

# HELIOSEISMOLOGY WITH HIGH DEGREE $p$ -MODES\*

(Invited Review)

F.-L. DEUBNER

*Institut für Astronomie und Astrophysik, D-8700 Würzburg, F.R.G.*

**Abstract.** The value of  $p$ -modes of high degree  $l$  as a diagnostic for the structure and dynamics of the solar envelope is reviewed.

## 1. Definition of the subject

For the purpose of this brief overview we shall define 'high degree' modes pragmatically as coherent non-radial oscillations which are not (well) observed in full disk observations of the Sun. Accordingly the spatial scale of these motions ranges typically from a few arc sec ( $3 \times 10^3$  km) to about  $1000''$  ( $\sim 10^6$  km). Incidentally, we cannot really quote any 'representative size' of the oscillations nor can it be defined in a meaningful way. It depends entirely on the spatial filter functions implied by the observational setup, since the full spectrum of  $p$ -modes extends from the radial modes ( $l = 0$ ) into the range of granular sizes ( $l \simeq 2000$ ) with monotonically decreasing amplitude (cf. Section 5). The contributions from various parts of the solar disk of such a small scale periodic wave field most effectively cancel in full disk observations.

Recognizing the global nature of the low degree  $p$ -modes reported earlier in this meeting, we wonder whether eventually we shall be able to trace somewhere within the range of sizes defined above a dividing line between truly globally coherent modes and only locally coherent oscillations. This might shed some light on the relative importance of various excitation mechanisms for  $p$ -modes presently under discussion.

## 2. Brief History

Most of the early observational work on the 5-min oscillations suffered severely from insufficient coverage of the large range of sizes and periods characterizing this phenomenon. The classic work of Leighton *et al.* (1962) – these authors were using two-dimensional photographic records of the Doppler shift – had the potential of revealing the large scale coherent properties of the oscillations. However, the observations were too short. On the other hand, subsequent investigations using photoelectric

\* Proceedings of the 66th IAU Colloquium: *Problems in Solar and Stellar Oscillations*, held at the Crimean Astrophysical Observatory, U.S.S.R., 1–5 September, 1981.

equipment, by Howard (1967), Deubner (1972), White and Cha (1973), and many others who took data from small areas on the solar surface for many hours at a time, completely failed to recognize the important spatial coherence of the modes.

Nevertheless, in 1970, Ulrich followed by Leibacher and Stein (1971) gave a basically correct description of the evanescent waves which are seen to cover the solar globe all the time and everywhere, in terms of resonant oscillations of the solar interior leaking to the surface.

Only later in the last decade, observers were ready to understand the requirements for really useful observations of the 5-min oscillations, which consisted of spatially extended records repeated as many times as daylight permitted.

### 3. Observing techniques

An ideal observation of solar  $p$ -modes obviously would cover the entire surface with sufficient spatial resolution ( $2''$ ) at roughly 100 s intervals for at least one solar revolution, i.e. about  $(3 \times 10^6) \times (2.5 \times 10^4) = 7.5 \times 10^{10}$  pixels per observation. Although this requirement appears somewhat unrealistic, the importance of uninterrupted observations from space assisted by ground based monitoring looking at the Sun from a different angle has been hardly enough emphasized yet, especially if transient events (flarequakes) as conceived some time ago by Wolff (1972) are to be evaluated.

These days observations are taken with single-channel receivers mostly by scanning the solar surface in one direction and taking account of the isotropic character of the wave field either by choosing the appropriate transformation of the spatial coordinate afterwards in the data analysis, or by spatially averaging in the other direction (long slit technique) during observation. (A brief summary of the various ways of sampling two-dimensional isotropic phenomena, and the relations among the pertinent data transforms is given in Aime and Ricort (1975).) Multi-channel receivers (linear and areal diode arrays) permit to obtain – at the cost of a largely increased data flow – two-dimensional images of the velocity (or intensity) distribution, which are of particular value if the assumption of angular isotropy has to be dismissed, e.g. in the investigation of latitude dependent effects such as differential rotation and other large-scale circulation systems (Hill *et al.*, 1983), and perhaps of low degree modes.

