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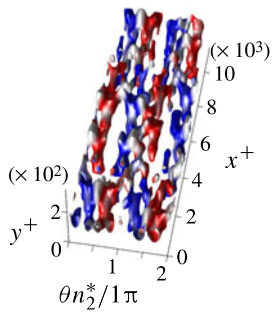
Toward coherently representing turbulent wall-flow dynamics



J. C. Klewicki†

Department of Mechanical Engineering, University of New Hampshire, Durham, NH 03824, USA

Department of Mechanical Engineering, University of Melbourne, Melbourne, Vic 3010, Australia

The complex dynamics of turbulent flow in the vicinity of a solid surface underlie numerous scientifically important processes, and pose persistently daunting challenges in many engineering applications. Since their discovery decades ago, coherent motions have presented a tantalizing prospective opportunity for constructing descriptions of wall-flow dynamics using only a relatively small number of elements. The veracity and reliability of such representations are, however, ultimately tied to their basis in the Navier–Stokes equations. In this regard, the study by Sharma & McKeon (*J. Fluid Mech.*, vol. 728, 2013, pp. 196–238) constitutes an important contribution, as it not only provides insights regarding the mechanisms underlying wall-flow coherent motion formation and evolution, but does so within a Navier–Stokes framework.

Key words: low-dimensional models, turbulence control, turbulent boundary layers

1. Introduction

While focusing on the flow through a pipe, the study by Sharma & McKeon (2013) broadly pertains to the flow over a no-slip material surface at Reynolds numbers in the fully turbulent regime. Describing the dynamics of these flows has proven to be notoriously difficult. Because of their scientific and technological importance, however, they remain the subject of intensive study, with special interest in their behaviour at high Reynolds number (Klewicki 2010; Marusic *et al.* 2010).

Within a wall-bounded turbulent flow, the largest eddies have a size characteristic of the overall width of the flow. Conversely, the smallest scales of motion are determined by the dynamics in the immediate vicinity of the surface, where the turbulence interacts directly with the surface to transfer momentum. Accordingly, the size of the smallest eddy is estimated by forming a length using the mean wall shear stress and the fluid's kinematic viscosity. By recognizing that the Reynolds number of the flow is approximately proportional to the ratio of these largest and smallest lengths, one comes to appreciate the vast range of scales of motion involved.

† Email address for correspondence: joe.klewicki@unh.edu

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The average properties of turbulent wall flows derive from the dynamical interactions across the range of motions, as dictated by the Navier–Stokes equations. The nonlinearity of these equations and the range of dynamically active motions renders the construction of tractable yet well-founded representations a discouraging proposition. Starting in the mid 1960s, however, researchers came to recognize that some of these motions, the coherent motions, display quasi-repeatable attributes, and carry with them the bulk of the dynamics (Cantwell 1981; Robinson 1991). Early coherent motion research focused on identification and classification. More recent studies, however, have sought to exploit their dynamical significance to construct low-dimensional models. Here is where Sharma & McKeon (2013) make a substantive contribution.

Early attempts at low-dimensional models empirically reconstructed spatial mode shapes using the proper orthogonal decomposition, e.g. Aubry *et al.* (1988). Over time, however, studies have increasingly sought to more formally connect to the Navier–Stokes equations. Notable among such attempts at low Reynolds number are those pertaining to exact coherent structures, e.g. Waleffe (2003), and those examining transitional and weakly turbulent flows by using state-space characterizations to identify, for example, fixed point and travelling wave solutions, e.g. Kerswell (2005), Gibson, Halcrow & Cvitanovic (2009). At the other extreme are studies that use global dynamical or geometric constraints to construct reduced-order models for flows approaching their asymptotic regimes, e.g. Julien & Knobloch (2007). The study by Sharma and McKeon is, however, perhaps the first to credibly employ a Navier–Stokes-based analysis (albeit under some assumptions and an intriguing hypothesis) to represent coherent motion dynamics on a mode-by-mode basis in the fully turbulent regime.

2. Overview

The approach of Sharma & McKeon (2013) borrows from wall-flow stability theory, as it develops a spectral formulation for the coherent motion mode shapes that stem from a critical layer mechanism associated with an assumed (e.g. measured) mean velocity profile. A spectral representation of the Navier–Stokes equations is cast into a form that segregates the nonlinear (forcing) and linear (response) terms. Determining the response behaviours is a primary aim of the analysis. These behaviours are determined by a resolvent operator that operates on the forcing function. Like in stability theory, the resolvent becomes large when the wave speed of a given mode is equal to the mean velocity. The most amplified modes are shown to be highly localized about regions of elevated vorticity where the wave speed of the given mode is equal to the local mean velocity. An intriguing and exciting element of this formulation is the implicit hypothesis that these nonlinear forcing and linear response mode combinations underlie the formation and evolution of the coherent motions observed in physical space.

