STELLAR WINDS AND SPINDOWN IN SOLAR TYPE STARS

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ABSTRACT

The angular momentum loss produced by stellar winds is reviewed and a simple model of angular momentum loss with multipole fields is presented in which the field is potential when the flow speed is less than the Alfvén speed, and radial when greater than the Alfvén speed. The simpler the magnetic geometry, the larger is the angular momentum loss rate. This result is used to explain the rotational discontinuity across the Vaughan-Preston gap as being due to a sudden increase in angular momentum loss when the dynamo field switches from a quadropole to a dipole geometry.

The evolution of the internal rotation of stars as a result of surface angular momentum loss is considered. In the absence of a magnetic field, differential rotation can drive instabilities which then transport angular momentum out from the interior down the angular velocity gradient. Other instabilities such as that caused by the build up of ^{3}He can also transport angular momentum outwards. If angular momentum is transported by such weak turbulence, it also makes the star more homogeneous than standard evolutionary models and lowers the predicted value of the solar neutrino flux.

The recent results on rotational splitting of solar oscillations are considered: these suggest that the inside of the sun is spinning faster than the surface and are compatible with models in which angular momentum is transported by mild turbulence. But data is scarce — and in such circumstances the speculations of the theorist must be viewed with caution!

1. INTRODUCTION

Mass loss in stellar winds from the surface of rotating stars will carry away angular momentum and thus lead to a decrease in the stars' angular momentum, but the details of mass and angular momentum loss are uncertain; so too are the dynamics of rotating stars. In particular the mechanism by which angular momentum is transported from the interior to the surface layers is still a matter for debate.

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A magnetic field can cause the material in the stellar wind to corotate with the star. The actual angular momentum loss therefore depends on the detailed magnetic structure of the open wind region. If the inside of a star contains a magnetic field, this can be very effective in causing nearly uniform rotation, and angular momentum is transported outwards by magnetic forces. On the other hand, if there is only a negligible internal magnetic field, the spindown of the surface layers by a stellar wind produces differential rotation which can be unstable - the resulting instabilities then transport angular momentum. If such instabilities exist they can lead to diffusion of chemical inhomogeneities and thus affect the evolution of the star.

The problems to be solved are difficult: this paper necessarily can only cover a few topics and even then only superficially.

2. ANGULAR MOMENTUM LOSS

The simplest model of a stellar mass loss is Parker's (1958) spherically symmetric isothermal solar wind. For a given temperature T and density ρ_0 at the base of a corona of radius r_0, the wind velocity V is given by

$$\frac{V^2}{V_s^2} - 2\ \ln \frac{V}{V_2} = 2\ \ln\left(\frac{r}{r_c}\right) + 4\ r_c\left(\frac{1}{r} - \frac{1}{r_c}\right) + 1 \ , \qquad (1)$$

where V_s is the isothermal sound speed, $r_c = GM/2V_s^2$ is the critical point where $V = V_s$ and M the mass of the star. More sophisticated models have been calculated including energy transport in the one fluid and multicomponent plasma approximations, but for illustrative purposes, the simple isothermal model will suffice. The mass loss in such a wind is

$$\frac{dM}{dt} = -\ 4\pi\ r_0^2\ V_0\ \rho_0 \ . \qquad (2)$$

The velocity V_0 at $r = r_0$ is given by the isothermal wind equation with $r = r_0$.

A naive model of angular momentum loss in such a wind was given by Weber and Davis (1967) taking a spherically symmetric flow along radial magnetic field lines. The essential features of their model was that the wind corotates with the star out to that distance where the wind speed equals the Alfvén speed $V_A = (B^2/4\pi\rho)^{1/2}$, which is larger than the radius of the star. If the magnetic field is radial, then $B \propto r^{-2}$ and the angular momentum loss is

$$\frac{dH}{dt} = -\ \frac{8\pi}{3}\ \rho_A\ V_A\ r_A^4\ \Omega = -\ \frac{2}{3}\ \frac{B_0^2\ \Omega}{V_A} \ , \qquad (3)$$

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where B_o is the strength of the field at the base of the corona.