### 4. Representation of Data

The  $k$ ,  $\omega$  or diagnostic diagram has become widely used as the most compact and to theory most directly applicable representation of the properties of oscillations. An explanation of its significance is contained in many standard texts (cf. Leibacher and Stein, 1981), and doesn't need to be repeated here. The discrete eigenfrequencies clearly separate the various modes of different radial index  $n$  in the  $k$ ,  $\omega$  plane as shown in

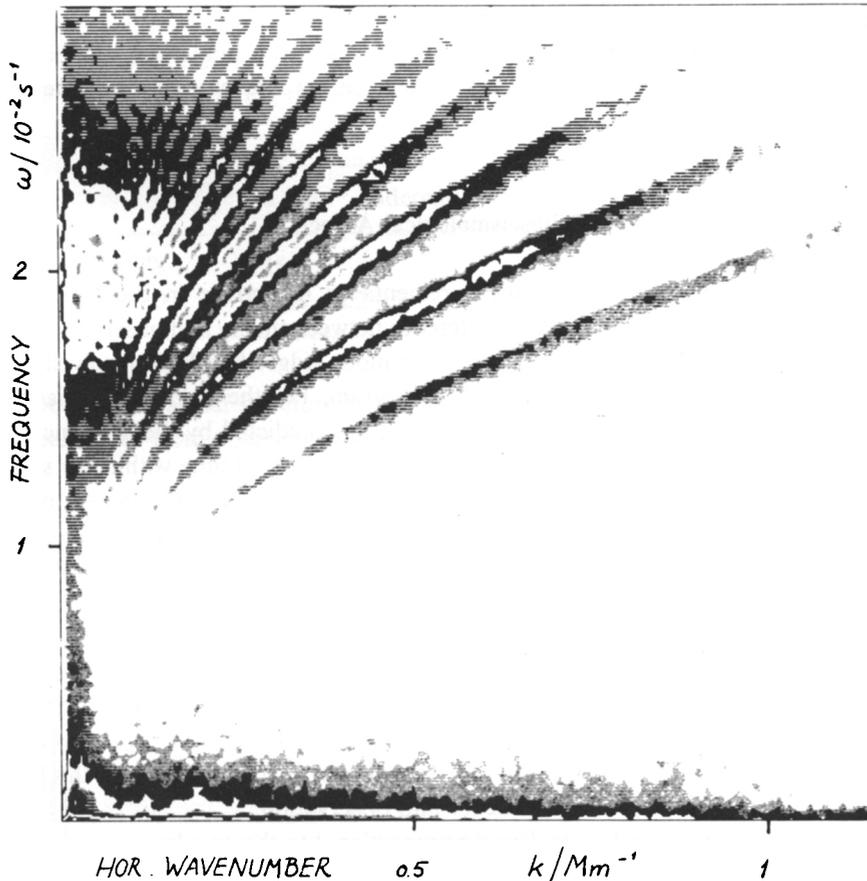


Fig. 1.  $k, \omega$  diagram of high degree solar  $p$ -modes obtained with the domeless Coudé refractor of the Anacapri station of the Kiepenheuer-Institut. Observed spectral line: Fe I 5576.099. Total duration of the observation:  $4 \times 7.54$  hr.

Figure 1, and the dependence of these frequencies on horizontal wavenumber  $k$  (or surface harmonic index  $l$ ) can be directly seen. Also, in this diagram, ranges of 'effective depth' of a mode of given order and degree may be delineated (Ulrich *et al.*, 1979).

For specific diagnostic purposes a number of modifications of the  $k, \omega$  diagram have been suggested: since asymptotically the  $p$ -mode ridges (see Figure 1) approach a parabolic shape with increasing wavenumber, a plot of  $\omega^2$  vs  $k$  immediately displays the deviations from the asymptotic law which is now represented by straight lines (Christensen-Dalsgaard, 1980). Division of  $\omega^2$  by  $k$  makes the straight lines all parallel and thus reveals in even more detail the structure of the asymptotic dependence of the eigenfrequencies on  $l$  and  $n$  (Gough, 1981).

Similar to these modified diagnostic diagrams is a plot of  $\omega/k$  vs  $k$ , which depicts horizontal phase velocity as function of spatial scale. This relates directly to the thermal

structure of the solar envelope and is useful in comparing 'effective depths' of various modes. Also, the amount of rotational splitting  $\Delta\omega = v k$  is independent of  $k$  in this kind of diagram which, therefore, lends itself as a convenient indicator of any systematic depth-dependence of rotation.

