BULL. AUSTRAL. MATH. SOC. 20M05, 20M07
VOL. 27 (1983), 231-242.

A SEMIGROUP WITH AN EPIMORPHICALLY EMBEDDED SUBBAND

PETER M. HIGGINS

We construct a semigroup S with an epimorphically embedded proper subband U. The band U furnishes an example of a regular semigroup which is not saturated, thus answering a question posed by Hall, *Semigroup Forum* (to appear).

1. Preliminaries and introduction

Let U, S be semigroups with U a subsemigroup of S. Following Howie and Isbell [*10*] we say that U *dominates* an element $d \in S$ if and only if for every semigroup T and all pairs of morphisms $\alpha, \beta : S \to T$, $\alpha|U = \beta|U$ implies that $d\alpha = d\beta$. The set of all elements of S dominated by U is called the *dominion* of U in S and is denoted by $\mathrm{Dom}(U, S)$. It is easily verified that $\mathrm{Dom}(U, S)$ is a subsemigroup of S containing U.

Let $\alpha : S \to T$ be a semigroup morphism. Then α is an *epimorphism* if for every pair of morphisms $\beta, \gamma : T \to V$, $\alpha\beta = \alpha\gamma$ implies $\beta = \gamma$. One can easily show that a morphism $\alpha : S \to T$ is an epimorphisms if and only if the inclusion $i : S\alpha \to T$ is en epimorphism, which is equivalent to the statement that $\mathrm{Dom}(S\alpha, T) = T$.

We say U is *epimorphically embedded* in S if $\mathrm{Dom}(U, S) = S$ and that U is *saturated* if this only occurs when $U = S$. A class of semigroups is *saturated* if all its members have that property. A class of semigroups closed under the taking of morphisms, such as a variety, is

Received 5 November 1982.

saturated if and only if every epimorphism from each member of the class is onto.

The main tool used in the field of semigroup dominions is Isbell's Zigzag Theorem.

RESULT 1 [11, Theorem 2.3 or 9, Chapter 7, Theorem 2.13]. Let U be a subsemigroup of a semigroup S and let $d \in S$. Then $d \in \mathrm{Dom}(U, S)$ if and only if $d \in U$ or there is a series of factorizations of d as follows:

$$d = u_0y_1 = x_1u_1y_1 = x_1u_2y_2 = x_2u_3y_2 = \dots = x_mu_{2m-1}y_m = x_mu_{2m}$$

where $m \geq 1$, $u_i \in U$, $x_i, y_i \in S$ and

$$u_0 = x_1u_1 , \quad u_{2i-1}y_i = u_{2i}y_{i+1} , \quad x_iu_{2i} = x_{i+1}u_{2i+1} \quad (1 \leq i \leq m-1)$$

and

$$u_{2m-1}y_m = u_{2m} .$$

Such a series of factorizations is called a *zigzag* in S over U with *value* d, *length* m and *spine* $u_0, u_1, \dots, u_m$. The proof of this theorem is difficult, but in this paper we only require the 'if' part of the statement, which follows by a straightforward manipulation of the zigzag equations.

The notations and conventions of Clifford and Preston [1, 2] and Howie [9], will be used throughout without explicit reference.

It was proved in 1975 by Gardner [3] that any epimorphism from a regular ring is onto, in the category of rings. It is natural therefore to consider the same question for semigroups, and indeed Hall [4] has explicitly posed the question: does there exist a regular semigroup which is not saturated? This is equivalent to asking the question: does there exist an epimorphism from a regular semigroup which is not onto (in the category of semigroups)? Some recent related results in this area are as follows: epimorphisms are onto for finite regular semigroups [5]; and epimorphisms are onto for generalised inverse semigroups [6].

In this paper we give an example which shows that, in general, regular semigroups are not saturated; indeed the example shows the same is true of orthodox semigroups (regular semigroups whose idempotents form a band), as the example is itself a band.

The example is relevant to other related problems. The problem of finding all saturated varieties of semigroups is open, although all the saturated commutative varieties [7], [12], and all the saturated heterotypical varieties [8] have been determined. A necessary condition for a variety V to be saturated is that it satisfies a homotypical identity of the form $x_1x_2 \ldots x_n = f(x_1, x_2, \ldots, x_n)$ with the x_i all distinct, and such that $|x_i|_f > 1$ for some i [8, Theorem 6]. The example shows that this condition is not in general sufficient, and allows us to strengthen this necessary condition, insofar as we may add that V must admit an identity which is not a consequence of the identity $x = x^2$.