Now the strength of the field B_o itself depends on Ω through the dynamo mechanism that is believed to operate in stellar convective zones. If the dynamo is such that $B_o \propto \Omega^2$, then with V_A approximately constant, one can reproduce (c.f.Durney 1972) the Skumanich (1972) relation

$$\Omega \propto t^{-1/2} \quad . \tag{4}$$

Such a model is clearly incorrect; the magnetic field on the sun is far from radial in the inner regions. Moreover the heating of the corona and hence the wind speed may themselves depend on the magnetic field and hence on the angular velocity.

3. MAGNETIC FIELD MODELLING

The classical model of the solar coronal magnetic field is the hairy ball model of Altschuler and Newkirk (1969) and Schatten(1970). This model takes the measured value of the line of sight component of the magnetic field, assumes the field is current free (so curl $\underline{B} = 0$) out to some spherical source surface r_s and radial thereafter. This is a very coarse approximation to the physics. In my opinion, the apparent fit between observations and modelling is just a consequence of conservation of magnetic flux ($\nabla . \underline{B} = 0$) . The detailed structure of the field cannot be given by a model that combines detailed data on the surface field with a simple global approximation of a spherical source surface.

Durney and Pneuman (1975) took the hairy ball model and solved the one fluid solar wind model along field lines. Not surprisingly they found that the Alfvenic surface where $4 \pi \rho V^2 = B^2$ was nothing like the spherical source surface.

Several authors have sought to find self-consistent models of a magnetic solar wind but for a much simpler magnetic structure. Pneuman and Kopp (1971) solved the problem of a steady isothermal wind with a dipole base field, and Endler (1971) solved the same problem dynamically. More recently Robertson (1983) and Steinolfson et al (1982) have examined and extended this to one fluid conducting and polytropic models. All these models require considerable effort and expense and still are far removed from the actual physical conditions in stellar coronae.

Approximate methods for dipole fields were developed by Mestel (1968) and by Rowse and Roxburgh (1981). In these models the magnetic field is assumed to be current free (curl $\underline{B} = 0$) within a surface S_A, and radial outside S_A, but the position of the surface is determined by the condition that the wind velocity which is along the field lines equals the Alfven speed. Rowse and Roxburgh (1981) included growing terms in the potential field and were able to make the potential field radial at S_A. These models satisfactorily reproduce the essential features of the full MHD solutions.

During the course of the meeting I extended these models to higher multipole fields in order to seek an explanation of the discontinuity of rotation speed across the Vaughan-Preston gap (see Vaughan's article in these proceedings). I therefore now give some details of the method.

4. APPROXIMATE MODELS OF ANGULAR MOMENTUM LOSS FOR MULTIPOLE FIELDS

I assume that when the velocity V is less than the Alfvénic velocity, V_A, the field is essentially current free and the flow follows the field lines, when $V > V_A$ the flow and the field lines are radial. With this approximation the tangential component of the field is given by

$$B_\theta = \frac{B_0}{(n+1)\, r^{n+2}} \left\{ 1 - \frac{[(n+1)\, r^{2n+1} + n]}{[(n+1)\, r_A^{2n+1} + n]} \right\} P_n^1 (\cos\theta) \tag{5}$$

and the radial component follows from $\nabla \cdot \underline{B} = 0$. Note that this field goes radial, $B_\theta = 0$ at $r = r_A$. The field strength $|B|$ varies with latitude but is given approximately by

$$B^2 = \frac{B_0^2}{r^{2n+4}} \tag{6}$$

where r is the distance in units of the radius of the star. The fraction of the star that has open field lines is $f = r_A^{-n}$, and the area of a flux tube increases as r^{n+2}.

If we now assume that the wind corotates out to the Alfvénic radius — which is a good approximation for calculating angular momentum loss (c.f. Mestel 1968) then this loss rate is

$$\frac{dH}{dt} = \frac{8\pi}{3}\, \rho_0\, V_0\, \Omega\, r_A^{2-n} \tag{7}$$

where r_A is given by the condition that the flow speed equals the Alfvénic speed

$$r_A^{n+2}\, V_A = \frac{B_0^2}{4\pi\rho_0 V_0} \tag{8}$$

It is worth pointing out that a strong field B_0 does not necessarily increase the angular momentum loss rate (c.f. Mestel 1968). Whilst r_A increases with B_0, the fraction of the star with open field lines decreases as B_0 increases. From equations (7) and (8) we see that (with V_A slowly varying) the angular momentum loss rate is proportional to B_0^a,

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where $a = (2-n)/(2+n)$. For radial ($n = 0$) and dipole ($n = 1$) fields, the loss rate does increase with B_0, for a quadropole field $n = 2$, it is independent of B_0, whilst for $n > 2$ the loss rate decreases as B_0 increases.

