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Global Bar–Forming Instabilities

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1. Introduction

We have recently obtained some interesting results on the global stability characteristics of gaseous and stellar systems using a free–energy variational formulation and Ginzburg–Landau phase transition theory (Christodoulou et al. 1995a, b). For $m = 2$ nonaxisymmetric modes in particular, we have been able to formulate a new, robust, global stability criterion (Christodoulou, Shlosman, & Tohline 1995, hereafter CST) that avoids many of the problems that plagued the well–known Ostriker & Peebles (1973) criterion. In this article, we briefly summarize the conclusions from the above–cited investigations and we proceed to comment on the stability properties of some interesting stellar models that were discussed during this conference and that are not trivially understood in terms of the new stability criterion.

2. Summary of Results

(a) Phase Transitions: Axisymmetric stellar systems suffer an $m = 2$ "bar mode" when the ratio of the rotational kinetic energy to the gravitational potential energy $T/|W| \gtrsim 0.14$ (Ostriker & Peebles 1973) but their fluid counterparts suffer a similar $m = 2$ instability when $T/|W| \gtrsim 0.27$. The difference between the two cases is caused by the different conservation laws that are valid in evolving stellar and gaseous systems: *(i)* Circulation is not conserved in stellar systems which are then free to undergo a dynamical second–order phase transition toward a nonaxisymmetric state of lower free energy and the same total mass and

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angular momentum. This equilibrium state becomes available to an axisymmetric "stellar Maclaurin" system beyond the bifurcation point of the "stellar Jacobi sequence" which consistently occurs at a value of $T/|W| \approx 0.14$ in a wide variety of axisymmetric stellar equilibrium sequences. *(ii)* In gaseous systems, the conservation law of circulation renders the above bifurcation point irrelevant. A dynamical second–order phase transition can then occur only beyond the bifurcation point of a sequence whose equilibria have lower free energies and conserve circulation along with total mass and angular momentum. A bar–like (ellipsoidal) sequence that exhibits all these properties bifurcates at a value of $T/|W| \approx 0.27$ in a wide variety of axisymmetric gaseous equilibrium sequences.

(b) Bar Formation As a Global Process: According to the above description, the formation of bar–like systems is the natural outcome of global $m = 2$ instabilities acting in axisymmetric stellar and gaseous systems beyond the corresponding bifurcation points and obeying the relevant conservation laws. When the bifurcating sequences exist, the unstable modes must operate and must complete the dynamical transition that leads to the bar–like equilibrium state of lower free energy. This fully nonlinear description of dynamical phase transitions is markedly different from the commonly discussed local linear analysis that relies on the absence of an inner Lindblad resonance for wave amplification by reflection at the center of a model.

(c) New Stability Criterion: The new stability indicator α is based on the angular momentum content of a system rather then on its energy content. It can be written in the interesting form

$$\alpha = \sqrt{\frac{1}{2} f t} \ , \qquad (0 \leq \alpha \leq 1/2) \ , \tag{1}$$

where $t \equiv T/|W|$ and f is a function that depends on both the *geometrical* and the *topological* structure of the system. For stability to $m = 2$ modes, we have found that stellar systems must have $\alpha \lesssim 0.25$ while gaseous systems must have $\alpha \lesssim 0.34$. In the case of uniformly rotating systems, eq. (1) implies that

$$\alpha = \frac{L \Omega_J}{2|W|} \ , \tag{2}$$

where L is the total angular momentum and Ω_J is the gravitational (Jeans) frequency introduced by self–gravity. Besides being a useful alternative expression for differentially rotating systems in which the form of function f is not known, eq. (2) also shows that the parameter α is not equivalent to the ratio $L^2/(2I|W|)$ (where I is the moment of inertia) introduced by Vandervoort (1982) in a pioneering attempt to take into account the total angular momentum content L of a differentially rotating system in place of its total kinetic energy T.

3. The Stability of Two Types of Stellar Models

During the conference, two types of stellar models were largely discussed because their $m = 2$ stability properties are not trivially understood in terms of the above–described bifurcation points and the corresponding behavior of the parameter α:

(a) Davies (1995) has recently studied in detail the stability of Toomre $n = 0$ and $n = 1$ stellar disks to bar formation. Various sequences of $n = 0$ models with different degrees of random–motion support and different numbers of particles in counter–rotation are easily understood since the disks become unstable to $m = 2$ modes for $\alpha \gtrsim 0.25$. However, for the sequences of $n = 1$ models, it is not obvious how a representative value of Ω_J (which is a function of radius) can be determined for use in eq. (2).
(b) Efstathiou, Lake, & Negroponte (1982) have studied stellar disk/halo models and formulated a stability criterion that does not depend on the core radius of the halo and that is understood in terms of the parameter α (see CST for details). On the other hand, Sellwood (1989) has performed computer simulations of such models and reported a dependence of the results on the core radius (just as had previously been found analytically by Toomre in an unpublished study). In particular, Sellwood argued that the instability could be suppressed in weakly perturbed models with small core radii.

Although the new stability criterion is backed by a physical understanding of the nonlinear processes that are responsible for bar formation, its application to the above two cases is not without ambiguities. Firstly, more work is needed to understand the "calibration" of Ω_J when it is a strong function of radius. Secondly, the numerical simulations of Sellwood (1989) have not been examined yet in terms of the parameter α. Contrary to expectations from linear analysis, Sellwood's models were unstable to a *nonlinear* $m = 2$ mode of instability. We believe then that these stellar models with a small compact core cannot be considered as pure exponential disks embedded in a halo; they are rather two–component systems with a central "point–mass." Thus, the observed nonlinear $m = 2$ mode must be the result of a discontinuous λ–transition or of a nonlinear resonance (see Christodoulou et al. 1995b). In such a case, Sellwood's models are not subject to the conventional "Maclaurin-to-Jacobi" phase transition. Instead, they suffer a nonlinear instability akin to those discussed by Christodoulou et al. (1995b) in relation to the "one–ring" equilibrium sequence, and their α–values should be determined along the lines introduced by CST for toroidal models.

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