The example is constructed using the same technique as was employed by the author in [7] and [8], namely taking a free semigroup S and factoring by a congruence ρ , generated by relations which ensure that S/ρ is dominated by a subsemigroup U , which is a member of a particular variety of interest.

2. The example

Let F be the free semigroup on $G = X \cup Y \cup A$ where $X = \{x_1, x_2, \ldots\}$, $Y = \{y_1, y_2, \ldots\}$ and $A = \{a_1, a_2, \ldots\}$. The subset $X \cup Y$ of G will be denoted by R . Let ρ_0 be the relation on F consisting of all pairs $\{(a, a^2) \mid a \in \langle A\rangle\}$, together with those defined by the zigzags

$$\begin{aligned} y_n = a_{6n+1}y_{2n+1} &= x_{2n+1}a_{6n+1}y_{2n+1} \\ &= x_{2n+1}a_{6n+2}y_{2n+1} = x_{2n+1}a_{6n+3}y_{2n+1} = x_{2n+1}a_{6n+3} , \end{aligned}$$

for $n = 0, 1, 2, \ldots$ and

$$x_n = a_{6n-2}y_{2n} = x_{2n}a_{6n-2}y_{2n} = x_{2n}a_{6n-1}y_{2n} = x_{2n}a_{6n}y_{2n} = x_{2n}y_{6n}$$

for $n = 1, 2, \ldots$. These are the pairs $(y_n, a_{6n+1}y_{2n+1})$, $(a_{6n+1}, x_{2n+1}a_{6n+1})$, $(a_{6n+1}y_{2n+1}, a_{6n+2}y_{2n+1})$, $(x_{2n+1}a_{6n+2}, x_{2n+1}a_{6n+3})$

https://doi.org/10.1017/S0004972700025715 Published online by Cambridge University Press

and $(a_{6n+3}y_{2n+1}, a_{6n+3})$ for $n = 0, 1, 2, \ldots$, together with a similar collection of pairs defined by the zigzags above with value x_n , $n = 1, 2, \ldots$. Note that y_0 is a name given for $a_1 y_1$ and is not a member of G . Let ρ be the congruence generated by ρ_0 and let $S = F/\rho$ and $U = \langle A \rangle \rho$. By construction, U is a band and $\mathrm{Dom}(U, S) = S$. We show $U \neq S$ by proving that $y_0 \rho \notin U$.

We introduce some definitions to facilitate discussion of the word problem which now arises.

We say an elementary ρ_0-transition $puq \to pvq$, where $p, q \in F^1$, has *base* u and *replacement* v . The transitions themselves are classified as follows: those of the form $paq \to pa^2q$, $a \in \langle A \rangle$, or a reversal of this type are *squaring transitions*; a transition of the form $px_nq \to pa_{6n-2}y_{2n}q$ or $py_nq \to pa_{6n+1}y_{2n+1}q$ is known as an *upward transition* while the corresponding reversal is a *downward transition*. In the general zigzag

$$Z_m : z = a_{3m-2}y_m = x_m a_{3m-2}y_m = x_m a_{3m-1}y_m = x_m a_{3m}y_m = x_m a_{3m}$$

with $z \quad R$, and $(a_{3m-2}, x_m a_{3m-2})$, $(a_{3m-2}y_m, a_{3m-1}y_m)$, $(x_m a_{3m-1}, x_m a_{3m})$ and $(a_{3m}y_m, a_{3m})$ pairs of ρ_0 , the corresponding transitions based on $a_{3m-2}, a_{3m-2}y_m, x_m a_{3m-1}$ and $a_{3m}y_m$ are *forward transitions*, while their reversals are *backward transitions*. Collectively, the upward and forward transitions are known as *positive transitions*, while their reversals are *negative transitions*.

The number of transitions in a sequence of elementary transitions I , is denoted by $|I|$, and I is said to be *positive* if it consists entirely of positive transitions. Given two sequences I_1, I_2 of elementary transitions in which the last word of I_1 coincides with the first of I_2 , we define their product, $I_1 \cdot I_2$ by concatenation.

We say that a generator $z \in G$ has *index* m if it occurs in the zigzag equations of Z_m and write $\mathrm{ind}\, z = m$ (note the value of Z_m does

https://doi.org/10.1017/S0004972700025715 Published online by Cambridge University Press

not have index m). We say that $y_m \in Y$ (respectively $x_m \in X$) has the letters a_{3m-2} (respectively a_{3m}) and a_{3m-1} as *associates* and the letter a_{3m} (respectively a_{3m-2}) as *annihilator*. The letters a_{3m-2}, a_{3m-1} (respectively a_{3m-1}, a_{3m}) are mutual *y-companions* (respectively *x-companions*).

