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Construction of biclosed categories

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The concept of a monoidal category is expressed relative to a suitably complete closed category V . It is then generalized to the concept of promonoidal category over V ; this comprises a V-category A together with three functors $P : A^{op} \otimes A^{op} \otimes A \to V$, $\mathrm{Hom} : A^{op} \otimes A \to V$, and $J : A \to V$, linked by natural associativities. If J is representable, one obtains a monoidal category precisely when P is representable as a covariant functor on A , a biclosed category when P is representable in each variable.

The first construction considered is convolution on the V-functor category $[A, V]$, see Day [1]. This provides, for each promonoidal structure $(P, J, \ldots)$ on A , a biclosed extension $(\otimes, /, \backslash, \ldots)$ on $[A, V]$ given by the formulas

$$F \otimes G = \int^{AA'} FA \otimes GA' \otimes P(AA'-)$$

and

$$G/F = \int_{AA'} [P(-AA'), [FA, GA']] , \quad F\backslash G = \int_{AA'} [P(A-A'), [FA, GA']] .$$

An example of convolution is the usual closed structure on the category of modules (respectively algebras) over a commutative ring (respectively theory).

The second construction deals with the existence of closed left retracts of a convolution. Various equivalent forms of the closure-under-exponentiation criterion are discussed and applied to the construction of cartesian closed categories of topological spaces

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(including the category of compactly generated spaces) and of sheaves.

Finally, the two constructions are "composed" so as to be independent of the functor category $[A, V]$. The resulting existence theorem is applied to the (large) theory of a commutative V-monad and produces a "bilinear" tensor product for the algebras over such a monad. This completes the construction given by Kock [2].

References

[1] Brian Day, "On closed categories of functors", *Reports of the Midwest Category Seminar IV*, 1-38. (Lecture Notes in Mathematics, 137, Springer-Verlag, Berlin, Heidelberg, New York, 1970).

[2] Anders Kock, "Closed categories generated by commutative monads", *J. Austral. Math. Soc.* (to appear).