The flow of the wind along these field lines is easy to calculate: the equations for an isothermal flow can be integrated to give

$$\frac{V^2}{V_s^2} = 2 \ln \frac{V}{V_s} = 2 (n + 2) \ln \frac{r}{r_c} + 4 X_c \left(\frac{1}{r} - \frac{1}{r_c} \right) + 1 \qquad (9)$$

where $X_c = GM/2 V_s^2 r_0$, $r_c = 2 X_c/(n+2)$ and the factor $(n+2)$ comes from the variation of the area of a flux tube with distance r. For given base conditions ρ_0, B_0, T_0 these equations can be solved to determine the angular momentum loss (Roxburgh 1983).

5. EFFECT OF CHANGING MAGNETIC FIELD GEOMETRY ON ANGULAR MOMENTUM LOSS

As a star's angular velocity decreases we may expect the dynamo generated field to go through a series of bifurcations to simpler magnetic geometries, with consequential sudden changes in the angular momentum loss rates. For a change from quadropole ($n = 2$) to dipole ($n = 1$) the ratio of angular momentum loss rates is

$$\frac{\dot{H}_1}{\dot{H}_2} = \left(\frac{V_{01}}{V_{02}} \right) r_{A1} \qquad (10)$$

Note that for a quadropole field the angular momentum loss rate is independent of the field strength. Now V_{01}, V_{02} are given by the wind equation and depend only on n, not on B_0. On the other hand, r_{A1} does depend on B_0 and increases as B_0 increases. Hence provided B_0 is large enough, the ratio $\dot{H}_1/\dot{H}_2 > 1$ and the change from quadropole to dipole magnetic field produces a sudden increase in angular momentum loss rate.

The response of a star to a sudden increase in angular momentum loss depends on the coupling between the interior and the surface convective zone. If this coupling is weak, the convective zone (with its small moment of inertia) would spin down on a short time scale and the rotation of the inner region would readjust on a longer time scale.

It is therefore tempting to use this as an explanation of the jump in rotational velocity across the Vaughan-Preston gap (see Vaughan, this volume). Provided the spindown time of the convective zone is short compared to the age of the star, we would expect to see such a jump as a consequence of a change in the magnetic field geometry. Indeed, the model could be refined to calculate the spindown time of the convective zone and hence to predict the change of finding stars crossing the gap.

6. THE INTERIOR OF SOLAR TYPE STARS

Models of the internal structure of solar type stars are based on the assumptions that energy is generated by hydrogen burning nuclear reactions (principally the proton-proton chain in the sun). This energy is transported outwards by radiation, but near the surface the lowering of specific heats due to ionisation and the temperature variation of the opacity, combine to produce a convectively unstable outer layer. It is further assumed that rotation, magnetic fields, oscillations, and mass loss, can be safely neglected. Such models can reproduce broad features of the observed main sequence.

Evolutionary sequences are then evaluated by calculating the change in chemical composition due to the nuclear reactions with the assumption that mixing only takes place in convectively unstable regions. Such models eventually develop a helium core surrounded by a hydrogen burning shell and become red giants.

I should however emphasise that we have no reliable theory of turbulent convection. Standard models use a local mixing length theory, which excludes the effect of overshooting into the stable layers (c.f. Roxburgh 1978) and the mixing length, ℓ is treated as a free parameter - adjusted to obtain agreement between models and the observed solar radius. Again, for the sun we do not know the helium abundance, Y, this is again taken as a free parameter and is adjusted so that models based on these assumptions give the observed solar luminosity for a model with an age of $t_o \simeq 4.7.10^9$ yrs. Clearly other assumptions could be made to fit the observations!

