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DILATON BLACK HOLES WITH A COSMOLOGICAL TERM

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Abstract

The properties of static spherically symmetric black holes, which carry electric and magnetic charges, and which are coupled to the dilaton in the presence of a cosmological constant, Λ, are reviewed.

1. Introduction

String theory [16] appears to provide the most promising framework we know of for constructing a quantum theory of gravity, a goal which has remained elusive despite the attentions of at least one generation of researchers. Unfortunately, although the mathematical structure of string theory is very rich, it has proved to be extremely difficult to extract a phenomenologically viable model of physics from this structure. In particular, in compactifying the extra spatial dimensions of the fundamental superstring theory on a Calabi-Yau manifold, one finds a plethora of possible 4-dimensional vacua, with no means of determining just which one should correspond to the ground-state of the universe. However, recent work suggests that this central problem of string phenomenology might be resolved through the introduction of a single large "universal" moduli space [1], whose various branches correspond to distinct Calabi-Yau vacua. To consistently deal with conifold singularities which arise in this moduli space, it is necessary to include certain black hole states in the spectrum of string excitations [17], which have the unusual property of being massless [37]. Thus there appears to be an intimate connection between string theory and black holes. Such a relationship has been suggested before on various grounds [38, 39], and over the last five years a number of exact black hole solutions of string theory have been found [40]. There would thus appear to be much insight to be gained from studying processes

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involving such stringy black hole states. At a more modest level, we might hope to gain some understanding of stringy black holes by studying the properties of solutions to the effective field theoretic models which arise in the low-energy limit of string theory – namely, the models one obtains when one keeps only the lowest-order terms in a perturbative expansion in powers of the Regge slope parameter, $\alpha' = 1/(2\pi T)$, T being the string tension. Indeed a number of such solutions can be promoted to be solutions of the exact string theory to all orders in α' [40], which justifies this approach.

In this paper I would like to discuss recent work (with S. Poletti and J. Twamley) [30–32] concerning the properties of black holes coupled to the dilaton field in the presence of a cosmological term. The scalar dilaton, along with the pseudoscalar axion, enters into the action of effective gravitational theories that arise in the low-energy limit of string theory. Although one can consistently truncate such models to consider, for example, solutions with a vanishing axion, the presence of the dilaton cannot be ignored if one is to consider the coupling of gravity to other gauge fields, and thus the dilaton can be considered to be an essential feature of stringy gravity.

The effective string action we will consider takes the form

$$S = \int d^4x \sqrt{-\hat{g}}\, e^{-2\phi} \left\{ \frac{\hat{\mathcal{R}}}{4} + \hat{g}^{\mu\nu}\partial_\mu\phi\, \partial_\nu\phi - \hat{\mathcal{V}}(\phi) - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \dots \right\} \qquad (1)$$

in four dimensions,[2] where we have included the dilaton ϕ, with potential $\mathcal{V}(\phi)$, and an Abelian $U(1)$ gauge field with field strength $F_{\mu\nu}$ as representative of matter content. The axion, which would enter with the matter degrees of freedom in a similar fashion to F, has been excluded. It is a characteristic of string theory that the dilaton, ϕ, couples universally to matter through the conformal factor $e^{-2\phi}$ in (1). The metric $\hat{g}_{\mu\nu}$ is thus said to correspond to the *string conformal frame*. To obtain an action resembling that of general relativity we conformally rescale to obtain the metric

$$g_{\mu\nu} = e^{-2\phi}\hat{g}_{\mu\nu} \qquad (2)$$

of the *Einstein conformal frame*, in which the action becomes

$$S = \int d^4x \sqrt{-g} \left\{ \frac{\mathcal{R}}{4} - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\, \partial_\nu\phi - \mathcal{V}(\phi) - \frac{1}{4} e^{-2g_0\phi} F_{\mu\nu}F^{\mu\nu} + \dots \right\}, \qquad (3)$$

where $\mathcal{V} \equiv e^{2\phi}\hat{\mathcal{V}}$ and $g_0 = 1$ in the case of string theory. It is the non-trivial coupling between ϕ and F in (3) which leads to a theory with rather different properties than that of the standard Einstein-Maxwell ($g_0 = 0$) theory.

[2] I will work exclusively in four dimensions here. The extension to arbitrary $D \geq 4$ is straightforward (at least as far as the non-numerical results are concerned), and is dealt with in [30–32].

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Although the value of the dilaton coupling parameter is fixed to be $g_0 = 1$ by the tree-level string action, it is nonetheless useful to leave this as an arbitrary parameter since a variety of similar theories can then be treated together. Another case of interest is $g_0 = \sqrt{3}$, which corresponds to the 4-dimensional theory obtained by dimensionally reducing the 5-dimensional Kaluza-Klein theory. In that case the scalar ϕ is associated with the radius of the fifth dimension rather than being a dilaton field. From the string theoretic point of view the potential $\mathcal{V}(\phi)$ can be expected to be present if one is dealing, for example, with a central charge deficit in which case the potential takes the Liouville form

$$\mathcal{V}_{\text{exp}} = \frac{\Lambda}{2} e^{-2g_1\phi}, \tag{4}$$

with $g_1 = -1$ and $\Lambda = (D_{\text{crit}} - D_{\text{eff}})/(3\alpha')$, α' being the Regge slope parameter. Alternatively, if one considers non-perturbative effects such as the generation of a dilaton mass by supersymmetry breaking then one might also expect a contribution to the potential of the form

$$\mathcal{V}_{\text{susy}} = \exp\left[-\alpha e^{-2\phi}\right]\left\{A_1 e^{2\phi} + A_2 + A_3 e^{-2\phi}\right\}, \tag{5}$$

where α, A_1, A_2 and A_3 are constants. This is in fact the potential which arises in four dimensions from supersymmetry breaking via one-gaugino condensation in the hidden sector of the string theory [5], and should be regarded as typical of the general type of potential one might expect.