## 5. Helioseismological Aspects

We now turn to the use of these measurements as diagnostic in the context of helioseismology. The most conspicuous difference between theory and observation was pointed out by Deubner (1975) soon after the  $p$ -mode ridges had been resolved. The position of the observed ridges in the  $k, \omega$  diagram, i.e. the eigenfrequencies are systematically lower by a few percent than the values predicted by theory. Since the  $p$ -mode eigenfrequencies decrease with increasing sound travel time within the stellar envelope, this can only mean that the solar convection zone is deeper than assumed in the model or, in terms of mixing length theory, that convection is more effective and the mixing length parameter  $\alpha = l/H$  is larger than 1 (Ulrich and Rhodes, 1977). Raising the value of  $\alpha$  to 2.5 brings theory and observation more closely together, however, a further increase of  $\alpha$  would create a convection zone reaching considerably deeper than 200 000 km, which is then in conflict with the observed surface abundance of lithium.

Another possibility of lowering the eigenfrequencies is by reducing the pressure, e.g. by taking into account electrostatic forces in the equation of state. Lubow *et al.* (1980) have shown that together with an increased mixing length parameter of  $\alpha = 2.5$  this leads to almost perfect agreement between theory and observation.

The splitting of  $p$ -mode ridges is directly proportional to the angular velocity of the standing wave pattern of a given mode. It has already been mentioned in the last section as a means to infer rotational velocities not only at the surfaces but also as a function of depth within the solar envelope (Rhodes *et al.*, 1979). First results published by Deubner *et al.* (1979) indicated an increase of the rotation rate with depth. This result has recently been challenged by the Kitt Peak observers (Rhodes *et al.*, 1983) who find no variation whatsoever within their limits of accuracy. If this difference is real, it may indicate a change of the dynamics of the convection zone at different phases of the solar cycle, since the latter observations were obtained during solar maximum, the first ones at the time of the minimum.

Close to the abscissa, in the diagnostic diagram of photospheric motions we find concentration of power due to quasistationary flow patterns, such as granules and supergranules, with time scales long compared to the characteristic period of  $p$ -modes. These are surface features, and the splitting, or rather inclination (which was suppressed in Figure 1) of this ridge of power at the abscissa reflects the surface velocity, which we can measure in this way with absolute accuracy comparable to the accuracy of line shift measurements, however without the susceptibility of the latter for systematic degradation of the results, as e.g. caused by straylight.

Surface rotation rates determined in this way in recent years with the Capri instrument come very close to the rotation rates of sunspots and, maybe, indicate a slight decrease of the rotation rate during the rising phase of the present solar cycle:

	rad day <sup>-1</sup> (sidereal)
1978	0.2563
1980	0.2527 ± 22

Rotation and stellar structure below, say, 50 000 km in the convection zone can only be determined from observation of modes with sufficiently low  $l$ -value (cf. Figure 2 in