The standard solar model has its problems - the predicted neutrino flux is some three times greater than the measured upper limit (Davis 1978) and as was first pointed out by Gough and his coworkers (Dilke and Gough 1972, Christensen-Dalsgaard et al 1974), this standard model predicts a build up of the ^{3}He away from the centre that after some 3.10^8 years is large enough to excite a global over-stability. These problems have not been resolved, and must inevitably call into question the simple spherical standard model.

One exciting possibility is that the observations of solar oscillations will give an empirical model of the internal density distribution which can then be compared with the models. I suspect we are in for a few surprises when this is done!

The standard model has been questioned by several authors, myself included (c.f. Rood 1978, Roxburgh 1976, 1978). One school of thought argues that rapid internal rotation or strong magnetic fields can so distort the star as to change its structure. Another school argues that there must be some internal mixing caused by the ^3He instability and/or rotation and magnetic fields that changes the evolutionary models (Dilke and Gough 1972, Roxburgh 1976, Maeder and Schatzman 1981). A mixed, or partially mixed model gives a lower neutrino flux, essentially because mixing brings hydrogen into the central regions and with a higher hydrogen abundance, a lower temperature is required to produce a given amount of energy.

7. INTERNAL ROTATION OF THE SUN

The first indications that the inside of the Sun is rotating more rapidly than the surface layers come from the rotational splitting of the oscillation modes as observed by Claverie et al (1982) and Hill (1982). The interpretation of these observations is a matter of debate, but suggest that the average internal rotation is some 2 - 10 times the surface value. A detailed discussion is given in Gough (1982), where he uses the splitting of 7 lines observed by Hill to estimate the internal distribution of angular velocity. A word of caution is required:before such an analysis can be carried through, it is necessary to identify the particular normal mode that is being observed - this is by no means a simple matter.

The analysis by Gough (1982) shows that the observed modes fall into two classes, those where the main effect is from central regions , $0.1 < r/R < 0.3$, and those where the main effect is from regions near the surface. This permits one to conclude that the central regions are spinning faster, but makes it difficult to determine the variation of angular velocity with radius $\Omega = \Omega(r)$, the details depending on the method used to invert from the data to $\Omega(r)$. All models used by Gough give an angular velocity at $r = 0.1\ R_\odot$ of between 6 and 10 times the surface value, and would be consistent with an even larger rotation in the very central regions, but were unable to distinguish between a steady inward increase in Ω, and a model with Ω constant $\simeq 6\ \Omega_o$ in a large central core $r < 0.7\ R_\odot$, and a sudden discontinuous drop to near the surface value in the outer regions. The various models gave a solar quadropole moment, J_2, between 10^{-6} and 5.10^{-6}

The situation will become clearer as and when more data is available.

8. ROTATIONAL INSTABILITIES

There are potentially a range of instabilities that could be excited in the stable regions of rotating stars. In particular, if the angular momentum per unit mass decreases outwards over a surface of constant entropy, the star would be dynamically unstable (Høiland 1941). Shear instabilities have been considered by Zahn (1975), who concludes that horizontal shear, i.e. along surfaces of constant entropy, is readily excited but has a long growth time unless there is some other turbulence already present. The question of shear instabilities in the radial direction is much more difficult as if it is present, it is a finite amplitude instability.

As was pointed out by Goldreich and Schubert (1967) and Fricke (1967, 1968), departures from cylindrical rotation laws drive an instability in which the stabilising effect of stratification is overcome by thermal diffusion. For a spherically symmetric angular velocity distribution $\Omega(r)$, instability sets in when

$$\left(\frac{d\Omega}{dr}\right)^2 > \frac{4\nu}{\kappa} \frac{N^2}{r^2} \tag{11}$$

where ν is the viscosity, κ the thermal diffusivity and N the Brunt Vaisala frequency. In solar conditions $\nu/\kappa \simeq 10^{-6}$ and $N \sim 10^{-3}$ sec^{-1}, and this condition gives instability if $d\Omega/dr$ is greater than about $5\Omega_\odot/R_\odot$. This leads to a very nice picture that as the sun spins down, the interior becomes unstable: the resulting turbulence then diffuses angular momentum outwards and the resulting model has an angular velocity gradient similar to that deduced from rotational splitting of solar oscillations.

