CORRESPONDENCE.

ON THE EFFECT OF A RISE, OR FALL, IN MARKET VALUES OF SECURITIES, ON THE FINANCIAL POSITION AND RESERVES OF A LIFE OFFICE.*

To the Editor of the Journal of the Institute of Actuaries.

DEAR SIR,—The heavy fall in marketable securities during recent years, has brought forward the question, to what extent a depreciation of the life investments means a real loss to a life company, and ought to be charged to the profit and loss account, in case the aggregate market value of the securities should be exceeded by the amount appearing in the balance sheet of the previous year.

The matter might be dealt with in the following way.

A fall of market prices is always accompanied by a rise in the average rate of interest, and if the valuation is made on the corresponding higher percentage basis, then the diminution of the value of the life assets is to a great extent counterbalanced, in some cases even exceeded, by the reduction of the value of the company's liabilities.

Assuming that by the depreciation of the investments the value of the life assets is fallen from B to B$'$, that the average rate of interest of these assets is risen from i to i', and that the company's liability calculated upon bases i and i' is V and V$'$, then the value of the company's real loss arising from depreciation of a unit in the market value of the assets is:

$$f=\frac{B-B'-(V-V')}{B-B'}.$$

* See also the remarks on this subject in the President's Address (pp. 16–18 of present volume).

Now, evidently $B = V$,

and, understanding that the amount of interest on the life assets has remained unaltered, then: $Bi = B'i'$,

whence $$f = \frac{V'i' - Vi}{V(i' - i)}$$

in which $$V = A - P\mathbf{a},$$

and $$V' = A' - P\mathbf{a}',$$

A and $\mathbf{a}$ being valued on basis i, A' and $\mathbf{a}'$ on basis i', whilst P denotes in both formulas the same quantity, because the premium payable under the policy is invariable.

Supposing $i = \cdot035$, $i' = \cdot04$ and $P =$ net premium $O^{M(5)}$ $3\frac{1}{2}$ per-cent, I find according to the $O^{M(5)}$ mortality table, and for age 30 at entry:

Years elapsed	Values of f				
	Whole Life	Endowment Assurance			
		30 years	25 years	20 years	15 years
5	−2·11	−1·20	−0·70	−0·18	0·31
10	−0·68	−0·11	0·18	0·49	0·77
15	−0·16	0·32	0·55	0·79	1·00
20	0·13	0·59	0·79	1·00	...
25	0·32	0·80	1·06	...	...
30	0·47	1·00	...	...	...

On the same bases of mortality and interest I find for life annuities:

Age attained	Value of f
	Life Annuity
40	0·53
50	0·62
60	0·71
70	0·79
80	0·87
90	0·93

In these tables f means, as has been said, the real loss to the Company, if the market value of its life securities has decreased by unity.

Using other mortality tables and rates of interest, the figures remain about the same.

It appears from these tables that the gains or losses caused to a life company by a rise or a fall of the market value of its securities are for the greater part imaginary, and have but little influence on the results of the year, provided the liabilities are valued in the manner described.

A valuation on a variable per-cent basis can be practically realised with sufficient accuracy, by making the valuation say, on a 3 and on a 4 per-cent basis, and by calculating the liabilities on the desired per-cent basis by interpolation.

Your obedient servant,

Dr. D. P. MOLL.

Actuary of The Netherlands Fire and Life Insurance Company.

The Hague, July 1908.

INSTITUTE TEXT-BOOK, PART I.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—It has been pointed out to me by Mr. H. B. Smither that by application of the general formula (21) of Chapter V the answer to the question discussed in § 30 of that chapter may be accurately and much more simply expressed in the form

$$\frac{zC}{1+g}a'_{\overline{n|}} + \frac{g(1+\sqrt{1+i})}{2i}\left(C - \frac{zC}{1+g}a'_{\overline{n|}}\right)$$

where a' is calculated at the rate given by $v' = v(1+g)$. This result follows at once from

$$K = vzC + v^2zC(1+g) + v^3zC(1+g)^2 + \ldots + v^nzC(1+g)^{n-1}$$

Its identity with the present value at rate c of the varying annuity of which the rth payment is

$$(g+z)C + g\frac{C}{2}(\sqrt{1+c}-1)(1-zs_{\overline{r-1|}}),$$

where $s_{\overline{r-1|}}$ is calculated at rate g, can of course be established by algebraical transformations.

I shall be much obliged if you can find space for these few lines, as they may save some unnecessary trouble to future students.

I am, Sir,

Your obedient servant,

R. TODHUNTER.

25, *Pall Mall, S.W.*
17 *December* 1908.

https://doi.org/10.1017/S0020268100024859 Published online by Cambridge University Press