There has been a great deal of interest in the past few years in black hole solutions of effective low-energy theories of gravity derived from string theory, such as those described above. Most interest has focussed in particular on black holes with a massless dilaton, $\mathcal{V} \equiv 0$. For $g_0 = 1$, which is the case of interest to string theory, the general solution is given by[3] [10, 11, 14]

$$ds^2 = -f\,dt^2 + f^{-1}dr^2 + R^2 d\Omega^2, \tag{6}$$

$$\mathbf{F} = \frac{Q}{R^2} e^{2\phi} dt \wedge dr + P \sin\theta_1 d\theta_1 \wedge d\theta_2, \tag{7}$$

$$e^{2\phi} = \frac{r + \Sigma}{r - \Sigma}, \tag{8}$$

where

$$R(r) \equiv \sqrt{(r + \Sigma)(r - \Sigma)}, \tag{9}$$

[3] The radial coordinate of Gibbons and Maeda [14] is obtained by $r \to r + M$, and that of Garfinkle *et al.* [10] by $r \to r - \Sigma$, relative to the coordinate chosen here. Our choice here is dictated by the requirement that $R(r)$ agrees with the gauge chosen later in asymptotic expansions.

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$$f(r) \equiv \frac{1}{R^2}\left[(r-M)^2 - (M^2+\Sigma^2-Q^2-P^2)\right], \tag{10}$$

$$2M\Sigma = P^2 - Q^2 \tag{11}$$

and $d\Omega^2$ is the standard round metric on a 2-sphere, with angular coordinates θ_1, θ_2. The scalar charge Σ has been normalised such that

$$\phi = \frac{\Sigma}{r} + \mathrm{O}\left(r^{-2}\right) \tag{12}$$

at spatial infinity. It is possible to add an arbitrary constant ϕ_0 to ϕ at the expense of normalising the electromagnetic field differently from the standard choice at spatial infinity; however, we shall not do so here. Similar exact solutions are known for $g_0 = \sqrt{3}$ in the case of dyonic solutions [7] and for all values of g_0 in the case of solutions with a single (electric or magnetic) charge [14].

Unlike the standard Reissner-Nordström solutions the global properties of the solutions vary considerably according to whether the solutions carry a single charge, or both electric and magnetic charges. In the case of the latter dyonic solutions the spacetime exhibits two horizons and the properties of the solutions are similar to those of the Reissner-Nordström solutions. In particular, the extremal limit, $|P| + |Q| = \sqrt{2}\,M$, [or $\Sigma = (|P| - |Q|)/\sqrt{2}$], corresponds to the two horizons becoming degenerate at a finite value of R, and the black holes have zero Hawking temperature in this limit. However, the singly charged solutions have a single horizon, $r_{\mathscr{H}}$, and the extreme solutions are obtained in the limit $R_{\mathscr{H}} \equiv R(r_{\mathscr{H}}) \to 0$, and $\phi_{\mathscr{H}} \equiv \phi(r_{\mathscr{H}}) \to \infty$ for magnetic solutions ($Q = 0$), (or $\phi_{\mathscr{H}} \to -\infty$ for electric solutions ($P = 0$)). As a result, the area of the event horizon and the entropy of the extreme solutions is zero. Furthermore, the extreme solutions have finite Hawking temperature if $g_0 = 1$, and infinite temperature if $g_0 > 1$ [14, 15]. The fact that the temperature is formally infinite of course merely signals the breakdown of the semi-classical approximation if one is considering the Hawking evaporation process. Holzhey and Wilczek [21] have in fact demonstrated that in the case of the $g_0 > 1$ solutions an infinite mass gap develops for quanta with a mass less than that of the black hole so that the Hawking radiation slows down and comes to an end at the extremal limit, despite the infinite temperature. Furthermore, since scalar waves with an energy below the mass of the black hole are repelled, the black holes behave in a fashion similar to that of elementary particles [21].

An interesting feature of this extreme limit is that in the case of the purely magnetic solutions the spacetime is in fact completely regular when viewed in terms of the metric $\hat{g}_{\mu\nu}$ of the string conformal frame [10]. Since strings couple to $\hat{g}_{\mu\nu}$, string physics is not affected by the singularity which appears at $R = 0$ in the Einstein conformal frame in the extreme case. In the string conformal frame the $R = 0$ singularity is replaced by an infinitely long throat [10]. The property that the singularity here resides entirely

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in the conformal factor is also a feature of other solutions. In particular, in the case $g_0 = \sqrt{3}$, the purely magnetic solution corresponds to the Kaluza-Klein monopole [19, 36], which is singular from the point of view of the 4-dimensional Einstein conformal frame but is completely regular in five dimensions. Further examples are discussed in [12]. Although arguments based on the positivity of energy favour the Einstein conformal frame as being the physical one in the context of Jordan-Brans-Dicke theories and higher derivative models [3, 25, 35], it is clear that singularities which can be removed by a conformal rescaling – so that roughly speaking the singularity is in the Ricci tensor rather than the Weyl tensor – may be milder than other types of singularities. However, some work remains to be done to put this on a firmer footing in some general framework if one does not wish to appeal to string theory or higher-dimensional physics.

The fact that classical solutions are obtained in dilaton gravity with properties that differ significantly from the standard Einstein-Maxwell theory has provided fertile ground for the development of new ideas concerning various quantum gravitational phenomena, such as pair creation [8, 9, 13, 20, 33] and fission [13] of black holes. These ideas have led to the realisation that the entropy of extreme black holes should be zero, even in the case of the Reissner-Nordström solutions which have a horizon of finite area in the extreme limit [20].

