80.3 A test for divisibility by seven

In this note, a bar is used to denote the digits of a number: e.g. \( \overline{abc} = 100a + 10b + c \). A three digit number \( A = \overline{abc} \) is divisible by 7 if and only if \( 2a + bc \) is divisible by 7.

For example, let \( A = 245 \). Then \( 2 \times 2 + 45 = 49 = 7 \times 7 \). If \( A \) has more than three digits and is divisible by 7, then the above test applies if we equate the remaining digits other than \( bc \) with \( a \). For example, let \( A = 7133 \) then \( 71 \times 2 + 33 = 142 + 33 \), and applying the basic result to this gives \( 142 + 33 \to 2 \times 1 + 42 + 33 = 77 = 7 \times 11 \).

Let \( A = 35133 \), then \( 2 \times 351 + 33 = 702 + 33 = 735 \to 2 \times 7 + 35 = 49 = 7 \times 7 \). To prove this we note that \( 100a + 10b + c \equiv 2a + 10b + c \) (mod 7).

I have given some other divisibility test for primes which were described in Notes 2548 [1], 2566 [2], 2762 [3] and 3179 [4].

References

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80.4 My favourite calculus problem

... is (currently) ...

What is the shortest distance (over the surface) between a given point on the ‘top’ rim of a solid right circular cylinder and a given point on the ‘bottom’ rim?

This is a particularly rich minimisation problem, well suited to exploration by any of the characteristically twentieth century aids to mathematics education – from graph paper to graphics calculator! Some cases of similar problems have cropped up in the literature from time to time, [1, 2], but the subtle aspects of this particular formulation do not seem to have been highlighted before. (The corresponding problem with a frustum of a cone is also worth investigating.)