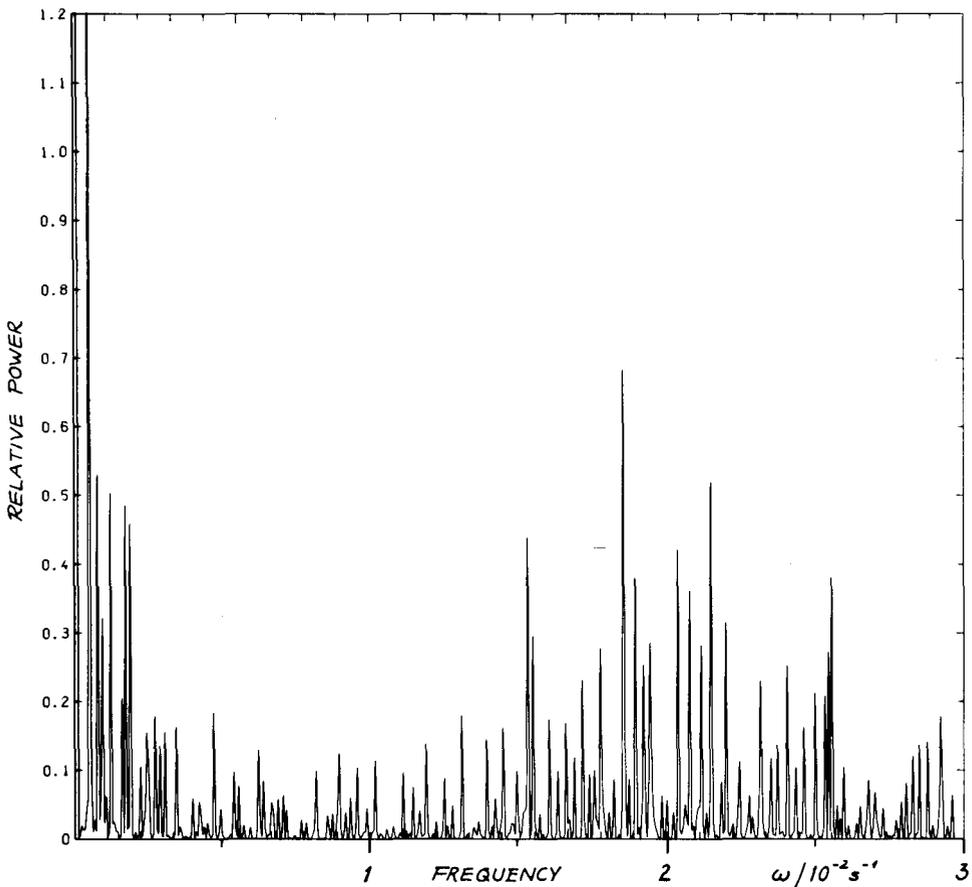


Fig. 2. Power spectrum of the slope of the velocity signal as a function of solar longitude, as measured at 106 s intervals in an area 1000" × 200" centered on the solar disk. The spectrum was obtained by sine-wave fitting of a data set from two contiguous days.

Ulrich *et al.*, 1979) which are extremely difficult to resolve in the  $k, \omega$  diagram with the current techniques of observation of high degree  $p$ -modes. Obviously the full disk observations presented earlier in this meeting by Isaak and Fossat are far superior in this important domain.

Nevertheless, we made an attempt to utilize the temporal variation of the slope of the Doppler signal across the solar disk observed during  $p$ -mode observations to detect the intermediate range of  $l$ -values from about 4 to 10. The resulting power spectrum in Figure 2, obtained by iterative sine-wave fitting looks similar to the spectra of low degree modes. It contains significant power in the 5-min range. Its diagnostic value is presently under study.

Whereas the position of the ridges, i.e. the eigenfrequencies can be determined with little ambiguity ( $\pm 10 \mu\text{Hz}$ ) the distribution of power along these ridges, and across the diagram as a function of  $\omega$  and  $k$  is more difficult to obtain, more subject to observational parameters (spatial and temporal window limitations, pixel size) and observing conditions (seeing and spectrograph stability, etc.).

The distribution of power contains information on excitation, damping, and propagation of the resonant modes, properties which are rather harder to understand quantitatively than the resonant cavity. In the spectrum (Figures 3–9 in Deubner, 1981), which displays oscillatory power per unit wavenumber, there is little variation of power in the low  $k$  range up to  $k \simeq 0.2 \text{ Mm}^{-1}$  (or  $l \simeq 150$ ), i.e. the typical scale of supergranulation, followed by a monotonic decrease at higher  $l$ -values. Does this finding imply that convective motions of this smaller scale interfere with oscillations of corresponding  $l$ -values? Does it imply that oscillations of this size are only stochastically excited by turbulent motions of a congruent scale? Or does dissipation become more effective at this size because it fits better to the chromospheric network scale, or simply because the resonant cavity becomes rougher there?

Accurate observation of the intrinsic ‘line width’ of the  $p$ -mode ridges will help to find an answer to these questions. Until today, the intrinsic ridge width of the high degree  $p$ -modes has not been resolved. Therefore the coherence or damping time  $\tau$  is at least equal to the available continuous observing time of the order of 8 to 10 h, and probably much larger. There is an indication that  $\tau$  increases with decreasing  $l$ -value (Deubner, unpublished). The difference among the values of the coherence time of low degree  $p$ -modes quoted by Grec *et al.* (1980,  $\simeq 2$  days) and Claverie *et al.* (1983, at least 30 days) illustrates the difficulty in assessing correct numbers from insufficiently resolved observations.

