Terminology was introduced when the need arose and theorems were interspersed with a generous dose of examples and applications. Exercises were plentiful and instructive. It was a book which offered the reader knowledge as well as pleasure. It was the first book from which I learnt graph theory.

The third edition appeared in 1985, followed by this fourth edition eleven years later. The content of this fourth edition remains basically the same (still only 171 pages but the size of a page slightly larger). The general lay-out has been much improved to make the book even more appealing. Helpful schematic diagrams have been added in some proofs. A few new applications (e.g. eight-circles problem, searching trees) have been added. A few new exercises have been added, and solutions to some selected exercises are provided at the end. One particularly welcome modification is the new bibliography, which is not only updated and substantially enlarged but is semi-annotated by incorporating the postscript in former editions into the bibliography for easier reference. As for terminology, not much has been changed from the third edition, but a ‘circuit’ now becomes a ‘cycle’.

I would still recommend this book to my students who want to learn graph theory. For reviews on the three earlier editions, readers can consult this Gazette: p. 348 of 57 (December 1973), p. 288 of 63 (December 1979) and p.75 of 70 (March 1986).

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The book is subtitled ‘A Modern Course of Classical Logic’, and this is a good summary of its intent. It is worth observing further that the book begins by introducing basic concepts of universal algebra, and bases its presentation of first-order logic and model theory on this machinery. The book requires some basic knowledge of set theory (up to facts about cardinalities of infinite sets), which is reviewed at the outset; otherwise, it does not so much require mathematical knowledge as mathematical maturity.

The book is intended as a graduate text for a first course in logic, and it demands careful attention of the reader. It rewards the reader's attention with a clear exposition (in Part One) of a line of development leading from the definition of relational structure (machinery from universal algebra), through a formal definition of language and theory of first-order logic, to a careful development of the notion of proof in first-order logic. The completeness theorem is proved (a first-order theory is consistent if and only if it has a model). Non-standard models of Peano arithmetic, ultraproducts, and the Löwenheim-Skolem theorems are also covered in Part One.

Part Two covers some more advanced topics, including proofs of Gödel’s incompleteness theorems, and also proofs of some more sophisticated results in the area of undecidability and computability (or the absence thereof) which are less familiar to the general mathematical audience. The proof that Goodstein's theorem, a natural theorem of arithmetic with a fairly easy set-theoretical proof, is unprovable in Peano arithmetic, is presented in detail.

The statement of Goodstein’s theorem, though probably not its proof, might be accessible and interesting to some high school students. One defines an expression in ‘pure base $p$’ as the result of expressing a number in base $p$ notation, then expressing each exponent $\geq p$ in the base $p$ expression in ‘pure base $p$’ (note that this
is a recursive definition!) The first term of the ‘Goodstein sequence’ of a number is the number itself. The \( n \)th term of the Goodstein sequence is obtained by expressing the previous term in ‘pure base \( n \)’, replacing all the bases \( n \) in the expression with \( n + 1 \), then subtracting one. Goodstein’s theorem is that every Goodstein sequence eventually terminates at zero. This is rather surprising, since the effect of incrementing the bases in the pure base \( n \) expansion would seem to be much greater than the effect of subtracting one. The proof requires an understanding of notation for infinite ordinals.

The book presents Tarski’s theorem on the elimination of quantifiers from the theory of ordered real closed fields, which shows that this theory is (in principle) mechanically decidable (a result which extends to elementary Euclidean geometry) and Matiyasevich’s theorem, which solves Hilbert’s Tenth Problem by showing that there cannot be an algorithm which determines for any Diophantine equation whether it has a solution.

The reviewer found the style to be clear (though demanding) and the treatment to be accurate. There are numerous exercises; the authors advise the reader to do all of them (if the reviewer had followed this advice, the review would have been somewhat delayed). The reviewer could imagine a lay reader with a strong undergraduate preparation in mathematics learning quite a lot from the book, though the topics at the end of Part One and in Part Two would be increasingly difficult; it would help to have access to some more informal treatment of propositional and first-order logic as well.

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Every mathematician would, I hope, be acquainted with the basic facts about the geometry of surfaces, of two-dimensional manifolds. However the theory of three-dimensional manifolds is nothing like so well-known; it is much more difficult and still only partly understood. For example, the Poincaré conjecture about the simply connected case remains an open question after nearly a hundred years of research. Yet what has already been discovered provides ample evidence that the theory of three-dimensional manifolds is one of the most beautiful in the whole of mathematics.

However, until this excellent introductory work appeared, this mathematical wonderland remained rather inaccessible to non-specialists. The author is both a leading researcher, with a formidable geometric intuition, and a gifted expositor. His vivid descriptions of what it might be like to live in this or that three-dimensional manifold bring the subject to life. Quite rightly he appeals a lot to intuition, as did Poincaré, but not unduly so. His enthusiasm is infectious and should make many converts for this kind of mathematics. There are some really good pictures, and plenty of exercises and problems.

A few years after Poincaré completed his great work, Hadamard described topology as ‘the revenge of geometry against analysis’, which had come to dominate the mathematics of the nineteenth century. It must have been this kind of topology, closely intertwined with geometry, which he had in mind. Fifty years earlier Riemann had seen that a new mathematical world lay in this direction, but alas did