11. STELLAR INTERIORS
1. Introduction

Massive stars are rapid rotators. Equatorial rotation velocities span the range \( v_{\text{rot}} = 100-400 \text{ km s}^{-1} \), with B stars rotating closest to their break-up speed \( v_{\text{crit}} \) (Howarth et al. 1997). During the last decade, many observations have revealed unusual surface abundances that may require additional internal mixing (beyond that of simple convection and overshooting) for their explanation, most important helium and nitrogen enrichment in main sequence O and B stars (Gies & Lambert 1992), in the SN 1987A progenitor (Fransson et al. 1989), and boron depletions in main sequence B stars (Venn et al. 1996). In particular the latter observations clearly point towards internal mixing and rule out a close binary origin of the abundance peculiarities (Fliegner et al. 1996). Altogether, the occurrence of some form of additional mixing responsible for altering the surface abundances in a large fraction, if not all massive stars appears to be beyond reasonable doubt, and mixing processes due to rotation are the most natural explanation.

Here we report on results obtained with a 1D implicit hydrodynamic stellar evolution code, which was modified to incorporate several effects of rotation. The angular momentum is treated as a local variable. The centrifugal force is included in latitude-averaged form, with non-spherical equipotential surfaces replacing the usual Lagrangian mass variable as independent spatial coordinate (Endal & Sofia 1978). The approximate constancy of all physical variables on equipotential surfaces is due to the action of the baroclinic instability (Zahn 1992). The transport of chemical elements...
Figure 1. Helium surface mass fraction at core hydrogen exhaustion for models with an initial mass of 5, 10, 20 and 40 $M_\odot$, as function of the ratio of rotation to critical rotation at the zero age main sequence.

and angular momentum is performed in the diffusion approximation, with diffusion coefficients appropriate for the following instabilities: convection, semiconvection, dynamic and secular shear instability, Goldreich-Schubert-Fricke instability and Eddington-Sweet circulation (cf. also Maeder 1997). Mass loss due to stellar winds is taken into account, which is important to maintain a small but significant angular momentum gradient and thus efficient rotational mixing (Zahn 1992).

2. Rotationally induced mixing

2.1. ENVELOPE POLLUTION AND SURFACE ENRICHMENT

Fig. 1 shows the effect of rotationally induced mixing on the surface helium abundance of galactic (Z=2%) massive main sequence stars. While the non-rotating models ($\omega/\omega_{\text{crit}} = 0$) maintain their initial helium surface abundance ($X_{\text{He}} = 0.30$) until core helium exhaustion, rotating models develop a helium enrichment which is more pronounced for larger initial rotation rates. Fig. 1 shows also that, for a fixed value of $\omega/\omega_{\text{crit}}$, stars with a higher initial mass obtain higher helium enrichment. This is due to the increasing contribution of radiation pressure at higher masses, which decreases the effect of mean molecular weight barriers on the mixing.

Like helium, all other abundances which are changed in the core due to hydrogen burning are altered at the stellar surface. This concerns in particular boron and carbon, which are depleted, and nitrogen, oxygen, sodium, and the radio-nuclide $^{26}$Al, which are all enhanced. Furthermore, the enrichment of helium not only at the surface but throughout the radiative stellar
Figure 2. Evolutionary tracks of three $10 M_{\odot}$ stars with different initial equatorial rotation velocities ($v_{eq} = 0$, 280, and 400 km/s; solid, dash-dotted, and dashed lines, respectively) during the core hydrogen burning phase. The track of a non-rotating $15 M_{\odot}$ star is also shown. The thick solid line marks the ZAMS position of non-rotating stars.

envelope increase significantly the mean molecular weight of the star. This leads to a strong increase of the luminosity (cf. Fig. 2), consequently to a larger convective core and to a larger helium core mass at core hydrogen exhaustion, compared to the non-rotating case (Langer 1992). It is obvious that all these effects significantly alter the nucleosynthesis predictions of massive star models (cf. Langer et al. 1997, for details).

2.2. ISOCHRONES, INITIAL MASS FUNCTION AND SUPERNova RATE

Figure 2 shows the effects of rotation on the main sequence evolution in the HR diagram for stars with an initial mass of $10 M_{\odot}$. Although the more rapid rotators are initially less luminous due to the reduced effective gravity, the increasing luminosity due to the mixing of helium (cf. Sect. 2.1) is soon the overwhelming effect. If the distribution of initial rotation velocities is not very strongly peaked, the tracks shown in Fig. 2 demonstrate that the concept of isochrones in the HR diagram changes its character. Due to the dependence of the tracks on the initial rotation rate, isochrones are no longer lines but become surfaces in the HR diagram.