One major defect with the dilaton black hole models studied to date is that the contribution of possible dilaton terms, $\mathscr{V}(\phi)$, has been largely ignored. Phenomenologically this could be regarded as a major defect, since it is widely believed that the dilaton must acquire a mass in order to avoid the generation of long range scalar forces, which are not observed in nature. This is by no means the only possibility – Damour and Polyakov have recently suggested an interesting scenario in which the dilaton remains massless but very weakly coupled to ordinary matter [4] – however, the "massive dilaton" scenario remains the conventional choice, and certainly one that deserves to be further studied. The reason why such models have not been studied very extensively is simply that the problem of finding appropriate exact solutions is a technically very difficult one. Gregory and Harvey [18] and Horne and Horowitz [22] have made an investigation of black hole models which include a mass term, in the form of a simple quadratic potential [18, 22], $\mathscr{V} = 2m^2 (\phi - \phi_0)^2$, or alternatively of the form [18] $\mathscr{V} = 2m^2 \cosh^2 (\phi - \phi_0)$. While a rigorous proof of the existence of black hole solutions in these models has still to be given, the arguments of Horne and Horowitz [22] based on numerical work are nonetheless compelling. The stability of the solutions remains an open question, however. Given that the putative black hole solution of [22] appears to be the envelope of a set of singular solutions for $R(r)$ and a separatrix between different singular solutions for $\phi(r)$, it seems doubtful that this solution could be stable – although the situation may be different for other potentials. However, it is clear from the arguments of [18, 22] that the properties of the solutions

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with a massive dilaton are essentially the same as those with a massless dilaton in the case of black holes which are small with respect to the Compton wavelength of the dilaton. This is a feature which one might expect to be generic of any potential, and it does provide some further justification for studying the solutions with a massless dilaton.

The aim of our recent series of papers [30–32] has been to study a problem which is similar to that of dilaton black holes with a mass-generating potential, but is technically somewhat simpler. In particular, we have set out to derive the properties of solutions with a cosmological term, which might be expected to be analogues of the Reissner-Nordström-(anti)-de Sitter solutions. I will outline our major results in turn.

2. Static spherically symmetric solutions with a Liouville potential

If one is to consider a cosmological term in the context of stringy gravity then the first question to be asked is what is the appropriate cosmological term to be considered. Rather than taking a simple cosmological constant, a Liouville-type potential (4) may be more appropriate, as one can thereby obtain the potential appropriate to a central charge deficit. Indeed, Kastor-Traschen type [24] cosmological multi-black hole solutions have been discussed in the context of such models by Horne and Horowitz [23], and by Maki and Shiraishi [26]. However, all exact solutions which have been constructed with non-zero dilaton couplings obtained involve a dilaton which depends on certain special powers of the time-dependent cosmic scale factor [26], and thus in particular they possess no static limit.

On the face of it, it would appear to be simple to rule out physically relevant black hole solutions in models with a potential (4), on the grounds that they do not possess "realistic" asymptotics. In particular, if by "realistic" asymptotics we require that black holes be asymptotically flat or asymptotically of constant curvature we might conjecture that:

(i) If $\mathscr{V}$ is non-zero then asymptotically (anti)-de Sitter solutions exist if and only if

$$\exists\ \phi_0 \text{ such that } \left.\frac{d\mathscr{V}}{d\phi}\right|_{\phi=\phi_0} = 0 \text{ and } \mathscr{V}(\phi_0) \neq 0. \tag{13}$$

The solutions are asymptotically de Sitter (anti-de Sitter) for $\mathscr{V}(\phi_0) > 0$ ($\mathscr{V}(\phi_0) < 0$).

(ii) If $\mathscr{V}$ is non-zero then asymptotically flat solutions exist if and only if

$$\exists\ \phi_0 \text{ such that } \left.\frac{d\mathscr{V}}{d\phi}\right|_{\phi=\phi_0} = 0 \text{ and } \mathscr{V}(\phi_0) = 0. \tag{14}$$

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The fact that trivial solutions exist under both these circumstances is quite obvious: if (13) holds then the Schwarzschild-(anti)-de Sitter solutions with constant dilaton, $\phi = \phi_0$, are solutions; while if (14) is satisfied then the Schwarzschild solution with constant dilaton, $\phi = \phi_0$, is a solution. Any particular potential may have many such solutions, depending on the number of different turning points.

The proof of necessity is much less clearly defined, since in the context of stringy gravity it is not obvious what are physically realistic restrictions to place on the dilaton asymptotically. The approach one would take in a conventional field theory setting is to demand that all fields have regular Taylor expansions at spatial infinity, which in terms of the coordinates (6) with $f = f(r)$, $R = R(r)$, are given by

$$\begin{aligned} \phi &= \phi_0 + \frac{\phi_1}{r} + \frac{\phi_2}{r^2} + \dots, \\ f &= \frac{-\Lambda r^2}{3} + f_{-1} r + f_0 + \frac{f_1}{r} + \frac{f_2}{r^2} + \dots, \\ R &= r + R_0 + \frac{R_1}{r} + \frac{R_2}{r^2} + \dots, \end{aligned} \tag{15}$$

if the solutions are assumed to be asymptotically flat or (anti)-de Sitter depending on the value of Λ. If we now take the independent field equations obtained by variation of the action (3), *viz.*

$$\left[R^2 f \phi'\right]' = \frac{d\mathcal{V}}{d\phi} R^2 + \frac{g_0}{R^2}\left[Q^2 e^{2g_0\phi} - P^2 e^{-2g_0\phi}\right], \tag{16}$$

$$\frac{R''}{R} = -\phi'^2, \tag{17}$$

$$\left[f\left(R^2\right)'\right]' = 2 - 4\mathcal{V} R^2 - \frac{2}{R^2}\left[Q^2 e^{2g_0\phi} + P^2 e^{-2g_0\phi}\right], \tag{18}$$

where $' \equiv d/dr$, and substitute the expansions (15), we obtain the result

$$\left.\frac{d\mathcal{V}}{d\phi}\right|_{\phi=\phi_0} = 0, \qquad \Lambda = 2\mathcal{V}(\phi_0), \tag{19}$$

from the lowest order terms, which proves the conjecture. "Realistic" black hole solutions are consequently ruled out for the Liouville-type potential (4), since it is monotonic in ϕ, except in the special case of a cosmological constant ($g_1 = 0$) when $\frac{d\mathcal{V}}{d\phi} \equiv 0$ identically.