LEMMA 2. *If* $z \in G$ *then there exists a unique shortest sequence* $I(z)$ *of elementary transitions* $I(z) : y_0 \to \ldots \to w$, *such that* $z \in C(w)$. *Further* $I(z)$ *is positive.*

Proof. We proceed by induction on $m = \operatorname{ind} z$. If $m = 1$ we see by inspection that $I(a_1) = I(y_1)$ is trivial and that the shortest sequences which introduce the other letters of index 1 are respectively,
$I(x_1) : a_1y_1 \to x_1a_1y_1$, $I(a_2) : a_1y_1 \to a_2y_1$, and
$I(a_3) : a_1y_1 \to x_1a_1y_1 \to x_1a_2y_1 \to x_1a_3y_1$.

We take as our inductive hypothesis that $I(v)$ has been constructed for all $v \in G$ such that $1 \le \operatorname{ind} v \le m$. Consider the unique $v \in R$ such that there exists an upward transition based on v with replacement $a_{3m-2}y_m$. Let the last word of $I(v)$ be pvq . We assert that

$$I(a_{3m-2}) = I(y_m) = I_v \cdot (pvq \to pa_{3m-2}y_mq)$$

and further that

$$I(x_m) = I(y_m) \cdot (pa_{3m-2}y_mq \to px_ma_{3m-2}y_mq) ,$$

$$I(a_{3m-1}) = I(y_m) \cdot (pa_{3m-2}y_mq \to pa_{3m-1}y_mq)$$

and that

$$I(a_{3m}) = I(x_m) \cdot (px_ma_{3m-2}y_mq \to px_ma_{3m-1}y_mq \to px_ma_{3m}y_m) .$$

Since every letter can be introduced by exactly one type of positive transition, in order to prove the assertion for $I(y_m)$ it suffices to show that if $J(y_m)$ is any shortest sequence beginning with a_1y_1 that introduces y_m , then $J(y_m)$ contains no negative transitions. Suppose that $puq \to pvq$ is the last transition of such a sequence $J(y_m)$. If

https://doi.org/10.1017/S0004972700025715 Published online by Cambridge University Press

$J(y_m)$ contains a negative transition, then it follows from the inductive hypothesis that $puq \to pvq$ is negative. If this transition were backward, it would be $pa_{3m}q \to pa_{3m}y_mq$, but since a_{3m} can only be introduced via x_ma_{3m-1}, and a_{3m-1} can only be introduced via $a_{3m-2}y_m$, it follows that this is not the case, so we may assume that $puq \to pvq$ is downward. Consider the letter $y \in Y$ appearing in u. Since y_m does not appear in $J(y_m)$ prior to pvq, it follows that y itself was introduced in $J(y_m)$ by a negative transition, and by the same argument as before, we conclude that y was introduced by a downward transition. Repetition of this argument yields the conclusion that $|J(y_m)|$ is arbitrarily large, and from this contradiction we conclude that $J(y_m)$ is positive and thus $J(y_m) = I(y_m)$ as given above.

Next, since the backward transition which introduces a_{3m-2} involves y_m, it follows that for any $J(a_{3m-2})$ (where $J(z)$ now denotes an arbitrary shortest sequence introducing z and beginning at a_1y_1) we have $J(a_{3m-2}) = I(y_m) = I(a_{3m-2})$. By a very similar argument to that used in the $J(y_m)$ case, it follows that any $J(x_m) = I(x_m)$. Since a_{3m} can only be introduced via the base x_ma_{3m-1}, it follows that any $J(a_{3m-1}) = I(a_{3m-1})$. By listing all possibilities, it can be seen that the shortest sequence introducing the word x_ma_{3m-1} is

$$I(y_m) \cdot (pa_{3m-2}y_mq \to px_ma_{3m-2}y_mq \to px_ma_{3m-1}y_mq)$$

from which it follows that any $J(a_{3m}) = I(a_{3m})$ as given above, and this completes the proof.

