THE EFFECT OF RADIATION PRESSURE ON THE RESTRICTED THREE-BODY PROBLEM

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ABSTRACT

The classical restricted three-body problem is generalized to include the force of radiation pressure and the Poynting-Robertson effect. The positions of the Lagrangian points L_4 and L_5 are found as functions of β, the ratio of radiational to gravitational forces. The Poynting-Robertson effect renders the L_4 and L_5 points unstable on a time scale (T) long compared to the period of rotation of the two massive bodies. For the solar system, T is given by $T = [(1-\beta)^{2/3}/\beta]\ 544\ a^2$ yr where a is the separation between the Sun and the planet in AU. Implications for space colonization and a mechanism for producing asymmetries in the interplanetary dust complex are discussed; testing the latter may be possible from the Zodiacal Light/Background Starlight Experiment aboard the International Solar Polar Mission spacecraft to be launched in 1983.

A steady state solution of the motion of a β particle (a particle whose surface to volume is such that the force of radiation pressure is non-negligible) in the gravitational field of two larger, orbiting bodies of masses M_1 and M_2 is sought. It is assumed that M_1 and M_2 are in circular orbits about their common center of mass, that their angular frequency (Ω) is given by $\Omega^2 = G(M_1 + M_2)/a^3$, and that the β particle remains in the plane in which M_1 and M_2 revolve. In the rotating reference frame whose origin is the center of mass and whose angular frequency is Ω, the two components of the equation of motion of a β particle with coordinates (X,Y) are

$$\frac{\ddot{X}}{\Omega^2} - \frac{2\dot{Y}}{\Omega} = X - \frac{(1-\beta_1)\mu(X+\mu-1)}{r_1^3} - \frac{(1-\beta_2)(1-\mu)(X+\mu)}{r_2^3} \tag{1}$$

$$\frac{\ddot{Y}}{\Omega^2} + \frac{2\dot{X}}{\Omega} = \left(1 - \frac{(1-\beta_1)\mu}{r_1^3} - \frac{(1-\beta_2)(1-\mu)}{r_2^3}\right)Y \quad . \tag{2}$$

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M_1 is located at $X = 1-\mu$, M_2 is located at $X = -\mu$ and $\mu = M_1/(M_1+M_2)$. The distances from the point (X,Y) to the centers of mass M_1 and M_2 are $r_1 = \{(x+\mu-1)^2 + Y^2\}^{1/2}$ and $r_2 = \{(x+\mu)^2 + Y^2\}^{1/2}$, respectively. The two terms on the left-hand side of equations (1) and (2) represent the acceleration and the coriolis force experienced by the β particle; the centrifugal acceleration has been placed on the right-hand side (RHS). The last two terms on the RHS's of equations (1) and (2) represent the components of the "effective" gravitational forces. These terms differ from those of the classical problem in that *each component has been reduced by a factor of the form* $1-\beta$.

The equilibrium points L_4 and L_5 are found by setting the RHS's of equations (1) and (2) equal to zero with $Y \neq 0$. The solution is

$$r_1 = (1-\beta_1)^{1/3} \equiv \delta_1 \quad , \quad r_2 = (1-\beta_2)^{1/3} \equiv \delta_2 \quad . \tag{3}$$

From solution (3) it follows that the L_4 and L_5 points exist only if all of the following conditions are satisfied: $\delta_1+\delta_2 \geq 1$, $0 < \delta_1 \leq 1$, and $0 < \delta_2 \leq 1$. These results are plotted in Fig. 1 which shows the positions of the L_4 and L_5 points as a function of β_1 and β_2 (and equivalently, δ_1 and δ_2). For application to the solar system, β_2 vanishes and δ_2 is unity; the locus of these solutions, parameterized by β_1, is the heavy arc shown in Fig. 1. (Henceforth, the subscripts on β_1 and δ_1 will be omitted). Furthermore, these solutions, when applied to the solar system ($\mu \simeq 1$) are *stable* in the sense that a small displacement of the β particle from equilibrium will result in oscillations whose amplitude *does not* increase with time.

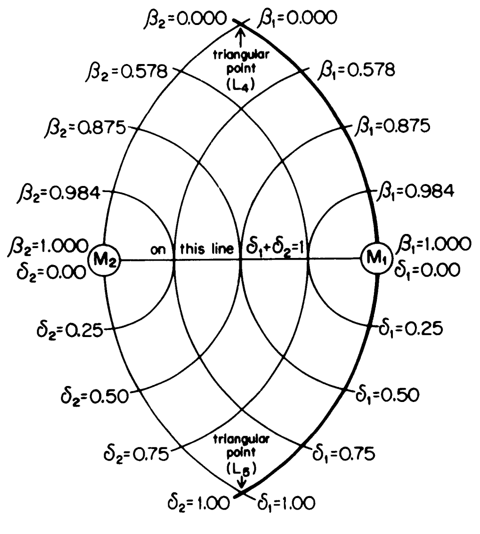

FIG. 1. The possible positions of the equilibrium points. The heavy line applies to systems with only *one* luminous body, M_1.