In the context of stringy gravity, however, the asymptotic condition on ϕ given in (15) is too restrictive. In particular, in string theory all couplings between the dilaton and matter fields involve powers of $e^{2\phi}$, so that provided the dilaton energy-momentum tensor is well-behaved at spatial infinity one would expect the *weak-coupling limit* $\phi \to -\infty$ to be physically admissible asymptotically. In [32] we have therefore given

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a proof of necessity of the above conjecture in the case of singly charged black hole solutions with a Liouville-type potential (4), without making any restrictions on the asymptotic behaviour of ϕ. Our approach generalises a similar method used to derive the global properties of uncharged static spherically symmetric solutions in a variety of models involving scalar fields with potentials [27, 28, 41]. Unfortunately, we have been unable to extend the method to deal with arbitrary potentials, $\mathscr{V}(\phi)$, and so are unable to offer a proof of the conjecture except on a case by case basis.

The argument of [32] proceeds by using an alternative radial coordinate [14], defined by $d\xi = f^{-1}R^{-2}dr$, and a slightly generalised metric, namely

$$ds^2 = f\left[-dt^2 + R^4 d\xi^2\right] + R^2 g_{ij}\,dx^i dx^j, \tag{20}$$

where now $f = f(\xi)$, $R = R(\xi)$ and g_{ij} is a 2-dimensional metric of constant curvature

$$\mathscr{R}_{ij} = \bar{\lambda}\, g_{ij}, \quad i = 1, 2. \tag{21}$$

Spherically symmetric solutions have $\bar{\lambda} > 0$ – however, integral curves in the $\bar{\lambda} > 0$ portion of the phase space can also have endpoints with $\bar{\lambda} = 0$, which is why this generalisation is necessary. It is then possible to convert the field equations into a 5-dimensional autonomous system of ordinary differential equations, and consequently various properties of the solutions can be derived from the 5-dimensional phase space using standard techniques from the theory of dynamical systems. The crucial results which concern us are:

(i) Critical points at finite values of the phase space parameters consist of a 2-parameter set with $\bar{\lambda} = 0$, $\Lambda = 0$ and $Q = 0$ (or $P = 0$ as appropriate). Solutions with $\bar{\lambda} > 0$ with endpoints at such critical points are found to correspond either to central singularities, or in the case of a 1-parameter subset, to horizons.

(ii) Critical points at phase space infinity can be divided up according to whether $R \to 0$ (central singularity), $R \to$ constant (Robinson-Bertotti solutions), or $R \to \infty$ (asymptotic region).

The asymptotic properties of the solutions are conveniently summarised in terms of the metric functions one obtains when using R as the radial variable, *viz.*

$$ds^2 = -f\,dt^2 + h^{-1}dR^2 + R^2 d\Omega^2, \tag{22}$$

where $h(R) \equiv f\left(\frac{dR}{dr}\right)^2$. The possible asymptotic behaviour of the solutions is summarised in Table 1.

The points $M_{1,2}$ correspond to the asymptotically flat solutions which lie in the subspace with a vanishing potential, $\mathscr{V}(\phi) \equiv 0$, and which of course include the black hole solutions of [10, 14]. The only other instance in which "realistic" asymptotics

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TABLE 1. Asymptotic form of singly charged solutions for trajectories approaching critical points at phase space infinity from within the sphere at infinity, in the case that $R \to \infty$. The points are labelled as in [32] – in some cases the values of g_0, g_1, $\bar{\lambda}$ and Λ are restricted if certain points are to exist. Here $\beta_1 = 1 + 2\,[2 - g_1\,(g_1 \pm g_0)]\,/\,(g_1 \pm g_0)^2$ and $\beta_2 = 2g_1\,/\,(g_1 \pm g_0) - 1$. In all instances the + sign refers to electric solutions and the − sign to magnetic solutions.

	$\|f\|$	$\|h\|$	$e^{2\phi}$
$K_{1,2}$	R^{2/g_0^2}	const.	$R^{\pm 2/g_0^2}$
$M_{1,2}$	const.	const.	const.
$N_{1,2}$	R^2	$R^{2(1-g_1^2)}$	R^{2g_1}
$P_{1,2}$	R^{2/g_1^2}	const.	R^{2/g_1}
$T_{1,2}$	$R^{2\beta_1}$	$R^{2\beta_2}$	$R^{4/(g_1 \pm g_0)}$

are obtained is the special case of a cosmological constant, $g_1 = 0$, when points $N_{1,2}$ correspond to asymptotically de Sitter or anti-de Sitter regions, according to the sign of Λ. The weak coupling limit for the dilaton is attained in various cases for points other than $M_{1,2}$, however, the metric is not asymptotically flat or of constant curvature in any of these instances.