Lemma 2 allows us to make the following definitions. For any $z \in G$ define the sets $X(z) \subset X$, $Y(z) \subset Y$ to consist of those members of X and Y respectively which occur as bases of upward transitions of $I(z)$, together with z itself ($z \in X(z)$ or $Y(z)$ according as $z \in X$ or $z \in Y$), and denote $X(z) \cup Y(z)$ by $R(z)$. A letter z is *derivable* from another letter z_1 (we write $z_1 \leq z$) if there exists a sequence

https://doi.org/10.1017/S0004972700025715 Published online by Cambridge University Press

$I : z_1 \to \ldots \to w$, containing no downward transition, such that $z \in C(w)$. We consider a letter to be derivable from itself by a trivial sequence of no transitions. Since $z_1 \le z$ implies $\operatorname{ind} z_1 \le \operatorname{ind} z$ and no two letters of the same index are mutually derivable from one another, it follows that derivability does indeed define a partial order on the members of G . Observe that the only letter derivable from any a_{3m-1} is itself, and that each member of R covers its annihilator with respect to the partial order $\le$. It follows from the proof of Lemma 2 that for any $z \in G$ the members of $R(z)$ form a chain (with respect to $\le$), beginning with x_1 or y_1 and ending with z . In fact for z , $z_1 \in R$, $z_1 \le z$ if and only if $z_1 \in R(z)$.

We may now state and prove the main lemma.

LEMMA 3. *Let* $w\rho y_0$. *Then* w *admits the following factorization*

$$w = w_1u_1w_2u_2 \ldots w_ku_kw_{k+1}zw_{k+2}u_{k+1}w_{k+3}u_{k+2} \ldots w_{l+1}u_lw_{l+2}$$

where each $w_i \in F^1$, $z \in R$, $l = |R(z)|$, *each* u_i *is an associate of a member of* $R(z)$, *and precisely one associate of each member of* $R(z)$ *occurs in the list* $u_1, u_2, \ldots, u_l$. *If* $r \in C(u_iw_{i+1} \ldots z)$ *or* $r \in C(zw_{k+2} \ldots u_i)$ *for some* u_i *then* $r \ge u$ *for some letter* u *such that* $\operatorname{ind} u = \operatorname{ind} u_i$. *However, for any* $x \in X(z)$ (*respectively* $y \in Y(z)$), *the annihilator of* x (*respectively* y), a_{3m-2} (*respectively* a_{3m}) *is not a member of* $C(zw_{k+2} \ldots u_x)$ *respectively* $C(u_y \ldots w_{k+1})$) *where* u_x, u_y *are the unique associates of* x *and* y *respectively in the list* $u_1, u_2, \ldots, u_l$.

REMARK. This lemma says in particular that $y_0\rho \notin U$ so that $U \ne S$.

Proof. We proceed by induction on $|I|$, where $I : a_1y_1 \to \ldots \to w' \to w$ is a sequence from y_0 to w . If $|I| = 0$ then the statements of the lemma are evidently satisfied. Consider an arbitrary such sequence I , and take as our inductive hypothesis that the lemma

https://doi.org/10.1017/S0004972700025715 Published online by Cambridge University Press

holds for the initial subsequence $J : a_1y_1 \to \dots \to w'$, with

$$w' = w'_1u'_1 \dots w'_ku'_kw'_{k+1}z'w'_{k+2}u'_{k+1} \dots u'_lw'_{l+2}$$

a factorization of w' satisfying the requirements of the lemma.

Before proceeding note that it follows from our inductive hypothesis that if a' and b' are associates of two distinct members of $X(z')$ occurring after z' , and $\operatorname{ind} a' > \operatorname{ind} b'$, then the first appearance of a' , or its x-companion, occurring after z' , is before the first appearance of b' , or its x-companion, after z' . Therefore, without loss we may assume that for any $i \geq k+1$, $C(z' \dots w'_{i+1})$ does not contain u'_i , nor the x-companion of u'_i . The preceding two sentences have duals which, of course, also hold.

We now consider the transition $w' \to w$ which may be

(i) a squaring transition,

(ii) an upward transition,

(iii) a downward transition,

(iv) a forward transition based on some a_{3m-2} or its reversal,

(v) a forward transition based on some $a_{3m-2}y_m$ or its reversal,

(vi) a forward transition based on some x_ma_{3m-1} or its reversal, or

(vii) a forward transition based on some $a_{3m}y_m$ or its reversal.

We show that in all cases the statements of the lemma continue to hold. for convenience, we will sometimes write the factorization given for w' in the abbreviated form $w' = v'_1z'v'_2$ where $v'_1 = w'_1u'_1 \dots u'_kw'_{k+1}$ and $v'_2 = w'_{k+2}u'_{k+1} \dots u'_lw'_{l+2}$.

