SELECTED TOPICS IN SPECTRAL GRAPH THEORY

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The purpose of this thesis is to study the spectra and several spectral properties of several classes of graphs.

Following the introduction in Chapter 1, the first part of the main body of the thesis consists of Chapters 2 and 3. In this part we determine the spectra of some classes of Cayley graphs, including unitary Cayley graphs [8] and quadratic unitary Cayley graphs [11]. We also investigate some spectral properties of these graphs, such as spectral moments [1], energies [2] and hyperenergeticities [3] of such graphs, and classify those which are Ramanujan graphs [13, 14].

In the second part, which consists of Chapters 4 and 5, we determine the spectra of graphs obtained by some graph operations, including neighbourhood coronae [9], subdivision-vertex neighbourhood coronae and subdivision-edge neighbourhood coronae [5]. By using the spectra, we construct infinitely many pairs of cospectral graphs [1] and new expanders [4, 12] from known ones.

In the last part, Chapters 6–8, we investigate spectral characterisations of some joins [6] and some bicyclic graphs including propeller graphs [10], dumbbell graphs and theta graphs [7]. We prove that all these graphs are determined by their spectra.

References

- [1] D. Cvetković, P. Rowlinson and S. K. Simić, *An Introduction to the Theory of Graph Spectra* (Cambridge University Press, Cambridge, 2010).
- [2] I. Gutman, 'The energy of a graph', *Ber. Math.-Statist. Sekt. Forsch. Graz* **103**(100–105) (1978), 1–22.
- [3] I. Gutman, 'Hyperenergetic molecular graphs', J. Serb. Chem. Soc. 64(3) (1999), 199–205.
- [4] S. Hoory, N. Linial and A. Wigderson, 'Expander graphs and their applications', Bull. Amer. Math. Soc. 43(4) (2006), 439–561.

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- [5] X. Liu and P. Lu, 'Spectra of subdivision-vertex and subdivision-edge neighbourhood coronae', *Linear Algebra Appl.* 438(8) (2013), 3547–3559.
- [6] X. Liu and P. Lu, 'Signless Laplacian spectral characterization of some joins', *Electron. J. Linear Algebra* **30**(1) (2015), 443–454.
- [7] X. Liu and P. Lu, 'Laplacian spectral characterization of dumbbell graphs and theta graphs', submitted.
- [8] X. Liu and S. Zhou, 'Spectral properties of unitary Cayley graphs of finite commutative rings', *Electron. J. Combin.* 19(4) (2012), #P13.
- [9] X. Liu and S. Zhou, 'Spectra of the neighbourhood corona of two graphs', *Linear Multilinear Algebra* 62(9) (2014), 1205–1219.
- [10] X. Liu and S. Zhou, 'Spectral characterizations of propeller graphs', *Electron. J. Linear Algebra* 27 (2014), 19–38.
- [11] X. Liu and S. Zhou, 'Quadratic unitary Cayley graphs of finite commutative rings', *Linear Algebra Appl.* 479 (2015), 73–90.
- [12] A. Lubotzky, 'Expander graphs in pure and applied mathematics', Bull. Amer. Math. Soc. 49(1) (2012), 113–162.
- [13] A. Lubotzky, R. Phillips and P. Sarnak, 'Ramanujan graphs', *Combinatorica* 8(3) (1988), 261–277.
- [14] M. Ram Murty, 'Ramanujan graphs', J. Ramanujan Math. Soc. 18(1) (2003), 33–52.

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