This picture is not correct since even a very small build up of chemical inhomogeneity can suppress the instability; the actual condition then becomes (Biermann 1968, Roxburgh 1975)

$$\left(\frac{d\Omega}{dr}\right)^2 > -\frac{4g}{r^2} \frac{1}{\mu} \frac{d\mu}{dr} \tag{12}$$

which permits very large angular velocity gradients to be stable.

However, the baroclynic instability is then effective, thermal diffusion destabilising what would otherwise be a stable state (Knobloch and Spruit 1982). This instability is not suppressed by a radial distribution of chemical composition $\mu = \mu(r)$ and the condition for instability becomes

$$\left(\frac{d\Omega}{dr}\right)^2 > \frac{8\nu}{\kappa} \frac{N^2}{r^2} \tag{13}$$

Even this analysis is not correct; whilst a radial distribution of chemical composition does not suppress the baroclinic instability, a particular non spherical distribution can. Thus the instability can redistribute matter until the system is stable again (Roxburgh 1982). However, as the chemical composition changes through nuclear reactions, the non-spherical distribution required to suppress instability changes leading to a continual slow turbulent diffusion (Roxburgh 1982) which will again transport angular momentum outwards.

The detailed analysis of rotational instabilities is still incomplete. Very little has been done on non axisymmetric perturbations, but it does seem that in the absence of a magnetic field, there will be some weak turbulence in the solar interior, but how effective it is remains to be determined.

9. ROTATIONAL SPINDOWN AND STELLAR STRUCTURE

We can now try to put together the previous discussion to try to build a plausible model (or models!) of the evolution of stars as they spin down, but such models are highly speculative.

Initially, stars are rapidly rotating, mass and angular momentum loss slow down the surface layers and produce differential rotation beneath the outer convective zone. This differential rotation is unstable, leading to mild turbulent diffusion which transports angular momentum from the interior to the convective zone. (cf Dicke 1972)

If the star has an oscillatory dynamo operating in the convective zone, or possibly just below the convective zone in the overshoot region, any central magnetic field is uncoupled from the dynamo field. If the star has an internal magnetic field, which is stable against 'floating' instabilities, this will keep the core uniformly rotating and there must then be a shear zone some distance beneath the convective zone, the gradient of angular velocity being determined by the efficiency of the rotationally driven turbulence.

If there is no internal magnetic field, we may expect a monotonic inwards increase in the angular velocity and some mild turbulence in the central regions which diffuses both angular momentum and chemical composition, leading to a greater degree of homogeneity in the central regions than predicted by the standard solar model, and hence a lowering of the neutrino flux. Attempts to model the effect of spin down with somewhat different assumptions have been made by Endal and Sofia (1981) and Schatzman and Maeder (1981).

There still remains the problem of the ^{3}He instability. One possibility is that this instability produces mild turbulence that keeps it close to marginal stability - again leading to diffusion of angular momentum and chemical composition. If this were so, we might expect to see the unstable mode - perhaps we do in the 160 min period global oscillation. The ^{3}He instability might be suppressed by rotationally driven turbulence diffusing ^{3}He and keeping the central regions stable. A magnetic field may also suppress this instability.

If some mixing is caused by rotational spin down, it will not suppress evolution to the red giant phase. The effect of mixing will decrease as the star slows down so that the main effect will be to increase the main sequence life time of such stars. This could lead to significant changes in estimates of the ages of globular clusters.

10. CONCLUSIONS

At the end of his book on the Internal Constitution of the Stars, Sir Arthur Eddington (1926) concluded: "It is reasonable to hope that in the not too distant future we shall be competent to understand so simple a thing as a star". We have not yet reached that position. Our knowledge is still very limited and where observational constraints are few there is much room for theoretical speculation. Let us hope that improved data, particularly of solar oscillations, will soon be available to help us reach that goal Eddington thought was close at hand, but which seems still to be remote.

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DISCUSSION

STIX: The shear layer near the bottom of the convection zone is just where the dynamo seems to need it (because of the magnetic buoyancy argument). On the other hand, the 20-fold angular velocity at the Sun's center that you showed on an earlier viewgraph is in conflict with the oblateness measurements, isn't it?

