots & \dots & \dots & \dots & \dots \\ n & n^2 & \dots & n^n & n^{n+1} \\ \dfrac{n}{2} & \dfrac{n^2}{3} & \dots & \dfrac{n^n}{n+1} & \dfrac{n^{n+1}}{n+2} \end{vmatrix}$$

vanishes when n is even. Below we give a simple proof.

2. Consider the integral

$$I \equiv \int_0^n x(x-1)(x-2)\ldots(x-n)\,dx$$

$$= \left\{\int_0^{\frac{n}{2}} + \int_{\frac{n}{2}}^{n}\right\} x(x-1)(x-2)\ldots(x-n)\,dx; \text{ (Put } y = n - x)$$

$$= \int_0^{\frac{n}{2}} x(x-1)\ldots(x-n)\,dx - (-)^n \int_0^{\frac{n}{2}} y(y-1)\ldots(y-n)\,dy$$

$$= 0, \text{ if } n \text{ is even.} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Now let $x(x-1)\ldots(x-n) = a_1 x + a_2 x^2 + \ldots + a_{n+1} x^{n+1}$. Then if n is even, we have a consistent set of $(n+1)$ equations in the a's,

$$0 = a_1 + a_2 + \ldots + a_{n+1}$$

$$0 = 2a_1 + 2^2 a_2 + \ldots + 2^{n+1} a_{n+1},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$0 = na_1 + n^2 a_2 + \ldots + n^{n+1} a_{n+1},$$

and by (1), $0 = \frac{n}{2} a_1 + \frac{n^2}{3} a_2 + \ldots + \frac{n^{n+1}}{n+2} a_{n+1}$,

and hence the determinant of the system is zero, which is Jung's result.

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On the Roots of a Symmetrical Determinant.

The six values of x which make the determinant

$$\Delta = \begin{vmatrix} x & 1 & \cdot & \cdot & \cdot & \cdot \\ 1 & x & 1 & \cdot & \cdot & \cdot \\ \cdot & 1 & x & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & x & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 & x & 1 \\ \cdot & \cdot & \cdot & \cdot & 1 & x \end{vmatrix}$$

vanish, are $-2\cos\frac{\pi}{7}$, $-2\cos\frac{2\pi}{7}$, ..., $-2\cos\frac{6\pi}{7}$. In general, the n values of x which make the corresponding determinant of order n vanish are given by $-2\cos\frac{r\pi}{n+1}$, $r = 1, 2, \ldots, n$. Each determinant has a diagonal filled with x's, bordered by adjacent parallels where each element is unity; and all other elements are zero.