# SELECTED TOPICS IN SPECTRAL GRAPH THEORY

## XIAOGANG LIU

The purpose of this thesis is to study the spectra and several spectral properties of several classes of graphs.

Following the introduction in Chapter 1, the first part of the main body of the thesis consists of Chapters 2 and 3. In this part we determine the spectra of some classes of Cayley graphs, including unitary Cayley graphs [8] and quadratic unitary Cayley graphs [11]. We also investigate some spectral properties of these graphs, such as spectral moments [1], energies [2] and hyperenergeticities [3] of such graphs, and classify those which are Ramanujan graphs [13, 14].

In the second part, which consists of Chapters 4 and 5, we determine the spectra of graphs obtained by some graph operations, including neighbourhood coronae [9], subdivision-vertex neighbourhood coronae and subdivision-edge neighbourhood coronae [5]. By using the spectra, we construct infinitely many pairs of cospectral graphs [1] and new expanders [4, 12] from known ones.

In the last part, Chapters 6–8, we investigate spectral characterisations of some joins [6] and some bicyclic graphs including propeller graphs [10], dumbbell graphs and theta graphs [7]. We prove that all these graphs are determined by their spectra.

## References

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XIAOGANG LIU, School of Mathematics and Statistics,
The University of Melbourne, Victoria 3010, Australia
e-mail: lxg.666@163.com