Finally, let us briefly consider another piece of information we have not yet discussed. In the optically thin parts of the outer atmosphere, directly accessible to dynamical probing in the visible, we may use phase relations of pairs of observed fluctuations as a tool to analyse the character of the waves and oscillations in situ, and thereby – taking a full dynamical model of the atmosphere with radiation into account – to infer its mean vertical structure. Extensive work in this vein is being done by a group of French workers (Schmieder, 1978; Mein, 1977; Gouttebroze and Leibacher, 1980; Leibacher *et al.*, 1982). Linear adiabatic theory predicts that in the regime of evanescent waves brightness

should precede upward velocity by about  $90^\circ$ , and vertical phase velocity should be very large for frequencies less than the acoustic cut-off frequency  $N_{ac} \sim 3 \times 10^{-2} \text{ s}^{-1}$ . This is approximately true in the Na D lines, however  $\Delta\phi_{1-V}$  is more like  $120^\circ$  in the medium strong Fe I 5383 line, and it is  $45^\circ$  in the C I 5380 line. At higher frequencies where waves become progressive and transport energy, brightness and velocity are expected to propagate in phase, and the phase velocity should approach the velocity of sound. Convincing evidence of the compliance of nature with these ideas is still missing; however, observation of these waves with periods shorter than  $\sim 150 \text{ s}$  is exceedingly difficult due to their strongly filtered amplitudes and comparatively high horizontal wavenumbers. For successful interpretation of the data a two-dimensional treatment of radiation transfer is likely to become important.  $k, \omega$  diagrams of short period oscillations may prove useful to distinguish between genuine solar and terrestrial atmospheric disturbances in the signal.

### References

- Aime, C. and Ricort, G.: 1975, *Astron. Astrophys.* **39**, 319.
- Christensen-Dalsgaard, J.: 1980, private communication.
- Claverie, A., Isaak, G. R., McLeod, C. P., van der Raay, H. B., and Roca Cortes, T.: 1983, *Solar Phys.* **82**, 233 (this volume).
- Deubner, F.-L.: 1972, *Solar Phys.* **22**, 263.
- Deubner, F.-L.: 1975, *Astron. Astrophys.* **44**, 371.
- Deubner, F.-L.: 1981, in S. Jordan (ed.), *The Sun as a Star*, NASA/CNRS monograph series on Nonthermal Phenomena in Stellar-Atmospheres, Washington, Paris, p. 65.
- Deubner, F.-L., Ulrich, R. K., and Rhodes, Jr., E. J.: 1979, *Astron. Astrophys.* **72**, 177.
- Gough, D.: 1981, private communication.
- Gouttebroze, P. and Leibacher, J. W.: 1980, *Astrophys. J.* **238**, 1134.
- Greig, G., Fossat, E., and Pomeranz, M.: 1980, *Nature* **288**, 541.
- Hill, F., Toomre, J., and November, L. J.: 1983, *Solar Phys.* **82**, 411 (this volume).
- Howard, R.: 1967, *Solar Phys.* **2**, 3.
- Leibacher, J. W. and Stein, R. F.: 1971, *Astrophys. Letters* **7**, 191.
- Leibacher, J. W. and Stein, R. F.: 1981, in S. Jordan (ed.), *The Sun as a Star*, NASA/CNRS monograph series on Nonthermal Phenomena in Stellar Atmospheres, Washington, Paris, p. 263.
- Leibacher, J. W., Gouttebroze, P., and Stein, R. F.: 1982, *Astrophys. J.* **258**, 393.
- Leighton, R. B., Noyes, R. W., and Simon, G. W.: 1962, *Astrophys. J.* **135**, 474.
- Lubow, S. H., Rhodes, Jr., E. J., and Ulrich, R. J.: 1980, in H. Hill and W. Dziembowski (eds.), *Non-Radial and Non-Linear Stellar Oscillation*, Springer Verlag, Berlin, p. 300.
- Mein, N.: 1977, *Solar Phys.* **52**, 283.
- Rhodes, Jr., E. J., Harvey, J. W., and Duvall, Jr., T. L.: *Solar Phys.* **82**, 111 (this volume).
- Rhodes, Jr., E. J., Deubner, F.-L., and Ulrich, R. K.: 1979, *Astrophys. J.* **227**, 629.
- Schmieder, B.: 1978, *Solar Phys.* **57**, 245.
- Ulrich, R. K.: 1970, *Astrophys. J.* **162**, 993.
- Ulrich, R. K. and Rhodes, Jr., E. J.: 1977, *Astrophys. J.* **218**, 521.
- Ulrich, R. K., Rhodes, Jr., E. J., and Deubner, F.-L.: 1979, *Astrophys. J.* **227**, 638.
- White, O. R. and Cha, M. Y.: 1973, *Solar Phys.* **31**, 23.
- Wolff, C. L.: 1972, *Astrophys. J.* **176**, 833.