ISERLES, A. *A first course in the numerical analysis of differential equations* (Cambridge University Press, Cambridge, 1996), xvi + 378pp., 0 521 55655 4 (paperback) £19.95 (US$27.95), 0 521 55376 8 (hardback) £55 (US$74.95).

This is a well written and exciting book on the analysis of the numerical solution of ordinary and partial differential equations. The target market is mainly final year undergraduate and beginning postgraduate students in mathematics, but engineers and scientists with a sufficient degree of mathematical maturity would also be well served by the lucid treatment and motivation.

The book is split into three main sections: ordinary differential equations, the Poisson equation and evolutionary partial differential equations. The first section deals with the derivation and analysis of multistep and Runge-Kutta methods. Also included is a discussion on the important issue of automatic step size control. In contrast to many other books in this area the reader will not find any guidance on writing computer codes to implement such techniques, the author preferring to emphasise the mathematical derivation and analysis of methods. At the end of each subsection are exercises of varying degrees of difficulty and a very useful comments and bibliography section. The references are up-to-date and the comments give the reader a good picture of many of the modern developments in each area and on outstanding issues. This aspect of the book is especially pleasing and hopefully will be inspirational to good students.

The second section of the book deals with Poisson's equation, which is chosen as a generic example of an elliptic boundary value problem. This is used as a vehicle to describe finite difference and finite element discretisations. Both of these approaches result in Poisson's equation being swapped for a large set of sparse linear algebraic equations. Various solution techniques are investigated from classical iterative methods to the more modern multigrid methods.

The final section deals with evolutionary PDEs with the diffusion and advection equations as the parabolic and hyperbolic test cases. This section includes a clear distinction between Fourier and eigenvalue techniques for establishing stability. At one point the author proves that $1 = 2$, using a slight of hand which emphasises the role of non-normality in the use of eigenvalue techniques. At the end of the book we find a short compendium of mathematics that would prove useful for the forgetful student.

The book contains few typographical errors, but an incorrect statement of Gerschgorin's circle theorem in the main text is reproduced in the exercise section. This small matter apart, the exposition throughout the book is clear and very lively. The author's enthusiasm and wit are obvious on almost every page and I recommend the text very strongly indeed.

J. MACKENZIE

AUDIN, M. *Spinning Tops* (Cambridge Studies in Advanced Mathematics Vol. 51, Cambridge University Press, Cambridge, 1996), viii + 139pp., 0 521 56129 9, (hardback) £25 (US$34.95).

The intimate relationship between geometry and physics has always been an important component of mathematical thought. Indeed, certain periods of activity have fuelled very significant developments across the two areas, one of the most important examples of which was the one lasting from the time of Newton until the early part of this century. It was during this era that many of the key ideas in classical mechanics and geometrical analysis were developed, the one discipline being closely intertwined with the other. One need only consider the work of L. Euler, J.-L. Lagrange, P.-S. Laplace, A.-M. Legendre, C. F. Gauss, S.-D. Poisson, C. G. J. Jacobi, W. R. Hamilton, P. G. Lejeune-Dirichlet, J. Liouville, G. F. B. Riemann, M. S. Lie, C. F. Klein, H. Poincaré, D. Hilbert, E. Cartan, C. Carathéodory, A. E. Noether and G. D. Birkhoff, much of which reflects the central rôle played by mechanics. Of course from a late twentieth century viewpoint another instance of the fruitful interaction between geometry and physics is provided by the rich and subtle interplay, seen especially over the past fifteen years or so, between topology on the one hand and quantum theories of fundamental forces such as gravity on the other. However, even within this contemporary arena it is well worth noting that many of the concepts from Lie theory, algebraic geometry, the calculus of variations, differential topology and symplectic geometry that are currently of central importance in work on quantum field theory and low-dimensional topology emerged to a large extent from the study of mechanics by the mathematicians listed above. Three important further examples of current strands illustrating the extent to which mathematics has been influenced by the geometric mechanics tradition are the following: firstly, recent studies in symplectic topology, including for example the study of J-holomorphic curves, capacities, the global theory of generating functions and the Selberg-Witten invariants; secondly, explorations of the way in which algebraic geometry, in particular algebraic curves, Abelian varieties and θ-functions, can shed light on integrable systems, both finite-dimensional and infinite-dimensional (in the case of the top this idea can be traced back to Euler!); thirdly, work on the topology of integrable systems, for instance on the bifurcations of Liouville tori. Of course the division of this general area of work into these three categories is somewhat arbitrary and there are important connections between them. Indeed, it is to the problem of providing a unified perspective on certain aspects of the second and third strands that the book under review is addressed.

In *Spinning Tops* Audin uses algebraic geometric techniques involving the eigenvector map associated with a spectral curve to extract in a natural fashion topological information about the level sets of integrable Lax systems based on a Poisson structure, these Lax systems being constructed using the powerful Adler-Kostant-Symes theorem. As an aside related to our introductory comments note that the structure at the heart of the AKS machinery is the canonical Poisson bracket on the smooth functions on the dual of a Lie algebra, thoroughly investigated by F. A. Berezin, A. Kirillov, B. Kostant and J.-M. Souriau in the 1970s, but discovered originally by Lie in the 1880s! The focus of Audin's book is indeed an analysis of spinning tops or, to be more precise, of the class of completely integrable rigid body systems (Euler-Poinsot, Lagrange, Kowalevski, Goryachev-Chaplygin). A major theme is the idea that algebraic curves are key players in these dynamical systems (indeed, as the author indicates, even the world of French literature is not immune to the importance of algebraic curves – see the quotation on page 1!).