One must add the caveat that the points $K_{1,2}$ provide an interesting special case in which solutions exist with a Riemann tensor whose components in an orthonormal frame vanish as $1/R^2$ at spatial infinity, even though the asymptotic structure of the spatial hypersurfaces at spatial infinity is not that of flat space [32]. Solutions with endpoints at $K_{1,2}$ will include a 1-parameter subset with regular horizons [2, 32], and in certain cases the dilaton will be asymptotically weakly coupled. Chan, Horne and Mann [2], who have studied these exotic black hole solutions in detail, argue that such solutions could be physical. Clearly, the physical properties of solutions with such unusual asymptotics deserve further study. However, we will put aside these interesting questions and will make the more conservative choice of demanding that the solutions be asymptotically flat or asymptotically (anti)-de Sitter. With this proviso the only possible "realistic" black hole solutions with a non-trivial dilaton potential would be those corresponding to integral curves from outside the $\mathcal{V} \equiv 0$ subspace which nonetheless have endpoints at $M_{1,2}$. An analysis of small perturbations about $M_{1,2}$ reveals that this is not the case, however. This completes the proof of the non-existence of charged black hole solutions for the Liouville potential (4) in all cases other the pure cosmological constant, $g_1 = 0$.

3. Static spherically symmetric solutions with a cosmological constant

In the case of a cosmological constant an analysis of small perturbations around $N_{1,2}$ reveals that such points attract or repel a 5-dimensional set of trajectories. To

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further show that black hole solutions exist it is necessary to show that there exist integral curves linking points $N_{1,2}$ to the 1-parameter set of critical points mentioned above which correspond to regular horizons. This seems reasonable given that the dimensionality of the set of solutions with endpoints at $N_{1,2}$ is greater than that of the other critical points. However, although it might be possible to give a rigorous argument simply from the properties of the phase space, the large dimensionality of the phase space makes it difficult. There is one further complication if $\Lambda > 0$ since in that case the asymptotic de Sitter region corresponds to one in which the Killing vector $\partial/\partial t$ is spacelike. The outermost horizon is therefore a cosmological one, and to obtain genuine black hole solutions at least two horizons are required. In terms of the phase space this means an additional requirement of the existence of integral curves which link different points in the 1-parameter family of solutions that correspond to horizons. Again the large dimensionality of the phase space makes the resolution of this question difficult. Instead we have found that the issue can be settled by a simple argument relating to the original dilaton field equation (16).

THEOREM 3.1. *The maximum number of regular horizons possessed by a static spherically symmetric spacetime coupled to the dilaton in the presence of a cosmological constant, $\mathcal{V} \equiv \Lambda/2$, and an electromagnetic field which is either electrically charged ($Q \neq 0$, $P = 0$) or magnetically charged ($P \neq 0$, $Q = 0$) with arbitrary coupling constant, g_0, is one.*[4]

PROOF. A straightforward proof by contradiction was given in [30]. Consider the case $P = 0$ and $g_0 > 0$. Suppose that the spacetime possesses at least two regular horizons, and let the two outermost horizons be labelled $r_\pm$, with $r_- < r_+$. Regularity implies that near $r = r_+$, $f \propto (r - r_+)$ and $\phi(r_+)$ and $R(r_+)$ are bounded with $R(r_+) \neq 0$, and similarly for r_-. From (16) it follows that at both horizons

$$\phi' f' \Big|_{r_\pm} = \frac{g_0 Q^2 e^{2g_0\phi}}{R^2}\Big|_{r_\pm} . \tag{23}$$

Suppose that $\partial/\partial t$ is timelike in the asymptotic region, as is appropriate for asymptotically flat or asymptotically anti-de Sitter solutions, so that $f(r) < 0$ on the interval (r_-, r_+), $f'(r_-) < 0$ and $f'(r_+) > 0$. For such solutions (23) implies that $\phi'(r_-) < 0$ and $\phi'(r_+) > 0$. These two values of ϕ' must be smoothly connected and thus $\phi'(r)$ must go through zero at least once in the interval (r_-, r_+) at a point r_0 such that $\phi''(r_0) > 0$. However, since $f(r) < 0$ on the interval (r_-, r_+) it follows from (16) that if $\phi'(r_0) = 0$ then $\operatorname{sgn}\phi''(r_0) = -\operatorname{sgn} g_0 < 0$. We thus obtain a contradiction.

[4]For $\Lambda > 0$ this result is more restrictive than that obtained by Okai [29], who also placed limits on the number of possible horizons by a different approach.

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If $\partial/\partial t$ is spacelike in the asymptotic region, as is appropriate for asymptotically de Sitter solutions, then $f'(r_-) > 0$ and $f'(r_+) < 0$, and thus the signs of $\phi'(r_-)$ and $\phi'(r_+)$ are reversed in the above argument. However, since $f(r) > 0$ now on the interval (r_-, r_+), the sign of $\phi''(r_0)$ must also be reversed and one still obtains a contradiction. Similar remarks apply if one takes the purely magnetic case ($Q = 0$), or if one reverses the sign of g_0, which completes the proof for all cases.

COROLLARY 3.2. *No static spherically symmetric asymptotically de Sitter black hole solutions exist when coupled to the dilaton in the presence of a cosmological constant, $\mathcal{V} \equiv \Lambda/2$, and an electromagnetic field which is either electrically charged ($Q \neq 0$, $P = 0$) or magnetically charged ($P \neq 0$, $Q = 0$) with arbitrary coupling constant, g_0.*

PROOF. As was noted above, black hole solutions with $\Lambda > 0$ must have at least two horizons, and so the result follows.

It is clear that the above results will not apply to the case of dyonic solutions, since then the right-hand side of (16) does not have a definite sign, which was a crucial requirement of the argument based on (23). Nevertheless, similar restrictions can be made.