The results mentioned thus far have an interesting application for artificial satellites and future space colonization. It has been suggested (*e.g.*, O'Neill, 1974) that the classical triangular points of the Sun-Jupiter or Sun-Earth system would be a convenient site to locate future space colonies. Present day technology permits the construction of spacecraft whose value of β can be made or varied in the range $0<\beta<1$. A space colony with a solar facing adjustable "sail" could thus "park" (*i.e.*, remain stationary with respect to the Sun and a planet without expending large amounts of energy) not only at the classical triangular positions but at *any* heliocentric distance out to that of the planet.

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When terms representing the Poynting-Robertson effect,

$$P_x = \frac{-\beta\mu a\Omega}{cr_1^2}\left[\frac{\dot{X}}{\Omega} - Y + \frac{(X+\mu-1)}{r_1^2}\left(\frac{\dot{X}}{\Omega}(X+\mu-1) + \frac{\dot{Y}}{\Omega}Y\right)\right] \tag{4}$$

$$P_y = \frac{-\beta\mu a\Omega}{cr_1^2}\left[\frac{\dot{Y}}{\Omega} + X+\mu-1 + \frac{Y}{r_1^2}\left(\frac{\dot{X}}{\Omega}(X+\mu-1) + \frac{\dot{Y}}{\Omega}Y\right)\right] \quad , \tag{5}$$

are included on the RHS's of equations (1) and (2), the problem can still be solved analytically (to first order terms in $a\Omega/c$) in the same manner as before. The locations of the equilibrium points differ in this treatment only by small corrections of the order of $a\Omega/c$. However, *all solutions are unstable* when the Poynting-Robertson effect is taken into account. The time scale of this instability, when applied to the solar system, is given by

$$T \simeq \frac{(1-\beta)^{2/3}ca^2}{3\beta M_o G} = \frac{(1-\beta)^{2/3}}{\beta} 544\ a^2\ \text{yr} \quad \text{(a in AU's)}. \tag{6}$$

A convenient and representative value for β of 0.57 makes $(1-\beta)^{2/3}/\beta = 1$. Thus, for example, the Sun-Jupiter system (a = 5.2 AU) results in $T \simeq 14{,}000$ yr. This means that if a particle with $\beta = 0.57$ is displaced slightly from its equilibrium position, it will oscillate about that position and the amplitude of that motion will increase by a factor of e in 14,000 yr. (A more complete derivation of these results is given by Schuerman, 1980).

It is not at all evident that the above mechanism could lead to a significant increase in the density of interplanetary β particles along the arc schematically shown as a heavy line in Fig. 1. Other forces act on these β particles which cannot be treated by the analytical methods employed here. Interactions with the gravitational fields of intervening planets would leave large gaps in the arc of β particles. Jupiter, because of its great mass, would be particularly effective in disrupting the dust in arcs associated with other planets; conversely, dust in the Sun-Jupiter arc would be the least affected by planetary encounters. The Lorentz force due to the solar magnetic field also acts on (charged) β particles. This conservative force is roughly 40% that of the solar gravitation (Greenberg and Schuerman, 1978). The uncertainties in the charges of the β particles and in the long term fluctuations and reversals of the solar magnetic field at distances greater than 1 AU make the inclusion of the Lorentz force in this problem particularly difficult. Finally, the ions in the solar wind impinging on the β particles and the Coulomb interaction between solar wind electrons and the β particles contribute non-conservative drag forces which together are estimated to be about 30% of the Poynting-Robertson effect (Bandermann, 1967; Misconi, 1976a). These forces must surely operate to shorten the time scale (T) in equation (6).

Even if planetary encounters do not disrupt the entire arc, and even if the large but conservative Lorentz force averages out to a small effect over many solar cycles, there still remains the need for a replenishment

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mechanism if an increase in the density of β particles along a planetary arc is to be expected on a "steady state" basis. The major source of replenishment for the interplanetary dust complex is thought to be cometary debris (Whipple, 1955 and 1967; Dohnanyi, 1970, 1972, and 1973). Comets shed both β particles (some with $\beta<1$) and larger ($\gtrsim 1$ μm) particles; light scattering by the latter is the main contribution to the zodiacal light. These larger particles ultimately fragment through self-collisions (Zook, 1975) or possibly by rotational bursting (Paddack and Rhee, 1975; Misconi, 1976b). It is not evident whether or not these sources of β particles provide a wide enough distribution in real space and velocity space so as to deposit a significant fraction of "zero-velocity" β particles at their appropriate equilibrium positions in times short compared to T in equation (6).

Fortunately, measurements made during the International Solar Polar Mission may determine whether or not this mechanism produces any "dust arcs" in the solar dust complex. A zodiacal light photopolarimeter (jointly designed by teams at the Ruhr-University, Bochum, FRG and the State University of New York at Albany) will map the brightness and polarization of the zodiacal light throughout the four and one-half year mission to Jupiter and back over the poles of the Sun. During the two periods of solar polar passage, the spacecraft will be in the center of the solar system but at 1.7 AU above (and below) the ecliptic plane. From these vantage points, isophotes of the zodiacal light may reveal any enhancements in the spatial density of the dust along arcs associated with planets.

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