Spinning Tops is very well written and contains a unified approach to ideas that have not been systematically presented in other texts. To attempt to give an idea of its scope and insight I present a brief synopsis of the various sections. The Introduction motivates the main material in the book and gives a brief introduction to Poisson manifolds, completely integrable systems on symplectic manifolds and the eigenvector map of a spectral curve. I particularly liked the discussion of the Arnold-Liouville theorem and of the questions it leaves unanswered as well as the impressive list of about fifty (classes of) examples of finite-dimensional integrable systems. The abstract algebraic geometric theory required for analysing the dynamical systems is covered clearly and with good examples in five appendices covering about forty pages at the end of the book. The main body is organized as follows. In Chapter 1 the Hamiltonian dynamics of the top is set up and the Euler-Poinsot case is analysed from a classical perspective. The following chapter deals firstly with the motion of the Lagrange top and its solution in terms of elliptic functions, but then applies in a precise and complete manner the Lax pair/eigenvector map technique to analyse the topology. Chapter 3 covers the remarkable Kowalevski case, first giving an indication of the methods and results of S. Kowalevski's 1889 article and subsequently bringing into play the eigenvector map associated with an $so(3, 2)$ Lax pair, the latter emerging from a general framework due to A. I. Bobenko, A. G. Reyman and M. A. Semenov-Tian-Shansky. Audin shows how the Kowalevski Lax pair may be modified in rather a mysterious manner to give a Lax system for the Goryachev-Chaplygin top and how the $so(3, 1)$ case of the Bobenko–Reyman–Semenov-Tian-Shansky method may be used to treat the Lagrange top. Chapter 4 sees a return to free rigid bodies, both the three-dimensional case already treated classically in Chapter 1 and the four-dimensional case. For the former the topological results are proved again using the eigenvector map, thereby emphasizing once more the importance of algebraic curves. For the four-dimensional situation the regular level sets are determined and an indication is given of the way in which the bifurcations of the Liouville tori may be obtained. The final chapter departs from the world of tops in order to show how the theory works even for incomplete flows (for rigid bodies the Hamiltonian is a proper map and so the level sets are always compact). The example given is a version of the periodic Toda lattice.

The author's style is precise, yet manages to be friendly, witty and informal as well. The mathematical exposition is complemented by a comprehensive bibliography and index and the text is illustrated by several clear diagrams (I admired in particular the pretty Lagrange top decorating the title page!). I noticed only a few small typographical errors: in line 24 on page 18 the first Γ should be $\Gamma + \epsilon M$; there is a time derivative dot omitted from the LHS of the equation in the statement of Proposition 2.1.1; on page 52 just after equation (8) $sp(4)(\mathrm{R})$ should be $sp(4, \mathrm{R})$; c should be C on line 21 of page 60; in the definition of Jac(X) on page 115 the RHS should be quotiented by the lattice Λ.

In conclusion I can thoroughly recommend Michèle Audin's *Spinning Tops*, which is an elegant and original contribution to the literature of geometric mechanics. It will be of interest to all workers in the area, especially those appreciating a sophisticated algebraic geometric flavour.

S. T. SWIFT

STUART, A. M. and HUMPHRIES, A. R. *Dynamical systems and numerical analysis* (Cambridge Monographs on Applied and Computational Mathematics No. 2, Cambridge University Press, 1996), xxii+685 pp., 0 521 49672 1, (hardback) £40 (US\$59.95).

This book is concerned with the modern dynamical systems approach to analysing the numerical approximation of initial-value autonomous ordinary differential equations, i.e.

$$\dot{u} = f(u) \quad u(0) = U_0 \qquad f : \Re^p \mapsto \Re^p. \tag{1}$$

This way of looking at (1) has grown rapidly in popularity during the last ten years and should be contrasted with the traditional numerical analysis of initial-value O.D.Es, e.g. in [1]. The fundamental idea of the new approach is that conditions are imposed on the vector field f so that (1) defines a dynamical system and then attention is focused on the *evolution semi-group operator* $S(t) : \Re^p \mapsto \Re^p$ defined by

$$S(t)u(0) = u(t). \tag{2}$$

When one moves on to consider a discretisation of (1) from this viewpoint, the first task is to set the discrete problem in a similar framework. Thus, if (1) is approximated using a one-step method with fixed time step Δt, then the discrete equation is either

$$U_{n+1} = F_{\Delta t}(U_n) \tag{3}$$

for an explicit method or

$$G_{\Delta t}(U_{n+1}, U_n) = 0 \tag{4}$$

for an implicit method. This book specifically considers Runge-Kutta methods and derives conditions on f plus restrictions on Δt so that (3) or (4) constitute dynamical systems. Then one may consider the discrete *evolution semi-group operator* $S_{\Delta t} : \Re^p \mapsto \Re^p$ defined by