A similar effect occurs for the mass-luminosity relation: the luminosity depends on mass and initial rotation rate. Stellar masses obtained from observed luminosities through standard M-L relations will in general result in too large masses. This has been found empirically by comparing evolutionary stellar masses to masses derived by other methods (e.g., Herrero et al. 1992). One consequence is that initial mass functions which are derived from observed luminosity functions using a mass-luminosity relation...
3. Mass loss of massive stars at the $\Omega$-limit

Massive stars have radiation driven winds. Friend & Abbott (1986) showed that rotation at a rate $\Omega := v_{\text{rot}}/v_{\text{crit}}$ leads to an enhancement of the mass loss rate by the factor $(1/(1 - \Omega))^{0.43}$ in O and B stars. This effect alone results in a dependence of the evolution of massive stars on their initial rotation rate.

Furthermore, it has been shown by Langer (1997) that massive main sequence stars may actually reach the $\Omega$-limit $\Omega \approx 1$. The coupling of mass

$$\begin{align*}
M_{\text{rot}} &= 10 M_\odot \\
T_{\text{rot}} &= \frac{v_{\text{rot}}}{200 \text{ km s}^{-1}} \\
T_{\text{crit}} &= \log T_c - \log \rho_c \\
\rho_{\text{ave}} &= \rho_c
\end{align*}$$

Figure 3. Evolutionary tracks of a rotating (solid line) and a non-rotating (dashed line) $10 M_\odot$ star in the $\log \rho_c - \log T_c$ diagram. The various core burning stages are indicated along the track of the rotating model.

for non-rotating stars will systematically overestimate the number of the more massive stars.

Finally, also the fate of a massive star may depend on its rotation rate. Fig. 3 compares the evolution of a rotating and a non-rotating $10 M_\odot$ star in the central temperature vs. central density diagram. Due to its larger helium core, the rotating star evolves through all the nuclear burning phases without being strongly affected by electron degeneracy, and explodes as a Type II supernova. The non-rotating star ignites carbon off-center, whereafter the core cools dramatically. Depending on the mass loss rates in the late stages, it may evolves into a ONeMg white dwarf rather than exploding. I.e., rotation decreases the critical initial mass for supernova explosion (which now is a function of the initial rotation rate) and therefore leads to a substantial increase of the supernova rate.
and angular momentum loss results in a well defined limiting mass loss rate $\dot{M}_{\Omega}$: larger $\dot{M}$ would result in so much angular momentum loss that the star would evolve away from the $\Omega$-limit. For a 60 $M_\odot$ star, Langer (1997) found $\dot{M}_{\Omega} \simeq 10^{-5} M_\odot$ yr$^{-1}$. Mass loss at the rate $\dot{M}_{\Omega}$ keeps the star at the $\Omega$-limit until the critical rotational velocity is decreasing again with time, i.e. for decreasing effective temperature during the main sequence evolution. Fig. 4 illustrates that the time spent at the $\Omega$-limit and therefore the total amount of mass lost depends strongly on the initial rotation rate. The stellar radiation field may not be strong enough to push all the mass lost at the $\Omega$-limit to infinity; rather a ring or disk may form around the star, which may thus resemble a Be or B[e] star.

During the post main sequence evolution, massive stars may again hit the $\Omega$-limit, but due to the much shorter evolutionary time scale $\dot{M}_{\Omega}$ may be much larger, mass and angular momentum loss decouple, and the mass loss becomes unstable. Garcia-Segura et al. (1997) showed that the assumption of Luminous Blue Variable eruptions being due to massive post main sequence stars hitting the $\Omega$-limit naturally explains the bipolarity of most LBV nebulae and in particular the shape of the Homunculus nebula around $\eta$ Carinae.

Clearly, the type of mass loss described here affects the initial-final mass relation of massive stars, which now depends on $\Omega$. The initial angular
momentum even determines whether a very massive star can lose enough mass to successfully explode as supernova, or instead rather collapses into a black hole (cf. Woosley et al. 1993).

4. Conclusions

We have shown how rotation can affect the structure, evolution and fate of massive stars. Internal mixing of matter and angular momentum shifts and disperses the mass-luminosity relation for massive main sequence stars, transports H-burning products to the surface of these stars, alters the character of isochrones, and makes more stars explode as supernovae. Rotation can enhance and produce mass loss, alter and disperse the initial-final mass relation, produce and shape LBV nebulae, and also lead to more supernovae. Many of these effects remain quantitatively to be explored. However, for most of them there is overwhelming observational evidence showing their qualitative significance. There is no doubt that the initial rotation rate is a truly fundamental parameter for massive stars, which is equally important as the initial mass and metallicity.

Acknowledgements

This work has been supported by the Deutsche Forschungsgemeinschaft through grant La 587/8-2.

References


Discussion of this paper appears at the end of these Proceedings.