Case (i). First suppose $w' \to w$ has the form $paq \to pa^2q$,

https://doi.org/10.1017/S0004972700025715 Published online by Cambridge University Press

$p, q \in F^1$, $a \in \langle A \rangle$, and suppose v_1' is a subword of p . There are no difficulties here; we consider the second 'a' to be inserted after the first and take $v_1 = v_1'$ in the required factorization of w . The dual comment applies if v_2' is a subword of q . Next consider the reverse squaring transition, and suppose v_1' is a subword of p . As explained above, we may assume without loss that the second 'a' contains no u_i' which occurs in the canonical factorization of w' , and thus no difficulties accompany its deletion. Again the dual comment applies if v_2' is a subword of q . This concludes case (i).

Case (ii). Suppose $w' \to w$ has the form $px_nq \to pa_{6n-2}y_{2n}q$ and suppose v_1' is a proper subword of p . The only difficulty which might conceivably arise is the introduction of an unwanted annihilator. However, a_{6n-2} annihilates x_{2n} , and if $x_{2n} \in X(z')$ all associates of x_{2n} must occur in p , because $\operatorname{ind} x_n < \operatorname{ind} x_{2n}$. Therefore we may indeed take $v_1 = v_1'$ in the required factorization for w . If $v_1' = p$, so that $z' = x_n$, then we factorize w as $\left(v_1'a_{6n-2}\right)y_{2n}v_2'$ and take $v_1 = v_1'a_{6n-2}$, $v_2 = v_2'$. If v_2' is a proper subword of q there are no difficulties. We may take $v_2 = v_2'$ in the factorization of w . The arguments used when $w' \to w$ is an upward transition based on some member of Y are the same as those above, thus completing case (ii).

Case (iii). Suppose $w' \to w$ has the form $pa_{6n-2}y_{2n}q \to px_nq$ and suppose v_1' is a subword of p . We may again take $v_1 = v_1'$. The only apparent difficulty is that if $x_{2n} \in X(z')$ and the first associate of x_{2n} after z' is in q , then the derivability hypothesis of the lemma would be violated (as x_n is not derivable from a letter of index $2n$ although a_{6n-2} and y_{2n} are). However, since a_{6n-2} annihilates x_{2n} this situation is excluded by the inductive hypothesis. If $v_1' = pa_{6n-2}$, then a required factorization of w is $w = v_1x_nv_2$, where $v_1 = p$,

https://doi.org/10.1017/S0004972700025715 Published online by Cambridge University Press

$v_2 = q$. If v_2' is a proper subword of q then we may take $v_2 = v_2'$, except in case the a_{6n-2} appearing in $pa_{6n-2}y_{2n}q$ is one of the u_i' appearing in the factorization of w' . We show that we can then factorize w' as $w' = v_1''y_n v_2''$ with $v_1'' = pa_{6n-2}$, $v_2'' = q$ and still satisfy the conditions demanded by the inductive hypothesis. We have

$$w' = w_1'u_1' \ldots w_i'u_i'w_{i+1}' \ldots u_k'w_{i+1}'z'w_{k+2}' \ldots w_{j+1}'u_j'w_{j+2}' \ldots u_l'w_{l+2}' ,$$

and $p = w_1'u_1' \ldots w_i'$, $u_i' = a_{6n-2}$, $w_{i+1}' = y_{2n}\bar{w}_{i+1}$ for some $\bar{w}_{i+1} \in F^1$, u_j' is the first of the u's after z of index less than $2n$. If no such u_j' occurs the following argument requires slight modification. We now factorize w' , taking $w_1'u_1' \ldots w_i'u_i'$ as before, followed by $w_{i+1}'' = 1$, $z'' = y_{2n}$, $w_{i+2}'' = \bar{w}_{i+1}u_{i+1}' \ldots w_{j+1}'$, followed by $u_j'w_{j+2}' \ldots u_l'w_{l+2}'$ as before (although we rename this last product as $u_{i+1}''w_{i+3}'' \ldots u_{l'}''w_{l'+2}'$ where $l' = l - j + i + 1$). Observe that the inductive hypotheses still hold with this new factorization of w' , the only one causing some complication is the statement that the annihilator of u_x'' is not a member of $C(z'' \ldots z' \ldots u_x'')$ for all $x \in X(z'')$. However, by the inductive hypothesis, the annihilator of u_x'' is not a member of $C(z' \ldots u_x'')$ while all members of $C(z'' \ldots z')$ have index > ind u_x'' , so are certainly not annihilators of u_x'' . Hence we have reduced this case to the case where $v_1' = pa_{6n-2}$. The arguments above also deal with the case where $w' \to w$ is a downward transition with replacement a member of Y , this completing the proof for case (iii).

