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The homology of groupnets

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The work of G. Higman, A. Karrass, Hanna Neumann, B.H. Neumann, D. Solitar, and many others has drawn attention to groups which are graph products - free products with amalgamation, HNN groups, tree products, and the like. They are fundamental groups of graphs of groups in the terminology of Bass and Serre. The theory of groupnets (Brandt groupoids) lends itself neatly to the study of such groups.

The bridge between topology and combinatorial group theory provided by the groupnet has been used more frequently since the appearance of Higgins' formalisation [5] of the theorems of Grushko, Kurosh, Nielsen, and Schreier in terms of groupnets. The category *Gpnet* of groupnets contains as a full subcategory the category *Gp* of groups, but also contains algebraically-determined constructs - homotopies, fibrations, and 'unit intervals' - which are either undefined or vacuous in *Gp* yet correspond closely to the topological definitions through the forgetful functor from *Gpnet* to the category of directed graphs. In this thesis the 'internal' algebraic structure of the groupnet detailed in [4] is used rather than Higgins' categorical approach.

In Chapter 2, the ringoids of Mitchell [6] are extended to form a category of ringnets, and the notion of a representation $(\mathcal{D}, R) \to S$ for a ringnet diagram $(\mathcal{D}, R)$ is defined. If G is the homotopy colimit of a groupnet diagram $(\mathcal{D}, A)$ then there is an induced groupringnet representation $(\mathcal{D}, ZA) \to ZG$.

Chapter 3 constructs an 'S-mapping cylinder' complex for each complex diagram $(\mathcal{D}, R, C)$ and each representation $(\mathcal{D}, R) \to S$. The principal result of the thesis is that

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If G is a graph product and each vertex complex of $(\mathcal{D}, \mathbb{Z}A, C)$ *is a free resolution of its trivial module, then the* $\mathbb{Z}G$*-mapping cylinder is a free resolution of its trivial module.*

In the course of proof the notion of chain homotopy is extended to a form strongly motivated by the topological definition of homotopy.

The mapping cylinder complex determines Mayer-Vietoris sequences for graph products which extend the results of Bieri [*1*] for HNN groups and Lyndon and Swan [*7*] for free products with amalgamation. The sequences are in turn used to extend several results on duality groups and cohomological dimension. Several of these results are shown to be equivalent to those of Chiswell [*3*], Bieri and Eckmann [*1*, *2*] derived by other means.

For each group in a certain class of groupnets with cohomological dimension two (including torsion-free one-relator groups and tree products of free groups) the mapping cylinder may be employed to evaluate a comultiplication which gives a coring structure to the integral homology module of the group. This comultiplication is connected with the lower central series of the group, and in Chapter 6 its canonical form is determined for several cases with homology modules of low rank.

References

[*1*] Robert Bieri, "Mayer-Vietoris sequences for *HNN*-groups and homological duality", *Math. Z.* **143** (1975), 123-130.

[*2*] R. Bieri and B. Eckmann, "Amalgamated free products of groups and homological duality", *Comment. Math. Helv.* **49** (1974), 460-478.

[*3*] I.M. Chiswell, "Exact sequences associated with a graph of groups", *J. Pure Appl. Algebra* 8 (1976), 63-74.

[*4*] R.H. Crowell and N. Smythe, "The subgroup theorem for amalgamated free products, HNN-constructions and colimits", *Proc. Second Internat. Conf. Theory of Groups*, Canberra 1973, 241-280 (Lecture Notes in Mathematics, **372**. Springer-Verlag, Berlin, Heidelberg, New York, 1974).

https://doi.org/10.1017/S0004972700010522 Published online by Cambridge University Press

[5] Philip J. Higgins, *Notes on categories and groupoids* (Van Nostrand Reinhold Mathematical Studies, 32. Van Nostrand, London, New York, Cincinnati, Toronto, Melbourne, 1971).

[6] Barry Mitchell, "Rings with several objects", *Advances in Math.* 8 (1972), 1-161.

[7] Richard G. Swan, "Groups of cohomological dimension one", *J. Algebra* 12 (1969), 585-610.

https://doi.org/10.1017/S0004972700010522 Published online by Cambridge University Press