THEOREM 3.3. *The maximum number of regular horizons possessed by a static spherically symmetric spacetime coupled to a non-constant dilaton field in the presence of a cosmological constant, $\mathcal{V} \equiv \Lambda/2$, and an electromagnetic field which is both electrically and magnetically charged, $Q \neq 0$ and $P \neq 0$, with arbitrary coupling constant, g_0, is:*

(i) *two, if $\partial/\partial t$ is timelike in the asymptotic region;*
(ii) *one, if $\partial/\partial t$ is spacelike in the asymptotic region.*

PROOF. As was shown in [31] a simple proof by contradiction is obtained by assuming the existence of at least two regular horizons, r_- and r_+, as previously. It is convenient to define a rescaled dilaton

$$\Phi \equiv \phi + \frac{1}{2g_0} \ln\left|\frac{Q}{P}\right|. \tag{24}$$

If the dilaton in (16) is rescaled in this fashion, and the resulting equation multiplied by Φ we obtain

$$\left[R^2 f\, \Phi\Phi'\right]' = R^2 f\, \Phi'^2 + 2|QP|\,\frac{g_0\Phi \sinh(2g_0\Phi)}{R^2}. \tag{25}$$

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This equation can be integrated with respect to r between the horizons, r_- and r_+, with the result that the integral of the left-hand side vanishes. If $f(r) > 0$ on the interval (r_-, r_+) then the right-hand side of (25) is positive-definite, and a contradiction is obtained in all cases except that of a trivial constant dilaton, $\Phi = 0$, $\Phi' = 0$. A succession of three regular horizons is thus ruled out in the case of non-trivial solutions since this would require that either $f(r) > 0$ between the first pair of horizons or else $f(r) > 0$ between the second pair. Furthermore, we can have $f(r) < 0$ between the outermost horizons only if $f(r) > 0$ on the interval (r_+, ∞), that is, if $\partial/\partial t$ is timelike in the asymptotic region, which is true for asymptotically flat and asymptotically anti-de Sitter solutions but not for asymptotically de Sitter solutions. The latter solutions can thus have at most one regular horizon if the dilaton is non-trivial.

COROLLARY 3.4. *The unique static spherically symmetric asymptotically de Sitter black hole spacetime coupled to the dilaton in the presence of a cosmological constant,* $\mathcal{V} \equiv \Lambda/2$, *and an electromagnetic field which is both electrically and magnetically charged,* $Q \neq 0$ *and* $P \neq 0$, *with arbitrary coupling constant,* g_0, *is the Reissner-Nordström-de Sitter solution with constant dilaton*:

$$\begin{aligned} R(r) &= r, \\ f(r) &= \frac{-\Lambda r^2}{3} + 1 - \frac{2M}{r} + \frac{2|QP|}{r^2}, \\ e^{2g_0\phi_0} &= \left|\frac{P}{Q}\right|. \end{aligned} \tag{26}$$

PROOF. Since black hole solutions with $\Lambda > 0$ must have at least two horizons and $\partial/\partial t$ is spacelike in the asymptotic region, it follows from (26) that $\Phi = 0$, $\Phi' = 0$ is the only possible dilaton solution. The solution (26) is then obtained by direct integration.

If we require that $\phi_0 = 0$, so as to obtain the conventional normalisation for the electromagnetic field at spatial infinity, we see that (26) is in fact the solution for equal electric and magnetic charges, $|Q| = |P|$. The exact solution (26) of course also applies to $\Lambda \leq 0$. In addition, there also exist Robinson-Bertotti-type solutions of the form $e^{2g_0\phi_0} = |P|/|Q|$, $R = R_{\text{ext}}$ and

$$f = \left[R_{\text{ext}}^{-2} - 4\Lambda\right] + c_1 r + c_2, \tag{27}$$

where c_1 and c_2 are arbitrary constants, and the constant R_{ext} is a solution of the quartic

$$-\Lambda R_{\text{ext}}^4 + R_{\text{ext}}^2 - 2|QP| = 0. \tag{28}$$

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4. Asymptotically anti-de Sitter black holes

In view of the results outlined in the last section it is clear that in the presence of a cosmological constant, Λ, charged or dyonic black holes with a non-trivial dilaton will only exist if $\Lambda < 0$. Unfortunately there is no obvious way of determining an exact solution in closed form. In [30, 31] we have therefore resorted to numerical integration.

If we solve for the series expansions (15) from the field equations we find

$$\begin{aligned} \phi &= \frac{\phi_3}{r^3} + \mathrm{O}\left(\frac{1}{r^4}\right), \\ f &= \frac{-\Lambda r^2}{3} + 1 - \frac{2M}{r} + \frac{Q^2+P^2}{r^2} + \frac{\Lambda {\phi_3}^2}{5r^4} + \mathrm{O}\left(\frac{1}{r^5}\right), \\ R &= r - \frac{3{\phi_3}^2}{20r^5} + \mathrm{O}\left(\frac{1}{r^6}\right), \end{aligned} \tag{29}$$

where we have made the gauge choice $R_0 = 0$, $\phi_0 = 0$, which is the same choice made in (6)–(10). The effect of the dilaton is therefore short range, as opposed to the asymptotically flat case (12). The black holes with a massive dilaton [18, 22] similarly give rise to a short range force – however, in that case $\phi = \phi_4^{-4} + \mathrm{O}\left(r^{-5}\right)$ asymptotically. Furthermore, ϕ_4 is fixed in terms of the other charges directly by the asymptotic series solution in the solutions of [18, 22], whereas here ϕ_3 is only fixed once we make the further demand that particular solutions with the asymptotic form (29) correspond to black holes.

The appropriate constraint on ϕ_3 is found by integrating the dilaton equation (16) between the outermost horizon, $r_{\mathscr{H}}$, and spatial infinity, to give

$$\phi_3 = \frac{g_0}{\Lambda}\int_{r_{\mathscr{H}}}^{\infty} \frac{dr}{R^2}\left(Q^2 e^{2g_0\phi} - P^2 e^{-2g_0\phi}\right). \tag{30}$$

This result is a direct analogue of (11) – it fixes ϕ_3 in terms of the other charges of the theory, so that ϕ_3 is not an independent "hair".