Case (iv). Suppose $w' \to w$ has the form $pa_{3m-2}q \to px_m a_{3m-2}q$. Since $a_{3m-2} < x_m$ there are no difficulties. For the reverse transition, the only case which would cause a problem is that in which $v_1' = p$; but this contradicts the inductive hypothesis as it says that $z' = x_m$ is immediately followed by its annihilator. This concludes case (iv) and the dual argument to this disposes of case (vii).

https://doi.org/10.1017/S0004972700025715 Published online by Cambridge University Press

Case (v). Suppose $w' \to w$ has the form $pa_{3m-2}y_mq \to pa_{3m-1}y_mq$ and that v_1' is a subword of p. Again a difficulty could only arise if $x_m \in X(z')$ and the first associate of x_m after z' were in q; but this is impossible as a_{3m-2} annihilates x_m. If $v_1' = pa_{3m-2}$ so that $pa_{3m-2} = pu_i'$, the role of u_i' is filled by taking $pa_{3m-1} = pu_i$ in w. Similarly there is no problem if v_2' is a proper subword of q; if necessary the role previously played by a_{3m-2} is now played by a_{3m-1}. Consider now that the reverse transition $w' \to w$ has the form $pa_{3m-1}y_mq \to pa_{3m-2}y_mq$. The only difficult case arises when $x_m' \in X(z')$ and $pa_{3m-1} = pu_i'$ where u_i' is an associate of x_m. We assert that we may then factorize w' as $w' = v_1''y_mv_2''$ with $v_1'' = pa_{3m-1}$ and obtain a factorization satisfying the inductive hypotheses. The argument involved is similar to that employed in case (iii) and is omitted. We may now take $v_1 = pa_{3m-2}$ in w, and thus dispose of case (v).

Case (vi), in which $w' \to w$ is a forward transition based on some x_ma_{3m-1} or its reversal, is dealt with using arguments dual to those of case (v), thus completing the proof of the lemma.

COROLLARY 4. *There exists a band which is not saturated.*

Further progress towards solving the problem of the determination of all the saturated varieties of semigroups would be made if we knew whether or not the variety defined by the identity $xy = xyx$ was saturated. If this variety is not saturated it follows that the same is true of the corresponding variety of bands. Since every non-normal band variety contains either this variety or its dual, it would follow that the saturated band varieties are exactly the normal varieties. Furthermore, a negative answer would allow a determination of all saturated varieties of monoids.

References

[1] A.H. Clifford and G.B. Preston, *The algebraic theory of semigroups*, Volume I (Mathematical Surveys, 7. American Mathematical Society, Providence, Rhode Island, 1961).

[2] A.H. Clifford and G.B. Preston, *The algebraic theory of semigroups*, Volume II (Mathematical Surveys, 7. American Mathematical Society, Providence, Rhode Island, 1967).

[3] B.J. Gardner, "Epimorphisms of regular rings", *Comment Math. Univ. Carolin.* **16** (1975), 151-160.

[4] T.E. Hall, "Epimorphisms and dominions", *Semigroup Forum* **24** (1982), 271-283.

[5] T.E. Hall and P.R. Jones, "Epis are onto for finite regular semigroups", *Proc. Edinburgh Math. Soc.* (to appear).

[6] P.M. Higgins, "Epis are onto for generalised inverse semigroups", *Semigroup Forum* **23** (1981), 255-259.

[7] Peter M. Higgins, "The varieties of commutative semigroups for which epis are onto", *Proc. Edinburgh Math. Soc.* (to appear).

[8] P.M. Higgins, "Saturated and epimorphically closed varieties of semigroups", *J. Austral. Math. Soc. Ser. A* (to appear).

[9] J.M. Howie, *An introduction to semigroup theory* (London Mathematical Society Monographs, 7. Academic Press, London, New York, San Francisco, 1976).

[10] J.M. Howie and J.R. Isbell, "Epimorphisms and dominions. II", *J. Algebra* **6** (1967), 7-21.

[11] John R. Isbell, "Epimorphisms and dominions", *Proceedings of the conference on categorical algebra*, La Jolla, 1965, 232-246 (Springer-Verlag, Berlin, Heidelberg, New York, 1966).

[12] N.M. Khan, "Epimorphisms, dominions and varieties of semigroups", *Semigroup Forum* (to appear).

Monash University,
Clayton, Victoria 3168,
Australia.