A discrete modal decomposition is used to demonstrate how the highest gain linear response modes are associated with the forcing modes. Given a mean velocity profile, the user selects the operative mode shapes and their amplitudes. Here the authors primarily focus on those that reflect the properties of hairpin vortex packets, and large-scale or 'superstructure' motions that are known to be prevalent. Hairpin-shaped vortices have long been postulated to be a basic building block of wall turbulence, and experimental studies over the past two decades provide evidence that collections of these motions (packets) form and evolve to generate larger-scale coherent features

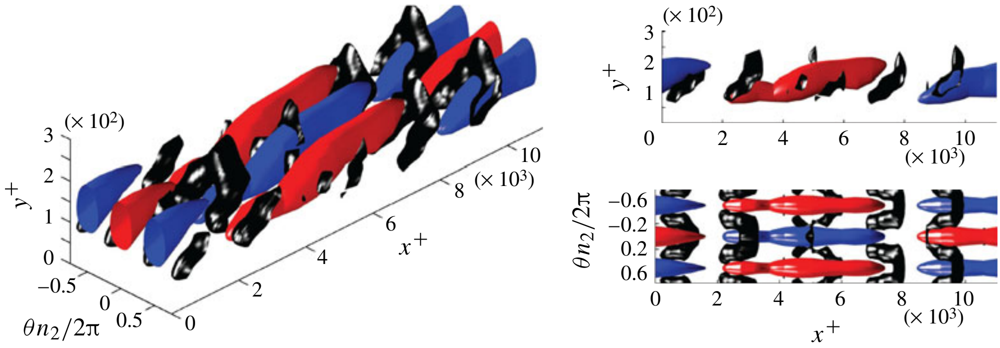

FIGURE 1. Visualization of the flow structure arising from an ideal packet, K_B, mode combination. This representation captures the spatial phase relationships between large-scale regions characteristic of elevated turbulent kinetic energy, and the smaller-scale vortical motions around their periphery. Adapted from figure 10 of Sharma & McKeon (2013).

e.g. Theodorsen (1952), Adrian, Meinhart & Tomkins (2000). Clear evidence has also recently emerged indicating the existence and importance of large-scale energetic motions, e.g. see Hutchins & Marusic (2007) and Bailey *et al.* (2008). Sharma and McKeon select mode shapes representative of the above motions, and use only one leading singular value. In general, however, the framework naturally embraces more complex coherent motion representations.

The study also shows the levels of complex organization that arise as an increasing yet small number of mode shapes are added to the menu of motions. Using just a single response mode at one streamwise wavelength and one spanwise wavelength results in a replicated array of hairpin-like vortices (see their figure 5). If an additional intrinsic wavelength that pertains to the streamwise spacing of the hairpin vortices is added, then figure 1 results. By adding this organizational feature, the response modes inherently generate larger-scale energetic regions of low vorticity. There is now considerable evidence supporting the existence of instantaneous flow field behaviours similar to that depicted, e.g. del Alamo *et al.* (2006). Remarkably, the depiction of figure 1 includes the spatial phase relationships between the energetic regions and the vortical motions at their periphery. Real turbulent wall flows have finite correlation lengths in the spanwise (azimuthal) direction. Their figure 17 nominally includes this scrambling effect by including a small number of modes that have a non-simple spanwise periodicity. With regard to the characteristic scales and their spatial arrangement, there is a striking correspondence between the structure they show, and that observed by visualizing vortices in experiments.

3. Future

The framework established by McKeon & Sharma (2010) and exemplified in Sharma & McKeon (2013) has a number of attractive features pertaining to future research. Notable among these is that it inherently allows one to postulate and explore mode combinations inspired by physical observations. Through this one not only learns about the coherent motions that result, but also about the veracity of the formulation itself. The former of these allows one to estimate, for example, the appropriate modal amplitudes. The latter is potentially much farther reaching. Namely, the formulation relies upon the expectation (hypothesis) that for sufficiently high

Reynolds number the regions of high gain are associated with the most amplified modes resulting from the singular-like values associated with the resolvent. This notion, physically underpins the expectation that the linear response modes will be highly selective with regard to the nonlinear forcing. This hypothesis speaks to the role of nonlinearity in the dynamics of wall turbulence, and thus deeper inquiry into its implications, and potential limitations, would seem to be fertile area for further research.

As more is learned about the framework's capabilities, one can envision its exploitation in numerous applications. These include the exploration of specific mode–mode interactions for the purposes of clarifying scale separation effects at high Reynolds numbers, and the use of the low-dimensional representation to devise efficient computational schemes. Perhaps the most exciting, however, is the potential of their framework for devising and characterizing turbulence control schemes. In this regard, the framework affords the apparent capacity to isolate the specific modes that are primarily responsible for specific mean flow features, and vice versa.

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