For the purposes of numerical integration it is convenient to use R as the radial variable, and to work with the metric (22). One can then solve for f in terms of h and ϕ: $f = h\exp\left[2\int^R d\bar{R}\,\dot{\phi}^2\bar{R}\right]$, where $\cdot \equiv d/dR$. Only two independent field equations then remain, *viz.*

$$-R\dot{h} + 1 - h = \left(\Lambda + h\dot{\phi}^2\right)R^2 + \left(Q^2 e^{2g_0\phi} + P^2 e^{-2g_0\phi}\right)\frac{1}{R^2}, \tag{31}$$

$$Rh\ddot{\phi} + s(h, \phi, \dot{\phi}) = 0, \tag{32}$$

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where

$$s(h,\phi,\dot{\phi}) \equiv \dot{\phi}\left[1 + h - \Lambda R^2 - \left(Q^2 e^{2g_0\phi} + P^2 e^{-2g_0\phi}\right)\frac{1}{R^2}\right] \\ - \frac{g_0}{R^3}\left(Q^2 e^{2g_0\phi} - P^2 e^{-2g_0\phi}\right). \tag{33}$$

The numerical procedure we adopted was to treat the problem as an initial value one, and to integrate outwards from a regular horizon, $R_{\mathscr{H}}$, the initial data being set a small distance away from $R_{\mathscr{H}}$ using the solutions obtained by substituting the series expansions

$$h = \sum_{i=1}^{\infty} \tilde{h}_i \,(R - R_{\mathscr{H}})^i\,, \\ \phi = \phi_{\mathscr{H}} + \sum_{i=1}^{\infty} \tilde{\phi}_i \,(R - R_{\mathscr{H}})^i \tag{34}$$

in the field equations (31) and (32). For the lowest order terms, for example,

$$\tilde{h}_1 = \frac{1}{R_{\mathscr{H}}^3}\left[R_{\mathscr{H}}^2 - \Lambda R_{\mathscr{H}}^4 - Q^2 e^{2g_0\phi_{\mathscr{H}}} - P^2 e^{-2g_0\phi_{\mathscr{H}}}\right], \\ \tilde{\phi}_1 = \frac{g_0}{\tilde{h}_1 R_{\mathscr{H}}^4}\left(Q^2 e^{2g_0\phi_{\mathscr{H}}} - P^2 e^{-2g_0\phi_{\mathscr{H}}}\right). \tag{35}$$

As the higher order terms are very lengthy we will not list them here. An outer horizon is obtained if we restrict to initial data such that $\tilde{h}_1 > 0$. We then integrate out until the solutions match the appropriate asymptotic series expressed in terms of R, rather than in terms of r as in (29). This is possible for a range of initial values, $R_{\mathscr{H}}$, $\phi_{\mathscr{H}}$. Details of the numerical procedures are discussed in [30, 31] and numerous resulting plots are presented there. Our results indicate that asymptotically anti-de Sitter black holes with a non-trivial dilaton do exist and with all possible numbers of horizons which are not excluded by Theorems 3.1 and 3.3. I will not reproduce all these results here but will outline the important features.

Dyonic solutions There is an additional numerical complication for dyonic solutions since it is possible, but not guaranteed, for asymptotically anti-de Sitter solutions to exist with two horizons in this case. For singly charged solutions, on the other hand, two horizons are ruled out by Theorem 3.1. To deal with the issue of the number of horizons, we first examined the case of the asymptotically flat solutions with arbitrary coupling constant, g_0, since it appears that exact dyonic solutions are only known in the cases $g_0 = 1$ [10, 11, 14] and $g_0 = \sqrt{3}$ [7, 15]. Although the singular nature of the differential equations (31) and (32) in general prevents one integrating through

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the horizons, it is possible to determine whether one is dealing with a genuine second horizon or not by examining the quantity $s(h, \phi, \dot{\phi})$ defined by (33). At a genuine horizon s should approach zero as h approaches zero. Interestingly enough, our numerical results for the asymptotically flat case indicate that we appear to be dealing with a non-linear eigenvalue problem. These results [31] are consistent with the existence of two horizons only if g_0 is the square root of a triangle number:

$$g_0 = 0, 1, \sqrt{3}, \sqrt{6}, \ldots, \sqrt{n(n+1)/2}, \ldots \tag{36}$$

In the case of the asymptotically anti-de Sitter solutions a similar qualitative behaviour is observed, with two horizons only existing for certain critical values of g_0, which differ slightly from the values (36). However, whereas the critical values of g_0 are independent of the initial conditions in the asymptotically flat case, this is no longer true for asymptotically anti-de Sitter solutions. Thus the critical values would appear to involve some complicated relationship between g_0, Λ, M and $|QP|$.

As far as the thermodynamics properties of the solutions are concerned, there is no qualitative difference between the cases of dyonic black holes with one or two horizons. In particular, the pattern of isotherms [31] is qualitatively the same in both cases, and the Hawking temperature of the solutions, which is given in general by

$$T = \frac{1}{4\pi R_{\mathcal{H}}^3}\left[R_{\mathcal{H}}^2 - \Lambda R_{\mathcal{H}}^4 - Q^2 e^{2g_0\phi_{\mathcal{H}}} - P^2 e^{-2g_0\phi_{\mathcal{H}}}\right], \tag{37}$$

is always zero in the extreme limit. This is related to the fact that the extreme solution always occurs at a finite value of R determined from (28):

$$R_{\text{ext}} = \frac{-1}{2\Lambda}\left[\sqrt{1 - 8\Lambda|QP|} - 1\right]. \tag{38}$$

The spacetime geometry of the extreme solutions approaches that of the Robinson-Bertotti-type solutions (27) and (28) in the neighbourhood of the degenerate horizon. Furthermore, the entropy – at least, the entropy naïvely defined as one quarter the area of the event horizon – is finite in the extreme limit, similarly to the Reissner-Nordström and Reissner-Nordström-anti de Sitter solutions.

