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CORRIGENDUM: C^*-ENVELOPES OF TENSOR ALGEBRAS FOR MULTIVARIABLE DYNAMICS

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Abstract There is an unfortunate error in Theorem 4.1 of our paper. However, the statement of the theorem remains true with a correct construction of adding a tail to enlarge the dynamical system.

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This error was pointed out by Elias Katsoulis, who, with Kakariadis, has found a correct version in the more general context of C^*-correspondences. The problem is that the procedure of adding a tail has to be more complicated. As Kakariadis and Katsoulis point out [**3**, Example 4.4], the problem arises in showing that the completely isometric embedding of $C^*_{\mathrm{e}}(\mathcal{A}(X,\sigma))$ into $C^*_{\mathrm{e}}(\mathcal{A}(X^T,\sigma^T))$ is a corner. We claimed that

$$C^*_{\mathrm{e}}(\mathcal{A}(X,\sigma)) = \chi_X C^*_{\mathrm{e}}(\mathcal{A}(X^T,\sigma^T))\chi_X.$$

But in order to prove this, one must show that terms of the form $\chi_X \mathfrak{t}_u \chi_T f \mathfrak{t}_v^* \chi_X$ lie in the span of terms of the form $\chi_X \mathfrak{t}_u \chi_X f \mathfrak{t}_v^* \chi_X$. This is false for the tail that we proposed.

A correct construction of such a tail is modelled on the infinite tail extensions of the left regular representation of the free semigroup $\mathbb{F}_n^+$ to a Cuntz representation from [**1**]. Kakariadis and Katsoulis [**3**] carry this out in greater generality, but we will briefly describe one way it can be done in our context.

Recall that X is a locally compact Hausdorff space and σ_i are proper maps of X into itself for $1 \leqslant i \leqslant n$, where $n \geqslant 2$. Set $U = X \setminus \bigcup_{1\leqslant i\leqslant n} \sigma_i(X)$. Let $F = \mathbb{F}_n^+ \setminus \mathbb{F}_n^+ 1$ be the words in $\mathbb{F}_n^+$ ending in $2,\dots,n$ together with the empty word $\varnothing$. Let $G = \mathbb{Z}_- \times F$ and

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define the tail to be $T = G \times U$. Define proper maps σ_i^T on $X^T = X \cup T$ by $\sigma_i^T|_X = \sigma_i$ and

$$\sigma_1^T((k,w,u)) = \begin{cases} u & \text{if } k=-1,\ w=\varnothing, \\ (k+1,\varnothing,u) & \text{if } k<-1,\ w=\varnothing, \\ (k,1w,u) & \text{if } |w| \geqslant 1 \end{cases}$$

and $\sigma_i^T((k,w,u)) = (k,iw,u)$ for $2 \leqslant i \leqslant n$.

Now follow through our proof of Theorem 4.1 with this new tail. Everything proceeds in the same way, except that when we claim that

$$C_{\mathrm{e}}^*(\mathcal{A}(X,\sigma)) = \chi_X C_{\mathrm{e}}^*(\mathcal{A}(X^T,\sigma^T))\chi_X,$$

this will be correct for the new tail. The reason is that the elements $\mathfrak{t}_i|_T$ are all isometries which carry each copy of U *onto* another copy because of the fact that the maps $\sigma_i^T|_T$ have disjoint ranges, and take each copy of U homeomorphically onto another. Thus, the terms in the corner which are not evidently in $C_{\mathrm{e}}^*(\mathcal{A}(X,\sigma))$ have the form $\chi_X \mathfrak{t}_u \chi_{(-k,w,U)} f \mathfrak{t}_v^* \chi_X$, where $f \in C_0(X^T)$ and $k>0$. A bit of reflection shows that this is 0 except when $u = u'1^k$ and $v = v'1^k$ for some $k>0$, and the term is

$$\chi_X \mathfrak{t}_{u'1^k} \chi_{(-k,\varnothing,U)} f \mathfrak{t}_{v'1^k}^* \chi_X = \mathfrak{t}_{u'}(\chi_X \mathfrak{t}_{1^k} \chi_{(-k,\varnothing,U)} f \mathfrak{t}_{1^k}^* \chi_X)\mathfrak{t}_{v'}^*.$$

But $\chi_X \mathfrak{t}_{1^k} \chi_{(-k,\varnothing,U)} f \mathfrak{t}_{1^k}^* \chi_X$ is just the restriction of f to $(-k,\emptyset,U)$ transferred to U, and hence lies in $C_0(U)$. Thus, all of these terms belong to $C_{\mathrm{e}}^*(\mathcal{A}(X,\sigma))$, and the rest follows.

We do not give details because a full discussion can be found in [**3**].

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