A NOTE ON THE DIFFERENTIAL FORMS ON EVERYWHERE NORMAL VARIETIES

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A. Weil proposed in his book "Foundations of algebraic geometry" several problems concerning differential forms on algebraic varieties. S. Koizumi[1] has proved that if ω is a differential form on a complete variety $\mathbf{U}$ without multiple point, which is finite at every point of $\mathbf{U}$, then ω is the differential form of the first kind. The following example shows that on everywhere normal varieties with multiple points this statement holds no more; that is: *A differential form on a everywhere normal variety which is finite on every simple point of its variety is not always the differential form of the first kind.*

In the projective space of dimension 3 with the field of characteristic 0 as universal domain, we consider the variety $\mathbf{V}^2$ with homogeneous equation $X_3^4 = X_1^4 + X_2^4$. Let k be a defining field of $\mathbf{V}$ and (x_0, x_1, x_2, x_3) a set of homogeneous coordinates of a generic point $\mathbf{P}$ of $\mathbf{V}$ over k.

1) Put $\frac{x_1}{x_0} = x,\ \frac{x_2}{x_0} = y,\ \frac{x_3}{x_0} = z;\ \frac{x_0}{x_1} = u,\ \frac{x_2}{x_1} = v,\ \frac{x_3}{x_1} = w$.

Since $k[1, x, y, z]$, $k[u, 1, v, w]$, $k[x_0/x_2, x_1/x_2, 1, x_3/x_2]$, $k[x_0/x_3, x_1/x_3, x_2/x_3, 1]$ are integrally closed, $\mathbf{V}$ is everywhere normal. And it is easily seen that $(1, 0, 0, 0)$ is the only singular point of $\mathbf{V}$.

2) Consider the differential form $\omega = 1/z^3\, dxdy$ on $\mathbf{V}$ defined over k; ω is finite on every point of $\mathbf{V}$ except $(1, 0, 0, 0)$.

$$z^3dz = x^3dx + y^3dy.$$

$$\frac{1}{z^3}dxdy = +\frac{1}{y^3}dzdx = \frac{1}{x^3}dxdz = -\frac{1}{2w^3}dudv = \frac{1}{2v^3}dudw \quad \text{etc.}$$

$$w^4 = 1 + v^4.$$

This shows the assertion.

3) ω is not the differential form of the first kind.

Put $x = r,\ y/x = s,\ z/x = t$.

$$k(\mathbf{P}) = k(r, s, t).$$

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[1] On the differential forms of the first kind on algebraic varieties. Journal of the Mathematical Society of Japan. Vol. 1 (1949).

On the locus U of (r, s, t) over k the point $(0, 0, 1)$ is the simple point of U with uniformizing parameters r, s.

$$\frac{1}{z^3}\,dxdy = \frac{1}{r^2t^4}\,drds\,.$$

$\dfrac{1}{r^2t^3}$ is not in the specialization ring of $(0, 0, 1)$ in $k(r, s, t)$.

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