Singly charged solutions The solutions with a single electric or magnetic charge are found to be qualitatively different to the dyonic solutions. This would appear to stem from the fact that there are no Robinson-Bertotti-type solutions in this case. As in the case of the asymptotically flat dilaton black holes there is a maximum of one horizon and the extreme limit corresponds to $R_{\mathcal{H}} \to 0$ and $\phi_{\mathcal{H}} \to \infty$ for magnetic solutions, (or $\phi_{\mathcal{H}} \to -\infty$ for electric solutions), independently of the value of Λ, assuming $\Lambda < 0$ and $g_0 > 0$. Consequently, from (37) we see that the extreme solutions will have a formally infinite Hawking temperature and zero entropy. This is

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indeed verified by the numerical solutions [30]. Of course, the infinite temperature is merely a signal of the breakdown of the semi-classical approximation.

In the case of the $g_0 = 1$ magnetic solutions, one can verify that the "throat structure" of the string conformal frame metric of the asymptotically flat extreme solutions is preserved for the asymptotically anti-de Sitter black holes. In particular, consider the independent field equations derived from the action (1), with $\hat{\mathscr{V}} = \Lambda e^{-2\phi}/2$, as is appropriate for an Einstein frame cosmological term. With a string conformal frame metric of the form

$$ds^2 = -\hat{f}\,d\tau^2 + \hat{f}^{-1}d\rho^2 + \hat{R}^2 d\Omega^2, \tag{39}$$

$\hat{f} = \hat{f}(\rho)$, $\hat{R} = \hat{R}(\rho)$, these are given by

$$\left[\hat{R}^2\hat{f}\left(e^{-2\phi}\right)'\right]' = 2e^{-2\phi}\frac{P^2}{\hat{R}^2}, \tag{40}$$

$$\frac{\hat{R}''}{\hat{R}} = -\phi'^2, \tag{41}$$

$$\left[\hat{f}\left(\hat{R}^2 e^{-2\phi}\right)'\right]' = 2e^{-2\phi} - 2\Lambda\hat{R}^2 e^{-4\phi} - 2e^{-2\phi}\frac{P^2}{\hat{R}^2}, \tag{42}$$

where now $' \equiv d/d\rho$. In the asymptotically flat case [10] the neighbourhood of the $\phi_{\mathscr{H}} \to \infty$ singularity is a so-called "linear dilaton vacuum", that is, of the form $\phi = -\alpha\rho$ as $\rho \to \infty$, ($\alpha < 0$ being a constant), with vanishing corrections of the form $e^{\alpha\rho}$. We thus look for similar solutions of the form

$$\begin{aligned} e^{-2\phi} &= \hat{\phi}_1 e^{\alpha\rho} + \hat{\phi}_2 e^{2\alpha\rho} + \cdots, \\ \hat{f} &= \hat{f}_0 + \hat{f}_1 e^{\alpha\rho} + \cdots, \\ \hat{R} &= \hat{R}_0 + \hat{R}_1 e^{\alpha\rho} + \cdots. \end{aligned} \tag{43}$$

It is straightforward to verify from the field equations (40)–(42) that the following leading order coefficients are unaffected[5] by the presence of Λ: $\hat{\phi}_1$ (free), $\hat{\phi}_2 = -2\hat{\phi}_1\hat{R}_1/\hat{R}_0$, $\hat{R}_0^2 = 2P^2$, $\hat{f}_0 = 1/(2P^2\alpha^2)$. Thus the extremal solution does indeed appear to be a direct analogue of the one of Garfinkle, Horowitz and Strominger [10]. The cosmological constant only affects the higher-order corrections: $\hat{f}_1 = -\Lambda\hat{\phi}_1/\alpha^2$, $\hat{R}_1 = c_1 - 2\sqrt{2}\Lambda\alpha P^3\hat{\phi}_1\rho/3$, *etc.*

5. Conclusion

If we neglect the possibility of black holes with an exotic asymptotic structure [2], then no physically interesting black hole spacetimes exist if the dilaton has a

[5]By contrast, it is possible that $\hat{f}_0$ is altered for certain non-trivial dilaton potentials [18].

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Liouville-type potential, except in the case that this potential is a simple cosmological constant in the Einstein conformal frame. Black hole solutions with a non-trivial dilaton only exist for a negative cosmological constant. These spacetimes are asymptotically anti-de Sitter with a short range correction due to the dilaton. The horizon structure and thermodynamic properties of the solutions are the same as corresponding asymptotically flat solutions with a massless dilaton in both the dyonic and singly charged cases.

Although a pure cosmological constant may not be the most natural cosmological term in the context of stringy gravity, we hope that the solutions we have studied here may nonetheless provide a useful approximation in some circumstances. In particular, if we have a non-trivial dilaton potential and if the minimum of that potential which corresponds to the groundstate of the dilaton has a value which is not precisely zero, then the universe would contain some (hopefully small) vacuum energy. If this vacuum energy is negative then the black hole solutions studied here (with $\Lambda < 0$) might provide a useful approximation to the complete solutions, just as the solutions of [18, 22] are well approximated by the solutions with a massless dilaton in certain regimes. If there is to be a non-zero vacuum energy then phenomenologically a small positive vacuum energy is the one favoured by current measurements of the Hubble constant. Although we find no black hole solutions with a non-trivial dilaton for $\Lambda > 0$, it is possible that this conclusion could be altered in the presence of a non-trivial dilaton potential, $\mathcal{V}(\phi)$. In particular, the dilaton equation (16), upon which Theorems 3.1 and 3.3 are based, acquires an additional $\frac{d\mathcal{V}}{d\phi}$ term which could provide an obstruction to these arguments for suitable potentials. Of course, for suitably complex potentials there are many additional features which could further complicate things.

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