ROXBURGH: **I consider the shear layer below the convective zone as a possible site for dynamo action. The diffusive rotation model is really only an illustration, and one would obtain different solutions by changing either or both the magnitude and radial variation of the diffusion coefficient.**

MOUSCHOVIAS: **The solar wind emanates primarily from coronal holes, which are found more frequently at higher latitudes. Higher latitudes, therefore, are magnetically braked faster than lower latitudes. Does a quantitative comparison of time scales (of magnetic braking and of redistribution of angular momentum on the solar surface) allow an explanation of the observed differential rotation in the above manner?**

ROXBURGH: **The diffusion time scale for the convective zone is of the order of years — very much shorter than the wind spindown time scale, so this effect should not be important in determining the differential rotation.**

HARTMANN: **Would you expect a difference in the way that stars with shallow convective zones would spin down as compared with the spindown of more fully convective stars?**

ROXBURGH: **The easy answer is yes! But in what way? First I would expect the differential rotation and the dynamo and the mass loss to all vary with convective zone depth. Even if all these were unchanged, the response of the star would be different as the diffusion coefficient, which is probably high in convective zones and probably smaller in radiative**

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zones, determines how the star responds to a given rate of mass loss. I would anticipate that the deeper the convective zone the longer would be the surface rotation spindown time. But it is not reasonable to assume that all other things are equal!

KÖMLE: If you change the magnetic-field structure in your model from a dipole to a quadrupole — what would the effect on the solar wind bulk velocity be?

ROXBURGH: It is increased. The quadrupole field diverges more rapidly than the dipole, and this leads to higher velocities.

KOUTCHMY: (1) My first question deals with the problem of calculating the angular momentum loss using the concept of a source surface. According to this concept, coronal structures outside the source surface should show the characteristic spiral pattern. This is clearly not observed, but the large-scale coronal structures are rather showing the rigid rotation properties up to distances of at least 12 - 15 $R_{\odot}$. Does this mean that the angular momentum loss of the sun is larger than what we think it is, and is this kind of observations relevant to the problem of angular momentum loss? (2) My second question is: Did you compute the solar-cycle related angular momentum loss due to coronal transient events?

ROXBURGH: (1) I believe it is misleading to refer to the source surface model for comparison with observations. At best it can only be an average of the real sun. The extent of individual corotating structures are determined by the actual properties of the magnetic field and corona — strong fields could cause corotation out to $15R_{\odot}$. (2) The answer to the second question is no!

ENDAL: A difficult problem that I encountered in my calculations (with S. Sofia) of solar spindown and internal rotation is that the composition gradient in the core prevents sufficient angular momentum removal as needed to satisfy the oblateness constraints. If the recent analysis of Knobloch and Spruit is correct, this may indicate the way out of this dilemma, and this would be a very important result.

ROXBURGH: I agree — indeed the model I calculated in 1971 for marginal stability of the Goldreich-Schubert-Fricke instability had similar problems. The triple diffusive instability does offer the possibility of increased diffusion — although it depends on the non-linear saturation of the instability.

TARBELL: Bill Press of Harvard has suggested another possible problem with the standard solar model. He suggests that gravity waves generated in the convection zone propagate into the core without too severe damping. The enhanced transverse gradients in temperature lead to enhanced radiative energy transport, simulating a reduced opacity. Are you aware of any calculated evolutionary sequences considering this effect? Do you think it is important?

ROXBURGH: In fact Henry Hill suggested this some years ago (see also Roxburgh, 1975). I doubt if this could be important on a thermal diffusion time for the sun — but perhaps in other stars with larger amplitude oscillations. But there is a need for a detailed study of the effect of oscillations on internal structure and angular momentum transport.

ROSNER: In response to Tarbell's question, I understand that W. Press and G. Rybicki have recently evaluated the diffusive properties of the modes in question. They find that as far as radiative transport is concerned, these modes probably do not resolve the neutrino problem. However, these modes probably do have important consequences for mixing, and hence may play an important role in the development of instabilities such as the triple-diffusive instability of Knobloch and Spruit (1983).

ROXBURGH: I would be pleased to receive a copy of that work.